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ON $K^+ \to \mu^+ \; \nu_\mu \; \nu_\mu \; \bar{\nu}_\mu$ DECAY

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Бардин Д.Ю., Биленький С.М. E2-5881

О распаде $K^+ \to \mu^+ \nu_\mu \nu_\mu \bar{\nu}_\mu$

При различных предположениях о виде гамильтониана $\nu_\mu \nu_\mu$ - взаимодействия рассмотрен распад $K^+ \to \mu^+ \nu_\mu \nu_\mu \bar{\nu}_\mu$. Вычислен спектр и поляризация μ^+ -мезонов, а также полная вероятность распада. Изучение на опыте этого процесса позволило бы получить информацию о $\nu_\mu \nu_\mu$ - взаимодействии.

Сообщения Объединенного института ядерных исследований

Дубна, 1971

Bardin D.Yu., Bilenky S.M. E2-5881

On $K^+ \to \mu^+ \nu_\mu \nu_\mu \bar{\nu}_\mu$ Decay

The decay $K^+ \to \mu^+ \nu_\mu \nu_\mu \bar{\nu}_\mu$ is considered under different assumptions on the form of the $\nu_\mu - \nu_\mu$ interaction Hamiltonian. The μ^+ -meson spectrum and polarization as well as the total decay probability are calculated. An experimental study of this process would allow information on $\nu_\mu - \nu_\mu$ interaction to be obtained.

Communications of the Joint Institute for Nuclear Research.

Dubna, 1971

In refs. [1,2] attention was drawn to the fact that the available experimental data do not contradict the presence of even strong interaction between neutrinos. If the Hamiltonian is taken in vector form then by analysing the appropriate experimental data we may conclude[2] that

$$F \lesssim 2.10^6 \ G \ ,$$

where F is the $\nu - \nu$ interaction constant, and $G \approx 10^{-5} \frac{1}{M_p^2}$ is the ordinary weak interaction constant. It is obvious that the problem of interaction between neutrinos is of fundamental interest in the physics of weak interactions *). Special experiments, the purpose of which is to investigate this interaction, seem to be important. Many such experiments are discussed in ref. /2/. Here we consider in detail the process

$$K^+ \to \mu^+ + \nu_\mu + \nu_\mu + \bar{\nu}_\mu \qquad (1)$$

the study of which makes it possible to obtain information on $\nu_\mu - \nu_\mu$ interaction. We consider this decay under different assump-

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*) In ref. /3/ the $\nu - \nu$ interaction was introduced in the framework of nonlocal field theory. This made it possible to construct a gauge invariant renormalizable theory.

tions on the type of the $\nu_\mu - \nu_\mu$ interaction Hamiltonian. The energy spectrum and the μ^+ -meson polarization as well as the total decay probability are calculated.

Let us analyse the process (1) under the assumption that the $\nu_\mu - \nu_\mu$ interaction Hamiltonian has the forms:

$$\mathcal{H}_{V,A} = F\,(\bar{\nu}_\mu\, O_\alpha\, \nu_\mu)\,(\bar{\nu}_\mu\, O_\alpha\, \nu_\mu)\,, \tag{2}$$

where

$$O_\alpha = a\,\gamma_\alpha\,(1+\gamma_5) + b\,\gamma_\alpha\,(1-\gamma_5) \tag{3}$$

and

$$\mathcal{H}_i = F_i\,(\bar{\nu}_\mu\, O^i\, \nu_\mu)\,(\bar{\nu}_\mu\, O^i\, \nu_\mu)\,, \tag{4}$$

where the index i takes the values of either S , or P , or T ;

$$O^S = 1\,; \qquad O^P = \gamma_5\,; \qquad O^T_{\alpha\beta} = \sigma_{\alpha\beta}\,.$$

The diagram of process (1) is given in Fig. 1. In the lowest order of perturbation theory in the weak and $\nu - \nu$ interaction the matrix element of the process is

$$\langle f\,|\,S\,|\,i\rangle = \frac{2}{(2\pi)^6}\left(\frac{m_\mu}{p_{10}}\right)^{1/2}\frac{G}{\sqrt{2}}\,F\,(2\pi)^4\,\delta\,(p_1+q-p)\,\times \tag{5}$$

$$[\,(\bar{u}\,(q_1)\,O\,u(-q_3))\,(\bar{u}\,(q_2)\,O\,\frac{1}{\hat{q}}\,\gamma_\beta(1+\gamma_5)\,u(-p_1)) - (q_1 \leftrightarrows q_2)\,]\langle 0\,|\,J_\beta|\,p\,\rangle\,.$$

Here q_1 , q_2 , p_1 and q_3 are the four-momenta of final neutrinos, μ^+ meson and antineitrino, p the four-momentum of the initial K^+ meson, m_μ the μ^+ -meson mass, J_β the weak interaction current, and

$$q = q_1 + q_2 + q_3\,. \tag{6}$$

We note that the factor 2 in expression (5) is associated with two possible chronological contractions of the $\bar{\nu}_\mu$ operator, entering the weak interaction Hamiltonian. The matrix element of the hadron current in eq. (5) is

$$< 0 \mid J_\beta \mid p > = \frac{1}{\sqrt{2p_0}} \frac{1}{(2\pi)^{3/2}} \, i f_K \, p_a \quad , \tag{7}$$

where f_K is the $K^+ \to \mu^+ \nu_\mu$ decay constant.

Integrating over the momenta of final neutrinos and summing over their polarization states we get the following expression for the μ^+ -meson density matrix (the K^+ -meson rest system):

$$\rho^i = - \frac{m_\mu G^2 F_i^2}{24(2\pi)^6 \, i} f_K^2 \frac{(q^2)^2}{m_K p_{10}} A_K \Lambda(-p_1) \hat{p} (1+\gamma_5)(1 - \frac{i m_\mu \hat{q}}{q^2}) \Lambda(-p_1). \tag{8}$$

Here $\Lambda(-p_1) = \frac{-\hat{p}_1 + i m_\mu}{2 i m_\mu}$, m_K is the K -meson mass. The values of the constants A_i for the types of $\nu_\mu - \nu_\mu$ interaction considered are given in the table

Table

$\nu_\mu - \nu_\mu$ interaction mode	A
Vector and axial (Hamiltonian (2))	$8a^2 (2a^2 + b^2)$
Scalar (pseudoscalar)	$3/16$
Tensor	9

Note that in obtaining eq. (8) we have used the relations:

$$\gamma_a \gamma_\rho \gamma_\beta (1+\gamma_5) \ldots \gamma_a \gamma_{\rho'} \gamma_\beta (1+\gamma_5) = 4\delta_{\rho\rho'} \gamma_\sigma (1+\gamma_5) \ldots \gamma_\sigma (1+\gamma_5) \tag{9}$$

$$\gamma_a \gamma_\rho \gamma_\beta (1+\gamma_5) \ldots \gamma_a \gamma_{\rho'} \gamma_\beta (1-\gamma_5) = 4\gamma_{\rho'} (1+\gamma_5) \ldots \gamma_\rho (1-\gamma_5) \, . \tag{10}$$

These relations hold in both cases when the matrices written down enter the same product which may contain in addition an arbitrary number of other matrices γ and when these matrices are included in different brackets. Eq. (9) and (10) can be easily obtained by means of the following relations:

$$\gamma_a \gamma_\rho \gamma_\beta (1 \pm \gamma_5) = (\mp e_{a\rho\beta\sigma} + \delta_{a\rho}\delta_{\beta\sigma} - \delta_{a\beta}\delta_{\rho\sigma} + \delta_{a\sigma}\delta_{\rho\beta})\gamma_\sigma (1 \pm \gamma_5) . \quad (11)$$

With the aid of eq. (11) we can also derive the relations:

$$\sigma_{a\beta}\gamma_\rho (1+\gamma_5)\sigma_{\mu\nu} \ldots \sigma_{a\beta}\gamma_\sigma (1+\gamma_5)\sigma_{\mu\nu} = \quad (12)$$

$$= 16 \, [\, 2\delta_{\rho\sigma}\gamma_a (1+\gamma_5) \ldots \gamma_a (1+\gamma_5) -$$

$$- \gamma_\rho (1+\gamma_5) \ldots \gamma_\sigma (1+\gamma_5) + 2\gamma_\sigma (1+\gamma_5) \ldots \gamma_\rho (1+\gamma_5) \quad (13)$$

$$\sigma_{a\beta}\gamma_\rho (1+\gamma_5)\sigma_{\mu\nu} \ldots \sigma_{a\beta}\gamma_\sigma (1-\gamma_5)\sigma_{\mu\nu} = 0$$

which simplify essentially the calculations when the $\nu - \nu$ interaction is of the tensor form.

From eq. (8) we get the following expression for the μ^+ -meson spectrum in the rest system of initial K^+ -mesons:

$$dw_i = \frac{1}{6(2\pi)^5} G^2 F_i^2 f_K^2 m_K^7 A_i (1+r^2-2x)(x^2-r^2)^{1/2} [(1-2x)x + r^2]\, dx \, . \quad (14)$$

Here $r = \frac{m_\mu}{m_K}$, $x = \frac{E}{m_K}$ (E is the total energy of a μ^+ -meson) and the A_i values for the considered types of $\nu_\mu - \nu_\mu$ interaction are given in the table.

The maximum total μ^+ -meson energy in the K^+ -meson rest system is

$$E_{max} = \frac{m_K^2 + m_\mu^2}{2m_K} .$$

Thus, the variable x changes in the limits

$$r \le x \le \frac{1}{2}(1+r^2) . \quad (15)$$

The spectrum of μ^+ meson resulting from the decay (1) is given in Fig. 2. Using eq. (14) we obtain the following expression for the total decay probability

$$w_i = \frac{1}{24(2\pi)^5} G^2 F_i^2 f_K^2 m_K^7 A_i \left[\frac{1}{60}(1-r^2)^3 + \frac{1}{40} r^2 (1-r^2)(15r^2+9r^4-r^6-3) + r^6 \ln r \right] . \tag{16}$$

For comparison we give the well-known expression for the decay probability $K^+ \to \mu^+ \nu_\mu$

$$w_{K\to\mu\nu} = \frac{1}{8\pi} G^2 f_K^2 m_\mu^2 m_K (1-r^2)^2 . \tag{17}$$

From eqs. (16) and (17) we get

$$\frac{w_i}{w_{K\to\mu\nu}} = 2{,}94 . 10^{-5} A_i F_i^2 m_K^4 . \tag{18}$$

As is seen from this relation the decay (1) is strongly suppressed (smallness of the phase space). This makes appropriate experiments very difficult. However, if using high-efficiency γ -quantum detectors one suppresses the background induced by the decay

$$K^+ \to \mu^+ + \nu_\mu + \gamma$$

it would be possible to lower essentially the upper boundary of the $\nu_\mu - \nu_\mu$ interaction constant. It is obvious that μ^+ -meson polarization measurements would allow to obtain additional information on the process (1).

We give the expressions for the μ^+ -meson polarization vector which are calculated by means of the density matrix (8). Thus we have

$$\vec{P} = \vec{\beta} \frac{1-2x}{1-2x+\frac{r^2}{x}} . \tag{19}$$

Here $\vec{\beta} = \frac{\vec{p}}{E}$ is the μ^+ -meson velocity in the K -meson rest system. Let us write the polarization vector in the form $\vec{P} = P\vec{n}$, where $\vec{n}$ is the unit vector in the direction of the μ^+ -meson momentum. Fig. 3 gives the polarization P as a function of energy.

For the maximum μ^+ -meson energy the polarization P is -1. For $x = \frac{1}{2}$, i.e. $E = \frac{1}{2} m_K$ the polarization vanishes. In the region $E < \frac{1}{2} m_K$ the polarization is positive.

In conclusion we make the following remark. As is seen from expressions (14) and (19) for all the considered $\nu_\mu - \nu_\mu$ interaction Hamiltonians the spectrum shape and μ^+ -meson polarization are independent of the interaction mode. It is not difficult to see that this property is also valid for a general local $\nu_\mu - \nu_\mu$ interaction without derivatives and is a consequence of the V-A structure of the usual weak interaction Hamiltonian.

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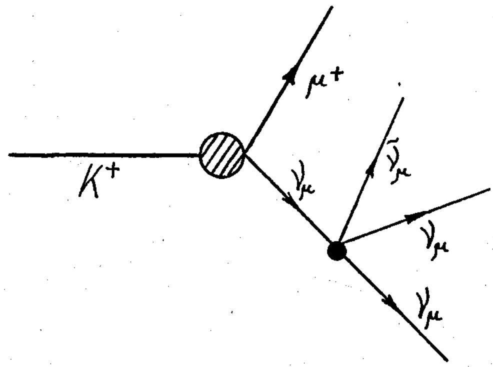

Fig. 1.

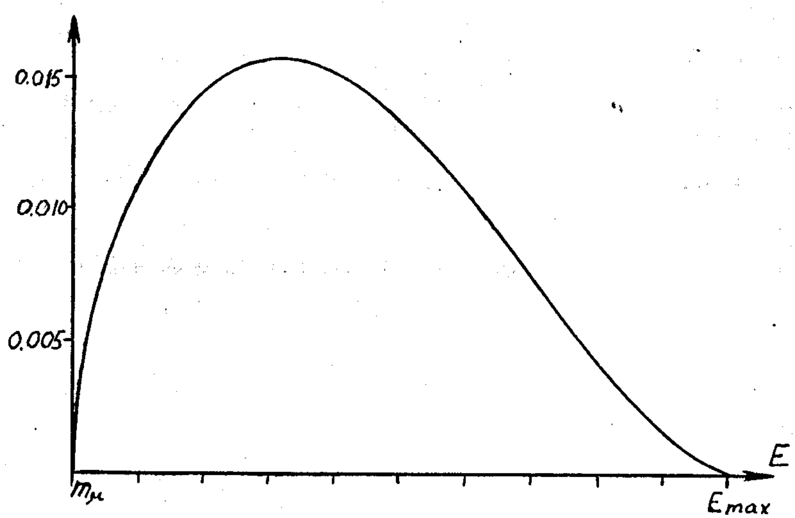

Fig. 2. μ^+ meson spectrum ($E_{max} = \dfrac{m_K^2 + m_\mu^2}{2m_K}$, the quantity $\dfrac{dw_i}{dx} \Big/ \dfrac{G^2 F_i^2 f_K^2 m_K^7 A_i}{6(2\pi)^5}$ is reconed on the ordinate axis).

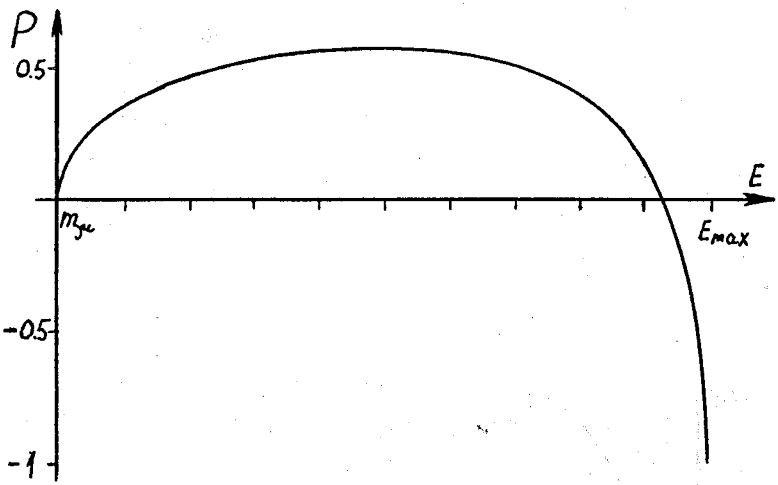

Fig. 3. μ^+ meson polarization as a function of the total energy E .