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THE AUTOMODELITY HYPOTHESIS AND THE ASYMPTOTIC BEHAVIOUR OF STRONG, ELECTROMAGNETIC AND WEAK INTERACTIONS AT HIGH ENERGIES

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THE AUTOMODELITY HYPOTHESIS AND THE ASYMPTOTIC BEHAVIOUR OF STRONG, ELECTROMAGNETIC AND WEAK INTERACTIONS AT HIGH ENERGIES

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Гипотеза автомодельности и асимптотическое поведение сильных, электромагнитных и слабых взаимодействий при высоких энергиях

Показано, что гипотеза автомодельности или масштабной инвариантности позволяет получить единым образом различные предсказания для асимптотического поведения сильных, электромагнитных и слабых процессов при высоких энергиях.

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The Automodelity Hypothesis and the Asymptotic Behaviour of Strong, Electromagnetic and Weak Interactions at High Energies

The hypothesis of automodelity or scale invariance is used to predict the asymptotic behaviour of strong, electromagnetic and weak interactions at high energies.

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Recently a great attention has been paid to experimental and theoretical studies of both elastic and essentially inelastic many-particle processes of hadron interactions at high energies.

The most important empirical properties of these processes are as follows: the limitedness of transverse momenta of outgoing particles and predominance of the longitudinal (along the collision axis) components, an approximate constancy of the total cross sections and diffraction behaviour of small-angle elastic scattering which appears to be due to the existence of a finite effective radius of strong interactions /1/.

On the other hand, in studying deep inelastic interactions of leptons with hadrons at high energies and large momentum transfers the so-called "point-like" picture of the behaviour of the total differential cross sections has been observed which is characterized by the vanishing of dimensional parameters defining the effective sizes of particles.

We note that the possibility of disappearing of the form factors when summing over all open channels in the processes of deep inelastic weak and electromagnetic lepton-hadron interactions was first indicated in ref. /2/. Further in refs. /3/ some arguments were given in favour of the "point-like" behaviour on the basis of the current algebra which then were proved in experiments on SLAC and at CERN.

At present to explain the dynamics of strong interactions and the elementary particle structure use is made of a number of models which are based on the essential experimental information on high-energy hadron-hadron and lepton-hadron interactions. These are the droplet model /4/ and the limiting fragmentation hypothesis /5/ based on it, the parton model /6/ and the coherent state model /7/.

In the present note we would like to draw attention to the fact that many results of these models are really model-independent and can be obtained on the basis of the dimensional analysis and automodelity or scale invariance principle.

In this connection we recall the situation which has occured in studying the process of deep inelastic electron-nucleon scattering. It turned out that the differential cross section of this process summed over all channels has a very simple asymptotic dependence on the kinematic variables which is defined from the analysis of the dimensionality and automodelity principle /8/.

In these processes the automodelity means that all the physical quantities like the total cross sections or the structure functions of deep inelastic lepton-hadron scattering must transform under scale transformations of all the momenta

$$q_\mu \to \lambda q_\mu, \quad p_\mu \to \lambda p_\mu, \qquad (1)$$

where q is the virtual photon momentum and p is the initial nucleon momentum, as homogeneous functions of the appropriate dimensionalities.

The range of application of the scale transformation (1) is defined by the conditions

$$|q^2|, \quad \nu = p \cdot q \gg m^2, \quad q^2/\nu - fixed. \qquad (2)$$

The automodelity principle makes it possible to come up to the problem of the asymptotic behaviour of various processes of deep inelastic lepton-hardon interaction in a unique manner. One of the current problems is to find its theoretical foundation in the framework of quantum field theory.

N.N. Bogolubov has attracted our attention to the importance of scale tramsformations in deep inelastic processes and in particular, to a possible analogy between deep inelastic electroproduction and dynamics of point explosion /9/.

From the physical point of view this means that in the processes of electromagnetic and weak interaction leptons play the role of point probing particles of zero sizes.

The sizes of hadrons vanish when summing over all open channels which leads to the above mentioned "point-like" behaviour in deep inelastic lepton-hadron interactions.

In the high-energy strong interaction processes the situation is somewhat different. Here the hadrons play the role of objects of finite effective sizes. The strong evidence for this is the limitedness of transverse momenta of secondaries and the exponential decrease of the elastic scattering probability at large momentum transfers.

Due to Lorentz contraction the hadron is represented at high-energies as an infinitely thin disk with finite transverse sizes. In contrast to the local lepton-hadron interactions, we shall ascribe to the interactions of such extended objects the automodel behaviour which is analogous to that of the plane explosion in hydrodynamics.

In order to formulate the automodelity principle as applied to the processes of strong interaction of hadrons at high energies we should use, however, an extended dimensional analysis. We introduce two different scales of measurement of lengths: along the particle collision axis L_z and in the transverse plane L_T. All the physical quantities in the high-energy limit may be identified as homogeneous functions of appropriate dimensionality, that is

$$[F] = L_Z^n L_T^m . \tag{3}$$

We note that the introduction of the vector units of length dimensionality which are different along three coordinate axes are well known in the theory of dimensional analysis[x/] and does not depend on dynamics.

However, we would like to stress here that the nonequivalence of different directions, along the hadron collision axis and perpendicularly to it, and the introduction of different dimensionalities L_Z and L_T associated with it are dictated by the strong interaction dynamics at high-energies.

By analogy with the plane explosion in hydrodynamics we assume that the processes of hadron interactions at extremely high energies are characterized by definite scale relations under transformations $p_z \to \lambda p_z$.

[x/] The vector units were first introduced in the dimensional analysis by Wiliams, in 1892. A detailed analysis of this method is given in the monograph/10/, where the improvement of its efficiency is demonstrated by a number of interesting examples from mechanics and hydrodynamics.

Physically this assumption is equivalent to the requirement that at high energies in the description of the asymptotic behaviour of various observable quantities characterizing the hadron interaction any dimensional parameters which fix the scale of the length along z-axis should vanish.

A special attention should be paid to the law of transformation of the particle energy at the longitudinal scale transformation. Assuming that all the particles in both the final and intermediate states in the unitarity condition have the limited transverse $\vec{p}_T$ and large longitudinal p_z components we get

$$p_0 = p_z + \frac{m^2 + \vec{p}_T^{\,2}}{2p_z} + \dots \tag{4}$$

Hence it follows that in the limit of infinite momentum the particle energy transforms at the longitudinal scale transformations in the same way as the momentum component along the collision axis.

Thus, we consider scale transformations of the form

$$p_z \to \lambda p_z , \quad p_0 \to \lambda p_0 , \quad \vec{p}_T \to \vec{p}_T . \tag{5}$$

The automodelity principle as applied to high energy strong interaction of hadrons means that all the physical quantities like the cross sections, angular distributions, etc. must transform under scale transformations (5) as homogeneous functions of appropriate dimensionality in the longitudinal scale, that is

$$F \to \lambda^{-n} F , \tag{6}$$

where n is the longitudinal dimensionality of the quantity F defined by the relation (3).

In what follows we consider the simplest consequences of the automodelity principle for the processes of inelastic and elastic hadron interaction at high energies.

1. Let us consider the <u>total cross section</u> of interaction of two particles $\sigma_{tot}(s)$. By definition the total cross section is characterized by a definite area transversal to the particle collision axis, i.e.

$$[\sigma_{tot}(s)] = L_T^2 . \tag{7}$$

It follows from the automodelity principle that $\sigma_{tot}(s)$ may not depend on s the only variable which has non-zero longitudinal dimensionality

$$[s] = [p_0^2] = L_z^{-2} . \tag{8}$$

2. Differential cross section for elastic scattering. The dimensionality of the differential elastic cross section in the limit of high energies and fixed momentum transfers when $[t] = [\vec{q}_T^{\,2}] = L_T^{-2}$ is defined by the relation

$$\left[\frac{d\sigma}{dt}\right] = L_T^4 \tag{9}$$

and, consequently, may not depend on s having longitudinal dimensionality.

Thus, it follows from the automodelity principle that

$$\lim_{\substack{s\to\infty \\ t - fixed}} \frac{d\sigma}{dt} = f(t). \tag{10}$$

It is interesting to note that relation (10) is a consequence of the droplet model and is analysed in detail in refs./11/. From the relation (10) it also follows that the total elastic cross section $\sigma_{el}(s)$ and the slope of the diffraction peak $b(s) = \frac{d}{dt} \ln \frac{d\sigma}{dt}|_{t=0}$ are constant

$$\sigma_{el}(s) = const, \quad b(s) = const . \tag{11}$$

It is obvious that the relation (11) can immediately be obtained from the dimensional analysis and automodelity requirement.

3. The ratio of the real and imaginary parts of the elastic scattering amplitude $a(s) = \frac{Re\, T(s,t=0)}{Im\, T(s,t=0)}$ is a dimensionless quantity and therefore in the high-energy limit it behaves like a constant (may be equal to zero)

$$a(s) = const. \tag{12}$$

We note that the relation (12) under the condition of constancy of the total cross section, on the basis of the general principles of quantum field theory, results in the asymptotic equality of the total cross sections of interaction of particles with antiparticles /12/.

4. Next, let us discuss the <u>single particle distribution</u> in the inclusive process $a + b = c + anything$ x/.

The differential cross section for this process can be represented in the form

$$d\sigma = \frac{d\vec{p}}{p_0} f(E, p_z, \vec{p}_T), \tag{13}$$

where E is the total energy of colliding particles; $\vec{p} = \{\vec{p}_T, p_z\}$ is the momentum of the particle c .

x/ This process as well as the more complicated one of two-particle distribution in hadron collisions were first studied theoretically in refs./3/.

Taking into account the dimensionalities of the cross section and the phase space $[d\sigma]=L_T^2, [\frac{d\vec{p}}{p_0}]=L_T^{-2}$ we find that the dimensionality of the function $f(E,p_z,\vec{p}_T)$ in eq. (13) is defined as

$$[f(E,p_z,\vec{p}_T)]=L_T^4 . \tag{14}$$

Hence it follows that the function f may depend only on the ratio of the two variables E and p_z having longitudinal dimensionality, i.e.

$$\lim_{\substack{E\to\infty \\ \frac{p_z}{E}-fixed}} f(E,p_z,\vec{p}_T)=f(\frac{p_z}{E},\vec{p}_T). \tag{15}$$

The relation (15) is usually regarded as a consequence of the multiperipheral and parton models and was conjectured by Feynman on the basis of heuristic arguments. In the rest frame of one of the colliding hadrons the relation (15) coincides under definite conditions with the prediction of limiting fragmentation hypothesis.

In a similar way, two-particle distribution in the process $a+b=c_1+c_2+anything$ can be written in the form

$$d\sigma=\frac{d\vec{p}_1}{p_{10}}\frac{d\vec{p}_2}{p_{20}}f(E,p_{1z},p_{2z},p_{1T},p_{2T}), \tag{16}$$

where $\vec{p}_1=\{\vec{p}_{1T},p_{1z}\}$ and $\vec{p}_2=\{\vec{p}_{2T},p_{2z}\}$ are the momenta of particles c_1 and c_2. The dimensionality of the quantity f is found to be

$$[f(E,\vec{p}_1,\vec{p}_2)]=L_T^6 \tag{17}$$

then according to the automodelity principle we find

$$\lim_{\substack{E\to\infty \\ \frac{p_{1z}}{E},\frac{p_{2z}}{E}-fixed}} f(E,p_{1z},p_{2z},p_{1T},p_{2T})=f(\frac{p_{1z}}{E},\frac{p_{12z}}{E},p_{1T},p_{2T}). \tag{18}$$

It is not difficult to generalize this result to the case of an arbitrary number of detected particles in the final state.

5. Consider now the process of production of two jets of particles with momenta directed predominantly along the collision axis. The total differential cross section for this process summed over the number of all the secondaries can be written in the following form

$$\frac{d^3\sigma}{dt\,d\nu_1\,d\nu_2}=F(s,\nu_1,\nu_2,t). \tag{19}$$

Here

$$\begin{aligned} s&=(p_1+p_2)^2\approx 4E^2\\ \nu_1&=p_1q\approx E(q_0-q_z)\\ \nu_2&=p_2q\approx E(q_0+q_z)\\ t&=q^2\approx\frac{\nu_1\nu_2}{E^2}-\vec{q}_T^{\,2}, \end{aligned} \tag{20}$$

where q is a momentum transferred from the initial hadron to the system of secondaries the momenta of which are co-linear to the initial hadron monentum, the components $q_z,\vec{q}_T$ being defined in the c.m.s. of colliding particles.

As is seen from formula (20) in the diffraction dissotiation region, where the condition $\frac{\nu_1\nu_2}{E^2}\ll\vec{q}_T^{\,2}$ is fulfilled, the variable t is mainly defined by the transverse momentum transfer and has the transverse dimensionality $[t]=L_T^{-2}$ while the variables s,ν_1,ν_2 will have the same dimensionality $[s]=[\nu_1]=[\nu_2]=L_z^{-2}$.

Thus, the differential cross section (19) can be represented in the following form

$$\lim_{\substack{s\to\infty \\ t - fixed \\ \frac{\nu_1 \nu_2}{s^2} \ll 1}} \frac{d^3\sigma}{dt\, d\nu_1\, d\nu_2} = \frac{1}{s^2} F\left(\frac{\nu_1}{s}, \frac{\nu_2}{s}, t\right). \tag{21}$$

Integrating the cross section (21) over the diffraction dissotiation region we obtain the total differential cross section for diffraction production of jets which, according to the automodelity principle, must not depend on energy

$$\lim_{\substack{s\to\infty \\ t - fixed}} \left(\frac{d\sigma}{dt}\right)_{tot} = F(t). \tag{22}$$

The behaviour of the differential cross sections (16),(19) and (22) was also studied in refs./14,15/ in the framework of various models of quantum field theory and the coherent state model/7/.

In refs. /7,15/ a possibility has been found for disappearing of the diffraction peak in the total diffraction dissotiation cross section (22) which is, in a certain sense, analogous to the point -like behaviour of the deep inelastic lepton-hadron cross sections.

The automodelity principle leads to interesting results for the asymptotic behaviour of different quantities measured in high energy inelastic experiments. In particular, the dimensionality considerations lead to the following asymptotic behaviour of the average multiplicity $\bar{n}(s)$, the average transverse momentum $\langle \vec{p}_T \rangle$ of secondaries and the inelasticity coefficient $k(s)$

$$\bar{n}(s) = const; \quad \langle \vec{p}_T \rangle = const; \quad k(s) = \left\langle \frac{E'}{E} \right\rangle = const, \tag{23}$$

where E' is the energy carried away by secondaries in processes $a + b \to a + b$ + anything.

The list of the automodelity predictions may, in principle, be extended.

We would like to discuss below the accuracy of the results obtained in this paper on the basis of the automodelity principle.

It is obvious that the automodelity hypothesis (6) is approximate and is applicable obly to the processes of strong interactions at high energies in which there is a physical nonequivalence of various directions, along the particle collision axis and perpendicularly to it. In particular, the automodelity principle is inapplicable in its original form to large-angle elastic scattering and charge exchange processes.

At present there is no definite way for estimating corrections due to violations of the automodelity principle. In the cases when the automodel behaviour hypothesis leads to the constancy of any quantity with energy we may expect in reality only a weak energy dependence of this quantity (in these cases one usually implies the logarithmic dependence, however the mechanism of appearance of such factors is unknown).

It is very interesting to construct models which would provide in a natural manner (let even at the expense of locality in the description of particles) the limitedness of transverse momentum transfers in strong processes at high energies and would lead in the framework of a certain approximation, for example the straight-line path approximation, to the automodel behaviour.

This work was repeatedly discussed with N.N. Bogolubov, A.M. Baldin, D.I. Blokhintsev, R.N. Faustov, O.A. Khrustalev, A.A. Logunov, M.A. Markov, D.V. Shirkov, Ya.A. Smorodinsky, L.D. Soloviev. The authors express their gratitude to all of them.

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