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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ON ESTIMATE OF THE AVERAGE MULTIPLICITY ASYMPTOTIC BEHAVIOUR AND DIFFRACTION MECHANISM OF PARTICLE INTERACTIONS AT HIGH ENERGIES

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ON ESTIMATE OF THE AVERAGE MULTIPLICITY ASYMPTOTIC BEHAVIOUR AND DIFFRACTION MECHANISM OF PARTICLE INTERACTIONS AT HIGH ENERGIES

It is quite evident that a consistent description of the elastic scattering of hadrons at high energies is impossible without deep understanding of dynamics of essentially inelastic processes which are followed by the production of a large number of secondary particles. One of the most important characteristics of such processes is the average multiplicity of secondary particles. The problem of the asymptotic behaviour of the average multiplicity in the limit of extremely high energies attracts now great attention.

In this note we would like to point to a possibility of existence of a definite simple relation which connects the average multiplicity asymptotic behaviour with the total cross section and the elastic diffraction slope parameter.

Our approach is based on the far-going idea on the close relationship between the diffraction character of elastic hadron scattering and the main regularities of multiparticle production processes [1-4]. To be more concrete we shall suppose the picture of high-energy diffraction processes which is based on the coherent state model developed in papers [5,6] as well as on the straightline path approximation method in quantum field theory models[7,8].

In recent papers a tendency has appeared to consider hadrons as composite systems with internal degrees of freedom in processes of strong interactions at very high energy (''droplet'' model[9], ''parton'' model[10], coherent state model[5,6]).

In the coherent state model it is suggested that the hadron states in strong interaction processes at high energies correspond to the coherent states of some complex system, the excitation spectrum of which is described in the simplest case by the four-dimensional relativistic oscillator.

As a remarkable fact we stress that predictions of the coherent state model are in qualitative agreement with the calculations performed in the framework of quantum field models using the functional integration method for the summation of the infinite series of the perturbation theory[6,7].

The coherent state method has lately been developed in papers dealing with the composite dynamic model of hadrons[11] and the problem of strong coupling of particles with quantum field[12]. The results obtained in these papers permit us to hope that the coherent state model gives a correct outline of the behaviour of composite quantum systems with infinite number of degrees of freedom.

As it was shown[5,6] the exponential decreasing of amplitudes with increasing momentum transfers is due in this model to the coherent excitement of transverse (in the center of mass system of colliding hadrons) modes of the oscillator, which is followed by emission of a large number of secondary particles. In the limit of extremely high energies, when the momentum transferred from an initial hadron to an excited system is fixed, the particle production process goes in a statistically independent way and is governed by the Poisson law.

Thus the experimentally observable diffraction behaviour of the differential cross section of the high-energy elastic scattering[x/]

$$\frac{d\sigma^{el}}{dt} = \left(\frac{d\sigma^{el}}{dt}\right)_0 e^{-A(s)t} \tag{1}$$

corresponds to the following dependence of the average number of secondary particles produced in inelastic hadron collision on s and t

$$\bar{n}(s,t) \underset{\substack{s\to\infty \\ t - fixed}}{\to} A(s)t \tag{2}$$

where $A(s)$ is the elastic diffraction slope parameter.

The differential cross section of inelastic processes, when n and m secondary particles are emitted respectively by each of two colliding hadrons, are determined by

$$\left(\frac{d\sigma}{dt}\right)_{n,m} = \left(\frac{d\sigma^{el}}{dt}\right)_0 W_n(s,t)\, W_m(s,t) . \tag{3}$$

Here

$$W_n(s,t) = e^{-A(s)\frac{t}{2}} \frac{1}{n!} \left[A(s) \frac{t}{2} \right]^n , \tag{4}$$

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x/ Here and after the variable t denotes the absolute value of the square of momentum transfer, i.e. $t = |q^2|$.

where the quantity $A(s)\frac{t}{2}$ has the sense of an average number of particles emitted by one of the two hadrons[x].

It follows from eqs. (3) and (4) that the differential cross section of the inelastic collision with excitement of only one of the colliding hadrons is characterized by the diffraction peak with the slope which is equal to a half of the elastic one:

$$\frac{d\sigma^*}{dt} = \sum_{n=0}^{\infty} \left(\frac{d\sigma}{dt}\right)_{n,0} = \left(\frac{d\sigma^{el}}{dt}\right)_0 e^{-A(s)\frac{t}{2}} . \qquad (5)$$

The total differential cross section for two hadron collision, after summing in eq. (3) over a number of all secondary particles, is given by

$$\frac{d\sigma^{tot}}{dt} = \left(\frac{d\sigma^{el}}{dt}\right)_0 = const \qquad (6)$$

and does not depend on t, which, in some respect, is analogous to the point-like of automodel behaviour of deep inelastic hadron-lepton scattering /13/. Obviously, the relation (6) is meaningful only for momentum transfers limited by the diffraction region. The real meaning of eq. (6) is that the total differential cross section can vary considerably in magnitude only with the variations $\Delta t \approx t_{eff}$ which are much greater than the size of the diffraction peak region i.e.

$$t_{eff} \gg \frac{1}{A(s)} . \qquad (7)$$

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[x] Note, that we consider the region of diffractive dissotiation in which $(M-m)^2 / 2m^2 \ll 1$, where M is the effective mass of the "stream".

To estimate the value t_{eff} we can use the unitarity condition. Integrating eq. (6) over the region $t \lesssim t_{eff}$ we must obtain a cross section which does not exceed the total interaction cross section

$$\int_0^{t_{eff}} \frac{d\sigma^{tot}}{dt}\, dt \approx t_{eff} \left(\frac{d\sigma^{el}}{dt}\right)_0 \leq \sigma^{tot} . \qquad (8)$$

Using the optical theorem and assumption that the forward elastic amplitude is pure imaginary, we get from eq. (8)

$$t_{eff} \lesssim \frac{16\pi}{\sigma^{tot}} . \qquad (9)$$

The consistency condition (7) can be represented in the form

$$\frac{\sigma^{el}}{\sigma^{tot}} = \frac{\sigma^{tot}}{16\pi A(s)} \ll 1 . \qquad (10)$$

The found value of t_{eff} can be used to estimate the average number of secondary particles $\bar{n}_{diffr}(s)$ produced in diffraction collision of two hadrons at high energies

$$\bar{n}_{diffr}(s) = \frac{1}{\sigma^{tot}} \int_0^{t_{eff}} \frac{d\sigma^{tot}}{dt} A(s)\, t\, dt \lesssim \frac{8\pi A(s)}{\sigma^{tot}} . \qquad (11)$$

Thus, the diffractive or peripheral part of the average multiplicity of secondary particles is determined by the parameters of the elastic scattering amplitudes. The conclusion about the behaviour of the total average number of secondary particles $\bar{n}(s)$ can be done only on the basis of some additional assumptions concerning the contribution of small distances to multi-particle production processes. In particular, if one assumes that "pionization" or production of particles with finite momenta in the center of mass system of colliding hadrons disappears in the limit of high energies, one gets from eq. (11) the following behaviour of the total average multiplicity

$$\bar{n}(s) \approx \frac{8\pi A(s)}{\sigma^{tot}} + \nu , \qquad (12)$$

where ν is the number of "leading" particles which is equal to 2 in this simplified approach[x/].

The analysis of relations (11) and (12) is of great interest because it can shed some light on the relative importance of the central and peripheral forces in processes of multiparticle production at high energies (see, e.g., ref.[/14/]).

It is interesting to note that relation (12) gives qualitatively the correct high-energy behaviour of the average multiplicity. Indeed the approximate constancy of the total cross section and the logarithmic shrinkage of the diffraction width, as is seen at the accessible low energies[/15/] corresponds to the logarithmic growth of the average multiplicity (12).

In a more general case, using the well-known limitation on the asymptotic behaviour of the diffraction slope parameter in quantum field theory[/16/] we get from eq. (12) the following upper bound on an asymptotic growth of the average multiplicity with increasing energy

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[x/] Eq. (12) disregards particle spins and isospins as well as the resonance contribution to the multi-particle production. In reality another fact is expected in the first term of eq. (12) which takes into account effectively the necessary number of degrees of freedom.

$$\bar{n}(s) \leq const \cdot \frac{\ln^2 s}{\sigma^{tot}} . \qquad (13)$$

Relation (12) gives an interesting physical interpretation of the growth of the effective radius of strong interactions with increasing energy/17/. Really, the diffraction slope parameter $A(s)$ determines the "visible" sizes of hadrons in elastic collisions. On the other hand, as it follows from eq. (9) the total cross section is determined by the minimal distance $R_0 \approx 1/\sqrt{t_{eff}}$ up to which the "point-like" behaviour of the total differential cross section of two hadron collision (6) has sense. One can see from eq. (12) that

$$A(s) \approx R^2 = \bar{n}(s) R_0^2 . \qquad (14)$$

Thus, under the condition of constancy of the total cross section the increasing of the effective radius of strong interaction R with increasing energy is due to an expansion of a cloud of secondary particles around the colliding hadrons.

Let us discuss now the question concerning the relation between the asymptotic behaviour of the average multiplicity and the inelasticity parameter $k(s)$ which defined the ratio of the average energy of secondary particles and the total energy of colliding hadrons[x].

Obviously, in the framework of an assumption on the statistical independence of secondary particles we have

$$k(s) = \bar{n}(s) \langle \frac{k_0}{\sqrt{s}} \rangle . \qquad (15)$$

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[x] Recently this question was considered in paper/18/.

Assume now that in the high-energy limit there exists the finite distribution of a number of particles produced in a given interval of the variable $x = \frac{2k_0}{\sqrt{s}}$ /10/, i.e.

$$d\bar{n}(s,x) \underset{\substack{s\to\infty \\ t-\text{fixed}}}{\to} \rho(x)dx, \quad \epsilon < x \leq 1, \tag{16}$$

where $\epsilon = \frac{2m_*}{\sqrt{s}}$ and m_* is some characteristic mass.

Hence in accordance with eq. (15) we get

$$\bar{n}(s) \to \int_\epsilon^1 \rho(x)\,dx$$
$$k(s) \to \frac{1}{2}\int_\epsilon^1 x\rho(x)\,dx \tag{17}$$

at $s \to \infty$.

It is easy to see that the asymptotic behaviour of the average multiplicity at high energies is determined by the behaviour of the distribution function $\rho(x)$ near the point $x=0$.

For example, if the average multiplicity grows as $(\ln s)^{1+\gamma}$ at $s\to\infty$ we get $\rho(x) \approx \frac{1}{x}(-\ln x)^\gamma$ at $x \approx 0$.

In the general case from requirement of the finitness of inelasticity it follows that the function $x\rho(x)$ should be integrable up to $x=0$.

One can easy get from eqs. (17) the following formula

$$k(s) = k(s_0) + m_* \int_{s_0}^{s} \frac{1}{\sqrt{s'}} \frac{d\bar{n}}{ds'}\,ds'. \tag{18}$$

Thus as it follows from this equation the inelasticity parameter at high energy is determined by the behaviour of the average multiplicity at all the foregoing energies.

Consider now the particular example when the average multiplicity grows beginning from some s_0 logarithmically with increasing energy. From eq. (18) we find that

$$k = \lim_{s \to \infty} k(s) \geq \epsilon_0 \frac{d\bar{n}}{d\xi}, \quad \xi = \ln \frac{s}{s_0}, \tag{19}$$

where

$$\epsilon_0 = \frac{2m_*}{\sqrt{s_0}} = x_{max} \leq 1.$$

So, we can see that in this particular case the upper bound on the growth of the average multiplicity with increasing energy is determined by the ratio of two parameters: the inelasticity k and the maximal fraction of the energy of an initial hadron on one secondary particle ϵ_0.

It should be noticed that in the framework of the statistical independence consideration when the particle production is described by the Poisson law, both the parameters k and ϵ_0 cannot deviate considerably from zero, i.e. $k \ll 1$, $\epsilon_0 \ll 1$.

Recently there has been suggested a number of models of multiparticle production where an assumption on the Poisson character of the distribution over the number of secondary particles was made in some or other form /19/.

The results obtained in these models depend crucially on the concrete conditions under which the particle production processes are assumed to obey the Poisson law. So in some cases the assumption of the Poisson distribution leads to the contradictions with some rigorous results of quantum field theory.

For example, as is well known, under some conditions from general principles of quantum field theory it follows that [16]

$$P_0(s) \equiv \frac{\sigma^{el}}{\sigma^{tot}} \geq const \cdot \frac{\sigma^{tot}}{\ln^2 s} . \quad (20)$$

If one assumes now that the integrated probabilities of multi-particle production are described by the Poisson distribution one gets $P_0(s) = \exp[-\bar{n}(s)]$ and hence the average multiplicity can not grow faster than $\ln \ln s$ [20].

In our approach the integrated probability of an elastic channel is given by $P_0(s) = \frac{1}{2\bar{n}(s)}$ and using formula (20) we just obtain the upper bound on the asymptotic growth of the average multiplicity (13) derived above.

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