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GENERAL ANALYSIS OF THE MESON-NUCLEON SCATTERING BASED ON THE FUNCTIONAL INTEGRATION METHOD

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Introduction

Well known are the results achieved by the quantum field theory with the help of the ordinary perturbation theory. But the difficulties which arise when one attempts to describe, for example, strong interactions are known not to a less extent. These difficulties have resulted in the development of a number of methods which go beyond the scope of perturbation theory. Among those one of the most distinguished is the functional integration method in quantum field theory[1]. This method has further been developed in papers[2] and later was successfully applied to the study of the infrared asymptotics in quantum electrodynamics and also in the research of the processes of hadron interactions at high energies (see e.g.[3,4]).

The principal merit of the functional integration method is the possibility of obtaining expressions of closed form for the complete Green functions and deriving with their help a number of general relations. Thus, the representation of the two-particle Green function in the form of the continuous integral gave the authors of paper[5] the opportunity to get the expressions of closed form for the elastic and inelastic two - nucleon scattering amplitudes, which may be used, for example, for the development of various approximations.

The present paper is devoted to the derivation of the expressions in the form of path integrals for the vertex function and the amplitude of the elastic meson-nucleon scattering.

1. Complete Nucleon Green Function

We consider the quantum-field model of the interaction between scalar nucleons and scalar mesons $L_{int} = g : \psi^* \psi \phi :$ x/.

Let us introduce the single-particle Green function of a nucleon in the external field $\phi(x)$ without taking into account the virtual effects. This function satisfies the following equation

$$[\Box + m^2 - g\,\phi(x)]\,G(x,y\,|\,\phi) = \delta(x-y) \tag{1.1}$$

the solution of which can be written in the form of continuous integral

$$G(x,y\,|\,\phi) = i\int_0^\infty d\tau\, e^{-i\tau m^2} \int [\delta\nu]_0^\tau \exp\{ig\int_0^\tau d\xi\, \phi[x + 2\int_0^\xi \nu(\eta)\,d\eta]\}\,\delta[x-y+2\int_0^\tau \nu(\eta)\,d\eta], \tag{1.2}$$

where

$$[\delta\nu]_0^\tau = \frac{\delta\nu \exp[-i\int_0^\tau \nu^2(\eta)\,d\eta]}{\int \delta\nu \exp[-i\int_0^\tau \nu^2(\eta)\,d\eta]} \tag{1.3}$$

x/ The obtained results can be easily generalized to the case of the model $L_{int} = ig : A_\mu \psi^* \overleftrightarrow{\partial}_\mu \psi : + \frac{g^2}{2} : A_\mu^2 \psi^* \psi :$

and $\delta\nu$ is the volume element in the functional space.

Performing the Fourier transforms we obtain

$$G(p,q\,|\,\phi) = \int dx\,dy\, e^{ipx-iqy}\, G(x,y\,|\,\phi) = i\int_0^\infty d\tau\, e^{-i\tau(m^2-q^2)} \int dx\, e^{ix(p-q)} \cdot \int [\delta\nu]_0^\tau \exp\{ig\int_0^\tau d\xi\, \phi[x - 2q\xi + 2\int_0^\xi \nu(\eta)\,d\eta]\}. \tag{1.4}$$

Now using the expression (1.4) we can find the complete Green function of a nucleon in the external field. We shall operate with the notations of the following type

$$\int DAB = \int dx_1\, dx_2\, D(x_1-x_2)A(x_1)B(x_2)$$
$$(DA)_{x_1}^{\{\alpha\}} = \int dx_2\, D(x_1-x_2)A(x_2,\{\alpha\}). \tag{1.5}$$

where $\{\alpha\}$ is the set of those arguments of A which do not take part in the contraction x/.

Let us denote, for example, the operator

$$[\exp\frac{i}{2}\int D\frac{\delta^2}{\delta\phi^2}] = [\exp\frac{i}{2}\int dx_1\,dx_2\, D(x_1-x_2)\frac{\delta^2}{\delta\phi(x_1)\,\delta\phi(x_2)}] \tag{1.6}$$

with the help of which we obtain the expression for the complete Green function

$$\bar{G}(p,q\,|\,\phi) = [\exp\frac{i}{2}\int D\frac{\delta^2}{\delta\phi^2}]\,G(p,q\,|\,\phi)\,S_0(\phi). \tag{1.7}$$

x/ In the case when $(DA)_x^{\{\alpha\}}$ as a function of x is included in the argument of a functional we shall omit, for the sake of simplicity, the symbol x.

Here $S_0(\phi)$ denotes the vacuum expectation value of the S-matrix in the presence of the external scalar field $\phi(x)$. Substituting (1.4) in (1.7) we obtain

$$\bar{G}(p,q\,|\,\phi) = i\int_0^\infty d\tau\, e^{i\tau(q^2-m^2)} \int dx\, e^{ix(p-q)} \int[\delta\nu]_0^\tau E(\{a\}\,|\,\phi), \qquad (1.8)$$

where

$$E(\{a\}|\phi) = \left[\exp\frac{i}{2}\int D\frac{\delta^2}{\delta\phi^2}\right]\exp\left\{ig\int_0^\tau d\xi\,\phi\left[x-2q\xi+2\int_0^\xi \nu(\eta)\,d\eta\right]\right\}S_0(\phi) = \left[\exp\frac{i}{2}\int D\frac{\delta^2}{\delta\phi^2}\right]e^{ig\int\phi j}S_0(\phi) \qquad (1.9)$$

$$\{a\} \equiv \{x,\tau,q;\nu\}.$$

In formula (1.9) we have employed the following notations

$$j(z,\{a\}) = \int_0^\tau d\xi\,\delta\left[x-z-2q\xi+2\int_0^\xi \nu(\eta)\,d\eta\right]$$
$$\int\phi j = \int dz\,\phi(z)\,j(z,\{a\}) = \int_0^\tau d\xi\,\phi\left[x-2q\xi+2\int_0^\xi\nu(\eta)\,d\eta\right] \qquad (1.10)$$

Using the techniques developed in the paper /5/ we can write (1.9) in the following form

$$E(\{a\}|\phi) = \exp\left\{ig\int\phi j - \frac{ig^2}{2}\int Dj^2 + i\Pi[\phi - g(Dj)^{\{a\}}]\right\}, \qquad (1.11)$$

where

$$\left[\exp\frac{i}{2}\int D\frac{\delta^2}{\delta\phi^2}\right] S_0(\phi) = e^{i\Pi[\phi]}. \qquad (1.12)$$

Note that $\Pi[\phi]$ corresponds to the sum of all the connected diagrams with the arbitrary number of closed nucleon loops and external meson lines, all possible internal meson pairing being taken into account.

Let us introduce the quantity

$$E_0 = E(\{a\}) = \exp\left\{-\frac{ig^2}{2}\int Dj^2 + i\Pi[-g(Dj)^{\{a\}}] - i\Pi[0]\right\}. \qquad (1.13)$$

Then

$$E(\{a\}|\phi) = e^{i\Pi[\phi]}E_0 R[\phi], \qquad (1.14)$$

where

$$R[\phi] = R(x,\{a\}|\phi) =$$
$$= \exp\left\{ig\int\phi j + i\Pi[\phi - g(Dj)^{\{a\}}] - i\Pi[\phi] - \right. \qquad (1.15)$$
$$\left. - i\Pi[-g(Dj)^{\{a\}}] + i\Pi[0]\right\} = \exp\left\{ig\int dx_1\,\phi(x_1)T(x_1,\{a\}|\phi)\right\}.$$

It is not difficult to convince oneself that in the expression (1.14) the terms corresponding to vacuum renormalization, radiative corrections to the nucleon line and effects of interaction of the nucleon with the external field, are factorized.

The functional $T(x_1,\{a\}|\phi)$ can be expressed in terms of the polarization operator of the meson field

$$P(x_1,x_2\,|\,\phi) = -\frac{\delta^2}{\delta\phi(x_1)\delta\phi(x_2)}\Pi[\phi]. \qquad (1.16)$$

In fact, let us perform the Fourier transform in the functional space

$$\Pi[\phi] = \int\delta\eta\,\Pi[\eta]\,e^{-i\int\phi\eta}. \qquad (1.17)$$

Then

$$\Pi[\phi - g(Dj)^{\{a\}}] - \Pi[\phi] - \Pi[-g(Dj)^{\{a\}}] + \Pi[0] =$$
$$= \int\delta\eta\,\Pi[\eta]\left[e^{-i\int\phi\eta} - 1\right]\left[e^{ig\int Dj\eta} - 1\right] =$$

$$g\int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int \delta\eta\, \Pi[\eta]\, e^{-i\lambda_1\int\phi\eta \;\; ig\lambda_2\int Dj\eta} \int\phi\eta \int Dj\eta \qquad (1.18)$$

$$g\int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dx_1\, dx_2\, \phi(x_1)\, (Dj)^{\{a\}}_{x_2}\, P(x_1, x_2 \;\; \lambda_1\phi - g\lambda_2 (Dj)^{\{a\}}).$$

Substituting (1.18) in (1.15) we obtain the expression for the functional $T(x_1, \{a\}|\phi)$

$$T(x_1,\{a\}|\phi) = j(x_1,\{a\}) - \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dx_2 (Dj)^{\{a\}}_{x_2}$$
$$P(x_1, x_2 \;\; \lambda_1\phi - g\lambda_2 (Dj)^{\{a\}}) \int dy\, Z(x_1, y, \{a\}|\phi)\, j(y, \{a\}) \qquad (1.19)$$

in which we have introduced a new functional

$$Z(x_1, y, \{a\}|\phi) = \delta(x_1 - y) + \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dx_2 D(x_2 - y)\cdot \qquad (1.20)$$

$$\cdot P(x_1, x_2 \;\; \lambda_1 \phi - g\lambda_2 (Dj)^{\{a\}}).$$

2. The Vertex Function and the Meson-Nucleon Scattering Amplitude

To find the vertex function and the scattering amplitude it is necessary, first of all, to pick out two poles and to pass on the mass shell in the expression (1.8). Omitting the factor $\exp\{i\Pi[\phi]\}$ and subtracting the free part of the Green function, we obtain

$$iF(p, q\,|\phi) = \lim_{p^2, q^2 \to m^2} (p^2 - m^2)(q^2 - m^2)\, i\int_0^\infty d\tau\, e^{i\tau(q^2 - m^2)}\cdot$$

$$\cdot \int dx\, e^{ix(p-q)} \int[\delta\nu]_0^\tau\, E_0\, ig \int_0^1 d\gamma \exp\{ig\gamma\int dx_1 \phi(x_1) T(x_1, \{a\}|\phi)\}\cdot \qquad (2.1)$$

$$\cdot \int dx_2\, \phi(x_2)\, T(x_2, \{a\}|\phi).$$

Using the expresssions (1.19) and (1..10)) we note that

$$\int dx_1 \phi(x_1) T(x_1, \{a\}|\phi) = \int_0^\tau d\xi \int dy\, \delta[x - y - 2q\xi + 2\int_0^\xi \nu(\eta)\, d\eta]\cdot$$
$$\cdot \int dx_1\, \phi(x_1)\, Z(x_1, y, \{a\}|\phi). \qquad (2.2)$$

Then after substituting (2.2) in (2.1) and replacing the ordinary and functional variables as follows

$$\tau \to \tau + \xi$$
$$x \to x + 2q\xi - 2\int_0^\xi \nu(\eta)\, d\eta \qquad (2.3)$$
$$\nu(\eta) \to \nu(\eta - \xi) - (p - q)\,\theta(\xi - \eta)$$

we obtain the expression

$$F(p, q\,|\phi) = \lim_{p^2, q^2 \to m^2} (p^2 - m^2)(q^2 - m^2) \int_0^\infty d\tau \int_0^\infty d\xi\,\cdot$$

$$\cdot\, e^{i\tau(q^2 - m^2) + i\xi(p^2 - m^2)} \int dx\, e^{ix(p-q)} \int[\delta\nu]^\tau_{-\xi}\, E_0\, ig\,\cdot \qquad (2.4)$$

$$\cdot \int_0^1 d\gamma \exp\{ig\gamma\int dx_1 \phi(x_1) T(x_1, \{a\}|\phi)\} \int dx_2 \phi(x_2) Z(x_2, x, \{a\}|\phi).$$

Performing the transition to the limit in (2.4) we have

$$F(p, q\,|\phi) = ig \int[\delta\nu]^{+\infty}_{-\infty} \int dx\, E^m_0(x; p, q; \nu)\, e^{ix(p-q)}\cdot$$
$$\cdot \int dx_1 \phi(x_1) Z^m(x_1, x; p, q; \nu|\phi) \int_0^1 d\gamma \exp\{ig\gamma\int dx_2 \phi(x_2) T^m(x_2, x; p, q; \nu|\phi)\} \qquad (2.5)$$

where E^m_0, Z^m and T^m are defined by the expressions (1.13), (1.20) and (1.19) accordingly, in which j of the formula (1.10) is replaced by

$$j^m(x - z; p, q; \nu) = \int_{-\infty}^{+\infty} d\zeta\, \delta[x - z - 2p\zeta\,\theta(-\zeta) - 2q\zeta\,\theta(\zeta) + \qquad (2.6)$$

$$+ 2\int_0^\zeta \nu(\eta)\, d\eta].$$

To find the vertex function $\Gamma(p,q;k)$ the expression (2.5) must be varied once with respect to ϕ.

$$(2\pi)^4\delta(q-p-k)\Gamma(p,q;k)=\int dy\, e^{iyk}\frac{\delta}{\delta\phi(y)}F(p,q|\phi)|_{\phi=0} \quad (2.7)$$

$$=ig\int[\delta\nu]_{-\infty}^{\infty}\int dx\, E_0(x;p,q;\nu)e^{ix(p-q)}\int dy\, e^{iky}Z_0^m(x,y;p,q;\nu),$$

where

$$Z_0^m(x,y;p,q;\nu)=Z^m(x,y;p,q;\nu|\phi=0)=\delta(x-y)\,\cdot$$

$$\cdot\int_0^1 d\lambda\int dz\, D(z-y)P(x,z|-\lambda g(D_j^m)^{\{x,p,q\}}). \quad (2.8)$$

Let us note that the functionals E_0^m and Z_0^m are invariant under translations, i.e. E_0^m does not depend at all on x, and Z_0^m depends only on the difference $(x-y)$

$$E_0^m=E_0^m(p,q;\nu)$$
$$(2.9)$$
$$Z_0^m=Z_0^m(x-y;p,q;\nu).$$

Then it follows from (2.7) the final expression for the vertex function

$$\Gamma(p,q;k)=ig\int[\delta\nu]_{-\infty}^{\infty}E_0^m(p,q;\nu)\int dz\, e^{-izk}Z_0^m(z;p,q;\nu). \quad (2.10)$$

Varying twice the expression (2.5) with respect to ϕ, it is not difficult to obtain the amplitude of the elastic meson-nucleon scattering. In fact,

$$(2\pi)^4\delta(q+r-p-k)F(p,k;q,r)=$$

$$=\int dy_1 dy_2\, e^{iy_1k-iy_2r}\frac{\delta^2}{\delta\phi(y_1)\delta\phi(y_2)}F(p,q|\phi)|_{\phi=0}=$$

$$=ig\int[\delta\nu]_{-\infty}^{\infty}E_0^m(p,q;\nu)\cdot\int dx\, dy_1\, dy_2\, e^{ix(p-q)+iy_1k-iy_2r}\,\cdot$$

$$\cdot\Big\{\frac{\delta Z^m(x,y_1;p,q;\nu|\phi)}{\delta\phi(y_2)}\Big|_{\phi=0}+\frac{\delta Z^m(x,y_2;p,q;\nu|\phi)}{\delta\phi(y_1)}\Big|_{\phi=0}+$$

$$+\frac{ig}{2}[Z_0^m(x-y_1;p,q;\nu)\cdot T_0^m(x-y_2;p,q;\nu)+ \quad (2.11)$$

$$+Z_0^m(x-y_2;p,q;\nu)T_0^m(x-y_1;p,q;\nu)]\Big\},$$

where

$$T_0^m=T^m(x,y_1;p,q;\nu|\phi=0) \quad (2.12)$$

and, because of the translation invariance, depends on $(x-y_1)$.

Introducing a translation invariant functional

$$Q^m(x-y_1,x-y_2;p,q;\nu)=\Big[\frac{\delta Z^m(x,y_1;p,q;\nu|\phi)}{\delta\phi(y_2)}+$$
$$(2.13)$$
$$+\frac{\delta Z^m(x,y_2;p,q;\nu|\phi)}{\delta\phi(y_1)}\Big]_{\phi=0}$$

we can write the expression for the scattering amplitude in the following form

$$F(p,k;q,r)=ig\int[\delta\nu]_{-\infty}^{+\infty}E_0^m(p,q;\nu)\int dz_1 dz_2\, e^{-ikz_1+irz_2}\,\cdot$$

$$\cdot\Big\{Q^m(z_1,z_2;p,q;\nu)+\frac{ig}{2}[Z_0^m(z_1;p,q;\nu)T_0^m(z_2;p,q;\nu)+ \quad (2.14)$$

$$+Z_0^m(z_2;p,q;\nu)T_0^m(z_1;p,q,\nu)]\Big\}.$$

The representation (2.14) can be used in the study of the asymptotic behaviour of the meson-nucleon scattering amplitude at high energies, and also in the investigation of the automodel behaviour of the nucleon structure functions /6/.

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