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PATH INTEGRATION METHOD AND SEMICLASSICAL APPROXIMATION FOR THE POTENTIAL SCATTERING AMPLITUDE

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1. Introduction

The semiclassical approach to the scattering problems within the framework of the nonrelativistic Schrödinger equation with the smooth potential /1,2,3/ is known to reproduce the main characteristics of the high-energy, elementary particle reactions. It thus represents an effective tool for analyzing the experimental data. This is rather similar to the quasipotential approach /4,5/ and other methods of the relativistic quantum field theory /6,7/.

In the present paper the semiclassical scattering is investigated by the functional integration method. Before stating the problems of this paper, the principal results of the conventional semiclassical W.K.B. method is briefly reviewed as applied to the potential scattering problems. To make things more clear, we consider only the following asymptotic domains:

$$
\begin{aligned}
&1)\ \frac{V_0}{E} \lesssim \frac{\lambda\!\!\!^{-}}{R} \quad ; (a)\ \theta \gg \frac{\lambda\!\!\!^{-}}{R} \quad ; (\delta)\ \theta \lesssim \frac{\lambda\!\!\!^{-}}{R} \\
&2)\ \frac{V_0}{E} \gg \frac{\lambda\!\!\!^{-}}{R} \quad ; (a)\ \theta \gg \frac{\lambda\!\!\!^{-}}{R} \quad ; (\delta)\ \theta \lesssim \frac{\lambda\!\!\!^{-}}{R}
\end{aligned}
\qquad (1.1)
$$

in the semiclassical limit[*)] $\frac{\lambda\!\!\!^{-}}{R} \ll 1$.

According to the W.K.B. method the scattering phase in the partial wave expansion of the amplitude is defined as the ratio of the classical action function to the Planck constant.

[*)] We use the following notations: V_0 is the average value of the potential energy, R is the potential size, $\lambda\!\!\!^{-} = \frac{\hbar}{k}$, k is the incident momentum modulus, $E = \frac{k^2}{2}$ is the kinetic energy (the scattering particle mass is assumed to be unity), θ is the scattering angle.

Consider the first case of (1.1). Here for the classical action function the Born approximation is valid because of the smallness of the quantity $\frac{V_0}{E}$. The range of angles (a) is classically forbidden, and in the W.K.B. approximation the amplitude falls exponentially with increasing momentum transfer, reproducing the Orear behaviour of the differential cross section at high energies. At small angles (b) the W.K.B. approximation coincides with the eikonal one for the scattering amplitude. The latter is extensively used nowadays for theoretical analysis of the experimental data. At very small angles and $\frac{V_0}{E} \ll \frac{\lambda}{R}$ the first Born approximation is true which describes the diffractional peak. The diffractional picture as a whole results from the multi-scattering. (Note that the smooth potential is interpreted by ref. [10] as a result of the interaction due to the reggeon exchange).

As to the second case, the classical action function is much greater than the Planck constant, and, generally speaking, is not approximated by the first Born term. As is easily seen from Eq.(1.1) only in the domain 2)(a) it is reasonable to say about the classical limit of the semiclassical expression for the differential cross section the calculation of which by the saddle point method leads to the classical mechanics formulae (except for some anomalous cases).

The diffractional domain 2)(b) has been investigated least of all. All the results, which can be found in the literature, have been obtained by the eikonal approximation, being in some sense the particular case of the W.K.B. method, as we have already noted.

Nevertheless, the question about the limits of applicability of the eikonal approximation is still open. The most acceptable condition for its validity is supposed to be as follows: $\frac{V_0}{E} \ll 1$. The rough estimation of the eikonal expansion terms gives the condition $\frac{V_0}{E} \ll \sqrt{\frac{\lambda}{R}}$, which, from the point of view of the classical limit $\lambda \to 0$, differs from first one in principle. On the other hand, there exist a number of papers, in which the applicability of the eikonal approach in the case[13] $\sqrt{\frac{\lambda}{R}} \ll \frac{V_0}{E} \ll 1$, or at an arbitrary value of $\frac{V_0}{E}$ [14], is justified by the physical results obtained by other, more general methods. In particular, the Yukawa potential scattering cross section at $V_0 \sim E$ obeys the Froissart bound[15].

It is very instructive, of course, to compute in the semiclassical approximation the scattering amplitude in the domain 2)(b) and study the question concerning the limits of eikonal approach validity.

In sect.2 the semiclassical approach is developed by using the path integration method. An advantage of such an approach over the conventional W.K.B. method is that it can be applied to the scattering on the spherically nonsymmetric potentials, as well.

In sect.3 the total cross section and the scattering amplitude with the Gaussian potential are calculated in the semiclassical approximation $\frac{\lambda}{R} \ll 1$, $\frac{RV_0}{\lambda E} \gg 1$. It is found that in the small angle diffractional domain the asymptotic values for the semiclassical and eikonal amplitudes coincide. The eikonal series is alternating one, therefore all its terms cancel and even in the case $\frac{V_0}{E} \to \infty$ do not violate an asymtotic behaviour of the eikonal amplitude, which describes the scattering analogous to the Fraunhofer diffraction on the black sphere with the radius rising logarithmically as $\frac{R}{\lambda}\frac{V_0}{E} \to \infty$. The change of the potential scattering behaviours with increasing potential energy is discussed. In particular, it is shown that at $\frac{V_0}{E} \gg \frac{\lambda}{R}$ the angle diffractional domain $\theta < \frac{\lambda}{R}$ (but not $\theta \sim \frac{V_0}{E}$, as is assumed usually in the semiclassical approach/9/), provides the main contribution to the total cross section.

2. The Semiclassical Approximation to the Scattering Amplitude

Let us now consider the semiclassical approximation to the scattering amplitude within the framework of the functional integration method. The closed continual representation for the relativistic potential scattering amplitude has been found by the author/12/. The nonrelativistic analog has the form*)

$$f(T^2) = \frac{1}{4\pi}\int d^3 z\, e^{i\vec{z}\vec{T}}\, V(z) \int_0^1 d\lambda \int\limits_{x_+(0)=z} \delta^3 x_+\, e^{iS_+(x_+)} \int\limits_{x_-(0)=z} \delta^3 x_-\, e^{iS_-(x_-)} , \tag{2.1}$$

where $\vec{T} = \vec{p} - \vec{q}$, $|\vec{p}| = |\vec{q}| = K$ and

$$S_+ = \int_0^\infty d\tau \left[\frac{\dot{x}_+^2}{2} - \lambda V(\vec{x}_+ + \vec{p}\tau)\right] ; \; S_- = \int_{-\infty}^0 d\tau \left[\frac{\dot{x}_-^2}{2} - \lambda V(\vec{x}_- + \vec{q}\tau)\right] \tag{2.2}$$

are the action functions which depend on arbitrary trajectories $x_\pm(\tau)$ passing at time $\tau = 0$ through the point $x_\pm(0) = z$; $\delta^3 x_\pm$ are the volume elements of functional space.

In the semiclassical approach $KR \gg 1$, to calculate the path integral the stationary phase method /16/ is valid, according

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*) We shall work in the system: $\hbar = m = 1$

to which[*]:

$$f_{\ell}(T^2)=\frac{1}{4\pi}\int d^3\tau\, e^{i\vec{\tau}\vec{T}}\,V(\tau)\int_0^1 d\lambda\, e^{iS_{\ell}(\tau,\lambda)}$$
$$S_{\ell}(\tau,\lambda) = S_+(X_{+\ell}) + S_-(X_{-\ell}) , \tag{2.3}$$

where $X_{\pm\ell}$ satisfy the classical equations of motion

$$\frac{\delta S_\pm}{\delta X_\pm}=0 \;\; ; \vec{X}_\pm(0)=\vec{\tau} \; ; \lim_{\tau\to\infty}\dot{\vec{X}}_+(\tau)=\vec{p} \;\; , \lim_{\tau\to-\infty}\dot{\vec{X}}_-(\tau)=\vec{q} \;\; . \tag{2.4}$$

By means of the motion integral (the energy conservation law)

$$\frac{(\dot{\vec{x}}_+ + \vec{p})^2}{2}+\lambda V(\vec{x}_+(\tau)+\vec{p}\,\tau)=\frac{K^2}{2} \; ; \;\; \frac{(\dot{\vec{x}}_- + \vec{q})^2}{2}+\lambda V(\vec{x}_-(\tau)+\vec{q}\,\tau)=\frac{K^2}{2} \tag{2.5}$$

it is easy to define /17/ the Hamilton-Jacobi equations for the function $S'_\pm$ replacing in Eq.(2.5) at $\tau=0$ $\dot{\vec{x}}_\pm(0)$ by $\mp\vec{\partial}_\tau S'_\pm(\tau)$ with the boundary conditions $\lim_{\vec{p}\vec{\tau}\to\infty} S'_+(\tau)=0$; $\lim_{\vec{q}\vec{\tau}\to-\infty} S'_-(\tau)=0$. Our considerations are confined to the case of small scattering angles when it is possible to neglect the dependence of the function $S'_{\ell}(\tau,\lambda)$ on the momentum transfer /12/. Then, choosing the direction of $\vec{p}$ along z-axis and denoting $\vec{\tau}=(\vec{\rho},z)$ for $S'_\pm$ we arrive at the Hamilton-Jacobi equations

$$\frac{1}{2}\left[(\vec{\partial}_\rho S_\pm)^2+(\partial_z S_\pm)^2\right] \mp K\,\partial_z S_\pm = -\lambda\, 2\, V(\rho,z) \tag{2.6}$$

of which the solution can be rather easily constructed in the

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[*] For the sake of simplicity suppose $S'_\pm(X_\ell)$ to be functions of a fixed sign, the unique solution for $\frac{\delta S'}{\delta X}$ to exist.

form of series

$$S_{\pm} = \sum_{\ell=1}^{\infty} \lambda^{\ell} S_{\ell\pm} \tag{2.7}$$

inserting the latter into Eq.(2.6) and putting the coefficients at the same powers of λ be zero.

In the case $\frac{V}{E} \ll 1$ the action function $S_{cl}(r,\lambda)$ is completely determined by the first term of the series (2.7) and coincides with the eikonal phase

$$S_{cl} = \lambda S_1(\rho) = \lambda \chi_c(\rho) = \lambda k \int_{-\infty}^{\infty} dz \, \frac{V(r)}{2E} \tag{2.8}$$

hence, for the amplitude at least for $e^{iS_2\lambda^2} \sim 1$ (in particular, this leads to the Schiff condition /14/: $\frac{V_0}{E} \ll \sqrt{\frac{r}{R}}$) the eikonal representation holds:

$$f(T^2) = \frac{k}{4\pi} \int d^2\rho \, e^{i\vec{\rho}\vec{T}} \left(e^{iS_1(\rho)} - 1 \right) . \tag{2.9}$$

Note that Eq.(2.8) is the solution of the Hamilton-Jacobi equations which follows from the conservation laws Eq.(2.5) when the change of particle momentum in magnitude and direction is significantly less than the momentum itself: $|\dot{X}|^2 \ll |\dot{X}_z \cdot k|$. Therefore the eikonal representation in classical mechanics corresponds to the quasiuniform motion.

In the straight-line approximation when a particle moves only along z-axis ($\vec{\partial}_\rho S = \dot{\vec{x}}_\rho = 0$), Eqs.(2.6) are easily solved and the action function becomes

$$S_{st}(\rho,\lambda) = k \int_{-\infty}^{\infty} dz \left[\sqrt{1 - \lambda \frac{V(r)}{E}} - 1 \right] . \tag{2.10}$$

Here the amplitude takes on the form of the impact-parameter representation

$$f_{st}(T_\perp^2)=\frac{K}{4\pi}\int d^2\rho\, e^{i\vec{\rho}\vec{T}_\perp} S_1(\rho)\int_0^1 d\lambda\, e^{iS_{st}(\rho,\lambda)} \qquad (2.11)$$

3. The Gaussian Potential Scattering

Here we will calculate in the semiclassical approximation the total cross section of the scattering on the Gaussian potential $V(r)=\frac{2V_0}{\sqrt{\pi}}e^{-\frac{r^2}{R^2}}$ in the limit $\frac{RV_0}{\hbar E}\to\infty$. For the sake of simplicity we shall consider the straight-line scheme (2.11). The asymptotic form of more exact expression (2.3) is calculated in the appendix.

According to Eq.(2.11) the total cross section has the form

$$\sigma_{t\,st}=\frac{4\pi}{K}\,\mathrm{Im}\, f(0)=4\pi\,\mathrm{Im}\int_0^\infty \rho\, d\rho\, S_1(\rho)\int_0^1 d\lambda\, e^{-iS_{st}(\rho,\lambda)}, \qquad (3.1)$$

where

$$S_{st}(\rho,\lambda)=-K\int_{-\infty}^{\infty}dz\left(\sqrt{1-\lambda\frac{V(r)}{E}}-1\right)=\sum_{m=1}^{\infty}S_m\lambda^m e^{-m\frac{\rho^2}{R^2}}$$

$$S_1=KR\frac{V_0}{E}\;;\quad S_2=KR\left(\frac{V_0}{E}\right)^2\frac{1}{2\sqrt{2\pi}}\;;\;\ldots \qquad (3.2)$$

Integrating by parts Eq.(3.1) over λ we arrive at

$$\sigma_{t\,st}=4\pi\,\mathrm{Im}\left[i\int_0^\infty \rho\, d\rho\left(1-e^{-iS_{st}(\rho,1)}\right)+A\right], \qquad (3.3)$$

$$A=-i\int_0^\infty \rho\, d\rho\int_0^\infty d\lambda\, e^{-iS_1(\rho)\lambda}\frac{\partial}{\partial\lambda}\left[e^{-iS_{st}(\rho,\lambda)+iS_1(\rho)\lambda}\right], \qquad (3.4)$$

The asymptotic form of the second term in Eq.(3.3) is computed by using the Watson-Sommerfeld transformation (see the appendix) and is found to be equal to $O\left(\frac{S_2}{S_1^2}\right)\sim O\left(\frac{1}{KR}\right)$ in the order of magnitude. Therefore in the semiclassical limit this term in Eq.(3.3) can be neglected. It should be noted that an analogous expression is derived in the W.K.B. approximation with an accuracy to the replacement of the function $S_{st}(\rho,l)$ by the scattering phase, in the semiclassical approximation.

Adding and subtracting the function $\exp\{iS_1(\rho)\}$ in the remaining expression of (3.3) under the integral sign we get

$$\sigma_{t.st} = \sigma_{t.eik} + \Delta\sigma_t \quad , \tag{3.5}$$

where

$$\sigma_{t.eik} = 4\pi \, I_m\Big[i\int_0^\infty \rho d\rho \left(1 - e^{-iS_1(\rho)}\right)\Big] \quad , \tag{3.6}$$

$$\Delta\sigma_t = 4\pi \, I_m\Big[i\int_0^\infty \rho d\rho \, e^{-iS_1(\rho)}\left(1 - e^{iS_1(\rho) - iS_{st}(\rho,l)}\right)\Big] \quad . \tag{3.7}$$

The asymptotic form of the eikonal cross section is as follows (see, e.g. ref. [9]):

$$\sigma_{t.eik} = 2\pi \rho_{max}^2 \quad . \tag{3.8}$$

Here

$$\rho_{max} = R\sqrt{\ln S_1} \quad ; \quad S_1 = KR\frac{V_0}{E} \to \infty \quad . \tag{3.9}$$

It should be stressed that the basic contribution to the integral Eq.(3.6) is given by the large impact parameters $\rho \sim \rho_{max}$, determined by the condition $S_1(\rho) \sim 1$.

To calculate Eq.(3.7) we substitute the integration variable $e^{-\frac{\rho^2}{R^2}} = x$, then Eq.(3.7) takes on the form

$$\Delta\sigma_t = 2\pi \, I_m\Big[\int_0^1 dx \, e^{-iS_1 x} g(x)\Big] \quad , \tag{3.10}$$

where

$$g(x) = \frac{\exp\{iS_1 x - iS_{st}(\sqrt{\ln\frac{1}{x}}R, 1)\} - 1}{ix} = \frac{\exp\{i\sum_{m\neq 0}^{\infty} S_m x^m\} - 1}{ix} \qquad (3.11)$$

is the infinitely differentiable and nonsingular function on the interval (0.1). Therefore according to ref./18/ the asymptotic expansion of Eq.(3.10) can be performed by means of the integration by parts. Taking into account the first term of this expansion we get for δ_{tst} the following expression

$$\delta_{tst} = \delta_{t,eik} + \frac{2\pi R^2}{S_1} I_m\left[e^{iS_{st}(0)} - e^{iS_1'} + O\left(\frac{g'(x)e^{iS_1 x}\Big|_0^1}{S_1}\right)\right]. \qquad (3.12)$$

Thus the asymptotic values of the semiclassical and eikonal expression coincide. From Eq.(3.3) the alternating nature of the eikonal series in $S_{st}'(0,1) - S_1' = \sum_{m=2}^{\infty} S_m$ is rather evident. An estimation of the first terms of this series in order to learn the limits of validity of the eikonal approximation can provide the upper limits on the quantity $\frac{V_0}{E}$ /11,12/; while using Eq.(3.10) it is not hard to prove the validity of the eikonal asymptotics at any $\frac{V_0}{E}$. Really, putting $X = Y\sqrt{S_2}$ in Eq.(3.10) we get

$$\lim_{\frac{V_0}{E}\sqrt{KR} \gg \frac{1}{\sqrt{KR}} \to 0} \Delta\delta_t \sim \int_0^{\sqrt{KR}\frac{V_0}{E}} dy\, e^{-iY\sqrt{KR}} \left[\frac{\exp\{-iy^2[1 + O(\frac{Y}{\sqrt{KR}})]\} - 1}{iY}\right] \sim$$

$$\sim O\left(\int_0^{\infty} dy\, Y e^{-i\sqrt{KR}Y}\right) \sim O\left(\frac{1}{KR}\right). \qquad (3.13)$$

The reason for validity of the eikonal approximation for the small angle scattering is that the strongly oscillating function $\exp\{iS_{st}(\rho,1)\}$ in (3.3) gives the contribution comparable with unity only in the large impact parameter region where $S_{el}(\rho) \sim 1$. However the latter condition is sufficient, as was already mentioned in the introduction, the classical action function to be approximated by the Born expansion (i.e. by the eikonal phase [*)]) with an accuracy to the quantities of the order $\frac{\lambda\!\!\!^{-}}{R}$.

Physically this means the strong interference of the diffracting part of the scattering particle plane wave, for which $\rho < \rho_{max}$, hence the diffractional picture is determined mainly by an peripheral domain $\rho \sim \rho_{max}$. Therefore the scattering at $\frac{R V_0}{\lambda\!\!\!^{-} E} \gg 1$ physically is adequate to the light-scattering on the black sphere (the Frauhofer diffraction) with logarithmically increasing radius (3.9). Hence, in particular, from the Babinet principle /19/ it follows that the values of elastic and inelastic cross sections coincide:

$$\sigma_{in} = \sigma_{el} = \pi \rho^2_{max} . \tag{3.14}$$

The value of ρ_{max} and, consequently, those of the total cross section and diffractional amplitude /19/

$$f_{eik}(k^2\theta^2) = i \frac{\rho_{max}}{2\theta} J_1(\rho_{max} \cdot k \cdot \theta) \tag{3.15}$$

in the classical limit $\lambda\!\!\!^{-} \to 0$ tend to infinity. This is due to the fact that in classical mechanics the scattering total cross section proves to be infinite for any field becoming zero only as $r \to \infty$.

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*) Note the contrast situation for the scattering through classical angles: the main contribution is furnished by those impact parameters for which $S_{cl}(\rho) \sim kR\theta_{cl} \gg 1$ /8/.

(We recall that the classical cross section at a fixed radius equals half of the quantum one). Therefore one gets that the main contribution to the total cross section at $\frac{V_0}{E} \gg \frac{\lambda}{R}$ just as in the case $\frac{V_0}{E} < \frac{\lambda}{R}$ is provided by the diffractional domain of the scattering angles. The typical angle of the predominant scattering is given not by the quantity $\theta \sim \frac{V_0}{E} \gg \frac{\lambda}{R}$, as is usually adopted in the semiclassical approach [8,9], but $\theta \sim \frac{\lambda}{\rho_{max}}$.

Thus, as the potential energy rises in passing from the domain 1)(b) $\frac{V_0}{E} < \frac{\lambda}{R}$ to the domain 2)(b) $\frac{V_0}{E} \gg \frac{\lambda}{R}$ (see (1.1)) there appear the following correlating qualitative distinctions: the logarithmical growth of the total cross section and the shrinkage of the diffractional peak ($R \to \rho_{max}$). While in the nondiffractional domain of angles $\frac{\lambda}{R} \ll \theta < \frac{V_0}{E}$ (see sect.1) for real potentials the Orear scattering behaviour is changed into the purely one, and in the case $V_0 \gtrsim E$ vanishes at all. A similar situation can occur, for instance, in the scattering of particles on the tensor potential (the gravitational fields)[14].

Conclusion

Within the framework of the path integration method the semiclassical representation Eq.(2.3) for the amplitude has been derived. It was applied then to calculating the scattering amplitude with the Gaussian potential in the diffractional domain of angles. By this it has been found that the smallness of the potential is not necessary condition for the simpler eikonal approximation to be valid, since the latter coincides with the semiclassical approximation even as $\frac{V_0}{E} \gg 1$.

The physical explanation of such a coincidence suggests an idea that this might be inherent in all potentials, quite rapidly decreasing at infinity.

If the latter is true, then at $KR\frac{V_0}{E}\gg 1$ the structure of such potentials in the finite region of origin, which defines the scattering amplitude in the Born approximation, is levelled by the strong interference of diffracting waves of the scattering particle. Therefore, potentials which are rather large in magnitude can be approximated by the black sphere with the radius of which the energy dependence is determined by the behaviour of an appropriate eikonal phase at infinity. It should be noted that the upper bounds on the interaction cross section at high energies, found by using the general principles of quantum field theory are consistent also with the semiclassical ideas on the scattering on strongly absorbing sphere /7/.

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Appendix

Let us calculate in the semiclassical approximation the total cross section of the scattering on the Gaussian potential, which has the form

$$\sigma_{tot} = \frac{4\pi}{K} \operatorname{Im} f(0) = \operatorname{Im} \frac{1}{K} \int d^3 r \, V(r) \int_0^1 d\lambda \, e^{\, i S_{cl}(r,\lambda)} \tag{1}$$

according to Eq(2.3).

S_{cl} is searched in the form of series in λ (see Eq.(2.7))

$$S_{cl}(r,\lambda) = \sum_{m=1}^{\infty} S_m(\rho, z) \left(\lambda e^{-\frac{\rho^2}{R^2}}\right)^m \tag{2}$$

it being known that

$$S_{cl}(0, z, \lambda) = S_{1f}(q\lambda) = \sum_{m=1}^{\infty} S_m \lambda^m \, . \tag{3}$$

The coefficients are defined by the solution of the iteration equations, which result from inserting the series (2.7) into the Hamilton-Jacoby Eq.(2.6)

$$\begin{aligned} &S_1(\rho, z) \equiv S_1 \\ &S_2(\rho, z) = S_2 \left[1 + \frac{\rho^2}{z^2} \psi_2\left(\frac{z}{R}\right)\right]; \; \psi_2(z) = \frac{4}{\sqrt{\pi}} \left[\int_z^{\infty} dz_1 \left(\int_{z_1}^{\infty} e^{-t_1^2} dz_1\right)^2 + \int_{-\infty}^{z} dz_1 \left(\int_{-\infty}^{z_1} e^{-z_2^2} dz_2\right)^2\right] \\ &\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &S_m(\rho, z) = S_m \left[1 + \sum_{n=1}^{m-1} \left(\frac{\rho}{R}\right)^{2n} \psi_m^n\left(\frac{z}{R}\right)\right] . \end{aligned} \tag{4}$$

It should be noted, that the functions ψ_n^m (7) are not zero throughout the whole z-domain. This makes it possible to perform the asymptotic calculation under the integral sign over z considering $\psi_m^n(z)$ as some constant coefficients.

After the variable replacement $e^{-\frac{\rho^2}{R^2}} = x$ integral (1) takes on the simple form

$$\sigma_{tot} = 2\pi R^2 \operatorname{Im} \left\{ \int_{-\infty}^{\infty} \frac{e^{-z^2}}{\sqrt{\pi}} dz \, \mathcal{J}(S_{cl}) \right\} \tag{5}$$

$$\frac{\mathcal{J}(S_{cl})}{S_1} = \int_0^1 dx \int_0^1 d\lambda \exp\left\{ i S_1 x \lambda \left[1 + x\lambda \frac{S_2}{S_1}(1 + \psi_2 \ln x) + \dots + (\lambda x)^m \frac{S_m}{S_1}(1 + \dots) + \dots\right]\right\} . \tag{6}$$

The integral of the type (6) can be found in the calculations of the Feynman diagram asymptotics and have been investigated in Ref./18/.

The asymptotic value of Eq.(6) as $S_1 \to \infty$ is determined by the most right singularity of the function $F(\xi)$. This function is an analytic continuation of the coefficient F_ξ of Taylor series in S_1 of the expression (3.6). In the case under consideration the most right singularity has the form

$$F(\xi) = \int_0^1\int_0^1 dx\, d\lambda\, (\lambda x)^\xi = \frac{1}{(\xi+1)^2} \tag{7}$$

which corresponds to the asymptotic behaviour /10/:

$$J(S_1 d) = i \ln S_1' + const\,. \tag{8}$$

Hence the value of the total cross section coincides with the eikonal expression (3.9).

R E F E R E N S

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