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ON THE REPRESENTATION OF SCATTERING AMPLITUDES AS THE CONTINUAL INTEGRALS OVER PATHS IN QUANTUM FIELD THEORY

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I. Introduction

In a number of recent papers the problem of the validity of the eikonal approximation in description of high-energy particle scattering in the framework of quantum field theory was discussed from various points of view [1].

The investigation of the quasipotential equation for an elastic scattering amplitude in quantum field theory [2] has shown that the validity of the eikonal approximation can be considered as a consequence of the assumption on smoothness of the local quasipotential [3].

We notice, that assumption on smoothness of the local quasipotential has been first done in paper [4] in connection with an analysis of the Orier's behaviour of large angle scattering at high energies.

The attempts for the theoretical explanation of the smoothness property of the local quasipotential were presented in papers [5] on the basis of probability analysis of the multiparticle processes in two-particle collisions, and in papers [6] in the framework of the coherent state model where high-energy hadron scattering is considered as an interaction of two complex systems with an infinite number of internal degrees of freedom [*)].

There is an interesting problem of justification of the eikonal approximation in the framework of various field-theoretical model of particle interaction and the familiar question on the status of the smoothness hypothesis of the two-particle quasipotential at high energies.

The great number of papers /8-17/ was devoted to the studying of the eikonal or Glauber representation for a scattering amplitude in the simplest models of quantum field theory.

In papers /8,9,11-16/ there was investigated essentially an asymptotic behaviour of a sum of the perturbation theory diagrams of the ladder type (with all the possible intersections of exchange meson lines between two nucleons) in the limit of high energies at fixed momentum transfers. It was shown that a sum of the principal asymptotic terms tends in the limit $s \to \infty$, $t =$ fixed, to the sum of the quasipotential graphs for the two-particle scattering amplitude.

It should be noticed that in papers /18/ some additional extra terms in asymptotic behaviour of some special diagrams of high orders have been found which can in principle change the results mentioned above in the simplest models such as the model of scalar nucleons with a scalar exchange.

*) We note also the recent paper /7/, in which the coherent state model is discussed in connection with the dynamical composite model of hadrons.

In papers /9-11/ in studying these problems the functional integration methods in quantum field theory were used. The important result of these studies was the formulation of the straight-line path approximation which gives an effective method of investigation of an asymptotic behaviour of scattering amplitudes in the framework of the functional integration technique. We note that the straight-line path approximation is a generalization of the so-called " $k_i k_j = 0$ " approximation proposed in papers /19-21/ in connection with the study of the infra-red asymptotics in quantum electrodynamics /22-23/.

The investigation of the radiative effects in two-particle scattering at high energies by means of the straight-line path approximation in papers /24-27/ leads to the qualitative understanding of the smoothness property of the complex quasipotential of two particles.

It should be stressed that the most hard problem is the role of the vacuum polarization effects, or speaking in the perturbation theory language, the contribution of diagrams with the closed nucleon loops. In this direction only some preliminary results /1/ are available.

We should stress here that an investigation of the structure of scattering amplitudes in the framework of the functional integration methods in general form presents up to now yet non-solved problem /28-30/.

In this paper we give the derivation of the representation of scattering amplitudes as the continual integrals over the particle paths for simple models of quantum field theory such as the models of scalar nucleons interacting with a scalar or a neutral vector field.

In particular case of the model with the scalar exchange the representation for two-nucleon scattering amplitude has the form:

$$F(p_1 p_2; q_1 q_2) = \int [\delta \nu_1]_{-\infty}^{\infty} E^{(1)}(p_1 q_1 | \nu_1) \int [\delta \nu_2]_{-\infty}^{\infty} E^{(2)}(p_2 q_2 | \nu_2) \cdot$$

$$\cdot \int dx\, e^{ix\Delta} g^2 D^*_{12}(x; p_i q_i | \nu_i) \int_0^1 d\gamma\, e^{-i\gamma g^2 \int D^*_{12} i_1 i_2} \qquad (1.1)$$

$$\Delta = (p_1 - q_1) = -(p_2 - q_2),$$

where the functional $E^{(i)}$ depends on the variables of the i-th nucleon and corresponds to the radiative effects, whereas the functional D^*_{12} depends on the variables of both nucleons and the relative coordinate $x = \frac{1}{2}(x_1 - x_2)$ and describes the interaction between nucleons.

All the functionals in eq. (1.1) can be expressed through the Green function of the meson field in the presence of the external sources

$$i_i = \int_{-\infty}^{\infty} d\zeta\, \delta[x - z + 2p_i \zeta \theta(\zeta) + 2q_i \zeta \theta(-\zeta) + 2\int_0^{\zeta} \nu_i\, d\eta]. \qquad (1.2)$$

One can see that the sources (1.2) correspond to the point-like classical particles moving along the paths which are determined by the functional variables $\nu_i(\eta)$.

2. The Construction of the Two-Particle Green Function

For simplicity we consider first the model of scalar nucleons, interacting with a scalar meson, with the interaction Lagrangian of the form $L_{int} = g : \psi^+ \psi \phi$:

The generalization of results to the model of scalar nucleons interacting with a neutral vector field will be given later.

The one-particle Green function of the nucleon in the given external scalar field $\phi(x)$ satisfies the equation

$$(\Box + m^2 - g\,\phi(x))\, G(xy|\phi) = \delta(x - y). \qquad (2.1)$$

The formal solution of eq. (2.1) can be represented by means of the functional integral /20/

$$G(xy|\phi) = i\int_0^{\infty} d\tau\, e^{-i\tau m^2} \int [\delta\nu]_0^{\tau} \exp\{ig \int_0^{\tau} d\xi\, \phi[x + 2\int_0^{\xi} \nu(\eta)\, d\eta]\} \cdot$$

$$\cdot\, \delta[x - y + 2\int_0^{\tau} \nu(\eta)\, d\eta], \qquad (2.2)$$

where

$$[\delta\nu]_{\tau_1}^{\tau_2} = \frac{\delta\nu \exp[-i \int_{\tau_1}^{\tau_2} \nu^2(\eta)\, d\eta]}{\int \delta\nu \exp[-i \int_{\tau_1}^{\tau_2} \nu^2(\eta)\, d\eta]} \qquad (2.3)$$

and $\delta\nu$ is a volume element of the functional space of the four-dimensional functions $\nu(\eta)$ defined on the interval $\tau_1 \le \eta \le \tau_2$.

The Fourier transform of the Green function (2.2) has the following form

$$G(pq|\phi)=\int dx\,dy\,e^{ipx-iqy}\,G(xy|\phi)=$$

$$=i\int_0^\infty d\tau\, e^{-i\tau(m^2-p^2)}\int dx\, e^{ix(p-q)}\int[\delta\nu]_0^\tau \exp[ig\int_0^\tau d\xi\,\phi(x+2p\xi+2\int_0^\xi \nu(\eta)\,d\eta)]. \tag{2.4}$$

Using the expression (2.4) we can find the two-particle Green function of nucleons in the form:

$$G(p_1q_1;p_2q_2)=[\exp\frac{i}{2}\int D\,\frac{\delta^2}{\delta\phi^2}]\cdot G(p_1q_1|\phi)\cdot G(p_2q_2|\phi)S_0(\phi)|_{\phi=0} \tag{2.5}$$

where

$$[\exp\frac{i}{2}\int D\,\frac{\delta^2}{\delta\phi^2}]\equiv \exp[\frac{i}{2}\int dx_1\,dx_2\,D(x_1-x_2)\,\frac{\delta^2}{\delta\phi_{x_1}\delta\phi_{x_2}}] \tag{2.6}$$

and $S_0(\phi)$ is the S-matrix averaged over the nucleon vacuum fluctuations in the presence of the external field ϕ. As is known, $S_0(\phi)$ can be written as

$$S_0(\phi)=\exp[i\pi(\phi)], \tag{2.7}$$

where the functional $\pi(\phi)$ in the models considered here corresponds to the sum of the connected diagrams, with one closed nucleon loop and an arbitrary number of external meson tails.

Introduce the notation

$$\int j_i\phi\equiv \int dz\,\phi(z)\,j(x_i-z;p_i;\tau_i|\nu_i), \tag{2.8}$$

where

$$j(x_i-z;p_i;\tau_i|\nu_i)=\int_0^{\tau_i} d\xi\,\delta(x_i-z+2p_i\xi+2\int_0^\xi \nu_i(\eta)\,d\eta). \tag{2.9}$$

Using these notations we can rewrite the expression for the two-particle Green function (2.5) in the following simple form:

$$G(p_1q_1;p_2q_2)=i^2\int_0^\infty d\tau_1\,d\tau_2\,e^{i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)}\int dx_1\,dx_2\,e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)}\cdot$$

$$\cdot\int[\delta\nu_1]_0^{\tau_1}[\delta\nu_2]_0^{\tau_2}\,E(x_i;p_i;\tau_i|\nu_i), \tag{2.10}$$

where

$$E\equiv[\exp\frac{i}{2}\int D\,\frac{\delta^2}{\delta\phi^2}]\cdot e^{ig\int\phi(j_1+j_2)}\cdot S_0(\phi)|_{\phi=0}. \tag{2.11}$$

Consider now in details the structure of the quantity E. Let us determine for each functional $A(\phi)$ the quantity

$$\bar{A}(\phi)=[\exp\frac{i}{2}\int D\,\frac{\delta^2}{\delta\phi^2}]\cdot A(\phi) \tag{2.12}$$

so that the average value $A(\phi)$ over the meson vacuum fluctuation is given by $\bar{A}=\bar{A}(\phi)|_{\phi=0}$.

Consider the average value of a product of two functionals

$$\overline{A\cdot B}=[\exp\frac{i}{2}\int D\,\frac{\delta^2}{\delta\phi^2}]\cdot A(\phi)\cdot B(\phi)|_{\phi=0}. \tag{2.13}$$

One can easily show that the following identity holds:

$$\overline{A\cdot B}=[\exp\frac{i}{2}\int D(\frac{\delta}{\delta\phi_1}+\frac{\delta}{\delta\phi_2})^2]\cdot A(\phi_1)B(\phi_2)|_{\phi_1=\phi_2=0}=$$

$$=[\exp i\int D\,\frac{\delta^2}{\delta\phi_1\,\delta\phi_2}]\,\bar{A}(\phi_1)\cdot\bar{B}(\phi_2)|_{\phi_1=\phi_2=0}= \tag{2.14}$$

$$= \bar{A}(i \int D \frac{\delta}{\delta\phi}) \cdot \bar{B}(\phi)|_{\phi=0} .$$

Choosing

$$A(\phi) = \exp[ig \int \phi(j_1+j_2)]; \quad B(\phi) = S_0(\phi) \tag{2.15}$$

we have

$$\bar{A}(\phi) = \exp[ig \int \phi \ (j_1+j_2) - ig^2/2 \int D(j_1+j_2)^2] \tag{2.16}$$

$$\bar{S}_0(\phi) = \exp[i \Pi(\phi)] , \tag{2.17}$$

where the quantity $\Pi(\phi)$ corresponds to the sum of all the connected Feynman diagrams with an arbitrary number of closed nucleon loops and internal meson lines (having in mind that the nucleons interact with the external field ϕ).

Using the identity (2.14) and eqs. (2.15–17) we find the foloowing expression for the quantity (2.11):

$$E = \exp[-\frac{ig^2}{2} \int D(j_1+j_2)^2 - g \int D(j_1+j_2) \frac{\delta}{\delta\phi}] \cdot e^{i\Pi(\phi)}|_{\phi=0} = \tag{2.18}$$

$$= \exp[-ig^2/2 \int D(j_1+j_2)^2 + i\Pi(-g \int D(j_1+j_2))] .$$

Expanding the quantity (2.18) in powers of the coupling constant and substituting the series in to eq. (2.10) we get, after performing simple functional integrations over ν_i , just the usual non-renormalized series of perturbation theory for the two-particle Green function.

We stress now the important fact, which will be used in the following. The expression (2.18) allows to separate in general form the contributions to an interaction between two nucleons (exchange effects), the self-interaction of the nucleons through the meson field (radiative corrections), and the vacuum renormalization.

Really, the first term, in the exponential in eq. (2.18) can be rewritten as

$$-ig^2/2 \int D(j_1+j_2)^2 = -ig^2 \int Dj_1 j_2 - ig^2/2 \int Dj_1^2 - ig^2/2 \int Dj_2^2 . \tag{2.19}$$

where the first term in the right hand side corresponds to the one-meson exchange between the nucleons and the rest terms lead to the radiative corrections.

Correspondingly, the second term in the exponential in eq. (2.18) can be represented in the form

$$\Pi = \Pi_{12} + \Pi_1 + \Pi_2 + \Pi(0) , \tag{2.20}$$

where

$$\Pi_{12} \equiv \Pi(-g \int D(j_1+j_2)) - \Pi(-g \int Dj_1) - \Pi(-g \int Dj_2) + \Pi(0) \tag{2.21}$$

$$\Pi_i \equiv \Pi(-g \int D j_i) - \Pi(0). \tag{2.22}$$

It can be shown that the quantities (2.21-22) may be expressed through the polarization operator of the meson field

$$P(x_1 x_2 | \phi) = -\frac{\delta^2}{\delta \phi_{x_1} \delta \phi_{x_2}} \Pi(\phi) \tag{2.23}$$

or through the full Green function of the meson field

$$G(x_1 x_2 | \phi) = G(x_1 - x_2) + \int dy_1 dy_2 G(x_1 - y_1) P(y_1 y_2 | \phi) G(y_2 - x_2) \tag{2.24}$$

in the presence of external sources. (see Appendix).

As a result we get for the quantity E the following expression

$$E = e^{i\Pi(0)} E^{(1)} E^{(2)} E^{(12)}, \tag{2.25}$$

where

$$E^{(i)} \equiv \exp[-ig^2/2 \int D_i^* \cdot j_i^2]; \quad i = 1,2 \tag{2.26}$$

$$E^{(12)} \equiv \exp[-ig^2 \int D_{12}^* \; j_1 \, j_2]. \tag{2.27}$$

Here we used the notations

$$D_i^* \equiv 2\int_0^1 d\sigma \int_0^\sigma d\lambda \, D(x_1 x_2 | -g\lambda \int D j_i); \quad i = 1,2 \tag{2.28}$$

$$D_{12}^* = \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \, D(x_1 x_2 | -g\lambda_1 \int D j_1 - g\lambda_2 \int D j_2). \tag{2.29}$$

We note, that D_i^* are connected with the Green function of the scalar meson interacting with the external sources, associated with the i-th nucleon, and D_{12}^* corresponds to the Green function of the scalar meson interacting simultaneously with the sources of both nucleons.

Thus, the quantity E which determines the two-particle Green function of nucleons is factorized into the terms, which describe correspondingly the interaction between two nucleons, the radiative connections and the vacuum renormalization.

3. The Representation for the Two-Nucleon Scattering Amplitude

The two-nucleon scattering amplitude is determined through the two-particle Green function (2.5) by

$$(2\pi)^4 \delta(p_1 + q_2 - q_1 - q_2) \, i F(p_1 p_2 ; q_1 q_2) =$$

$$= \lim_{p_i^2, q_i^2 \to m^2} \prod_{i=1,2} (p_i^2 - m^2)(q_i^2 - m^2) G(p_1 p_2 ; q_1 q_2). \tag{3.1}$$

We ignore the renormalization problem, which will be discussed in the following section, and drop in the right hand side of eq. (3.1) the factor $\exp i\Pi(0)$ so as it does not contribute to scattering processes.

As was mentioned in introduction the developing of the correct procedure of passing to the mass shell in constructing the scattering amplitude in general form, before any approximations are made, is very important. Many approximations being reasonable from a physical point of view, when are applied before the transition to the mass shell, disturb the positions of poles of the Green function and make all the procedure mathematically incorrect. In this paper we develop the method for extracting poles of the Green function, which is a generalization of the method, which has been used in papers /9,10,28/ in finding a scattering amplitude in the model of scalar nucleons interacting with a scalar meson in approximation where the contributions of the closed nucleon loops are neglected.

Using the expression for two-particle Green function (2.5) and eq. (2.9) and (2.25-29), we represent the definition of the scattering amplitude (3.1.) in the form:

$$(2\pi)^4\,\delta(p_1+q_2-q_1-q_2)\,iF(p_1p_2;q_1q_2)=\lim_{p_l^2,\,q_l^2\to m^2}\ \prod_{l=1,2}(p_l^2-m^2)(q_l^2-m^2)\,\cdot$$

$$\cdot\int_0^\infty d\tau_1\int_0^\infty d\tau_2\int_0^{\tau_1}d\xi_1\int_0^{\tau_2}d\xi_2\,e^{i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)}\int dx_1\,dx_2\,e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)}\cdot$$

$$\cdot\int[\delta\nu]_0^{\tau_1}[\delta\nu]_0^{\tau_2}\int dz_1dz_2\,\delta\Big(x_1-z_1+2p_1\xi_1+2\int_0^{\xi_1}\nu_1d\eta\Big)\,\delta\Big(x_2-z_2+2p_2\xi_2+2\int_0^{\xi_2}\nu_2\,d\eta\Big)\cdot \tag{3.2}$$

$$\cdot\,iF(z_1-z_2\,;\,i_1i_2),$$

where

$$F(z_1-z_2|i_1i_2)=g^2E^{(1)}E^{(2)}D^*_{12}\int_0^1 d\gamma\,e^{-i\gamma g^2\int D^*_{12}\,i_1i_2}\quad. \tag{3.3}$$

In getting the eq. (3.2-3) we have used the fact, that the free part of the Green function which is not connected with an interaction between nucleons can be subtracted by the formula:

$$E^{(12)}\to E^{(12)}-1=ig^2\int D^*_{12}\,i_1i_2\cdot\int_0^1 d\gamma\,e^{-i\gamma g^2\int D^*_{12}\,i_1i_1}\quad. \tag{3.4}$$

Using the identity

$$\int_0^\infty d\tau_1\int_0^\infty d\tau_2\int_0^{\tau_1}d\xi_1\cdot\int_0^{\tau_2}d\xi_2\ \ldots=\int_0^\infty d\xi_1\int_0^\infty d\xi_2\int_{\xi_1}^\infty d\tau_1\int_{\xi_2}^\infty d\tau_2\ \ldots \tag{3.5}$$

and making the change of the ordinary and the functional variables

$$\tau_l\to\tau_l+\xi_l$$

$$x_l\to x_l-2p_l\xi_l-2\int_0^{\xi_l}\nu_l\,d\eta \tag{3.6}$$

$$\nu_l(\eta)\to\nu_l(\eta-\xi_l)-(p-q)_l\,\theta(\eta-\tau_l)$$

we get the expression

$$(2\pi)^4\delta(p_1+q_2-q_1-q_2)F(p_1p_2;q_1q_2)=\lim_{p_l^2,\,q_l^2\to m^2}\ \prod_{l=1,2}(p_l^2-m^2)(q_l^2-m^2)\,\cdot$$

$$\cdot\int_0^\infty d\tau_1\,d\tau_2\,d\xi_1\,d\xi_2\,e^{i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)+i\xi_1(q_1^2-m^2)+i\xi_2(q_2^2-m^2)}\,\cdot \tag{3.7}$$

$$\cdot\int dx_1dx_2\,e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)}\int[\delta\nu_1]_{-\xi_1}^{\tau_1}[\delta\nu_2]_{-\xi_2}^{\tau_2}\,F(x_1-x_2|i_1i_2).$$

Going to the limit in eq. (3.7) and taking into account the translation symmetry of the quantity F x) we get the final result for the scattering amplitude:

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x) We remind that under translations $x_l\to x_l+h$ the functional variables of F namely the "current densities"

$$i_l\equiv\int_{-\xi_1}^{\tau_1}d\zeta\,\delta\Big[x_l-z+2p_l\zeta\theta(\zeta)+2q_l\zeta\theta(-\zeta)+2\int_0^{\zeta}\nu_l\,d\eta\Big] \tag{3.8}$$

undergo the transformation as well.

$$F(p_1 p_2; q_1 q_2) = \int [\delta\nu_1]_{-\infty}^{\infty} E^{(1)}(p_1 q_1|\nu_1) \int [\delta\nu_2]_{-\infty}^{\infty} \cdot E^{(2)}(p_2 q_2|\nu_2) \cdot$$

$$\cdot \int dx\, e^{ix\Delta} g^2 D^*_{12}(x; p_1 q_1|\nu_1) \int_0^1 d\gamma\, e^{-i\gamma g^2 \int D^*_{12} i_1 i_2}, \tag{3.9}$$

where $\Delta (= (p_1 - q_1) = -(p_2 - q_2)$; $x = \frac{1}{2}(x_1 - x_2)$ and all the quantities in this expression are the functionals of the limiting sources:

$$i_1 \equiv i(x_1 - z; p_1 q_1|\nu_1) = \int_{-\infty}^{\infty} d\zeta\, \delta[x_1 - z + 2p_1\zeta\theta(\zeta) + 2q_1\zeta\theta(-\zeta) + 2\int_0^{\zeta} \nu_1\, d\eta]. \tag{3.10}$$

One can note that the expression (3.10) determines the scalar density of the point-like particle, moving along the classical path $x_1(s)$, which depends on the proper time $s = 2m\zeta$ and satisfies the equation

$$m\frac{dx_1(s)}{ds} = p_1\theta(\zeta) + q_1\theta(-\zeta) + \nu_1(\zeta) \tag{3.11}$$

under the condition $x_1(0) = x_1$; $i = 1,2$.

Now we generalize the consideration given above to the case of the vector-exchange model with the interaction Lagragian

$$L_{int} = ig : A_\mu \psi^+ \overleftrightarrow{\partial}_\mu \psi : + g^2 : A^2_\mu \psi^+ \psi : \tag{3.12}$$

We shall not go into all the details but only summerize briefly the final results.

The scattering amplitude in this model is given by

$$F(p_1 p_2 : q_1 q_2) = \int [\delta\nu_1]_{-\infty}^{\infty} E^{(1)}(p_1 q_1|\nu_1) \cdot \int [\delta\nu_2]_{-\infty}^{\infty} \cdot E^{(2)}(p_2 q_2|\nu_2) \cdot \tag{3.13}$$

$$\cdot \ell_a^{(1)} \ell_\beta^{(2)} \int dx\, e^{ix\Delta} g^2 D^{*\alpha\beta}_{12}(x; p_1 q_1|\nu_1) \cdot \int_0^1 d\gamma\, e^{-i\gamma g^2 \int D^{*\alpha\beta}_{12} i_a^{(1)} i_\beta^{(2)}},$$

where

$$\ell_a^{(i)} = (p_i + q_i + 2\nu_i(0))_a \, ; \quad i = 1,2 . \tag{3.14}$$

Similarly to the previous case all the quantities in eq.(3.13) are expressed through the Green function of the vector meson field interacting with the external sources

$$i_a^{(i)} = \int_{-\infty}^{\infty} d\zeta (2p_i\theta(\zeta) + 2q_i\theta(-\zeta) + 2\nu_i(\zeta))_a\, \delta[x_i - z + 2p_i\zeta\theta(\zeta) + 2q_i\zeta\theta(-\zeta) + 2\int_0^{\zeta} \nu_i\, d\eta]. \tag{3.15}$$

It is easy to see that eq. (3.15) determines the current density of the point-like particle moving along the classical path (3.11), and obeys the condition

$$\partial_a i_a^{(i)} = 0 . \tag{3.16}$$

4. Discussion of the Renormalization Problems

It is evident that the one-particle Green function of interacting nucleons $G(p)$ which is determined by

$$(2\pi)^4 \delta(p-q) G(p) = [\exp \frac{i}{2} \int D \frac{\delta^2}{\delta\phi^2}] G(pq)\phi) S_0(\phi)|_{\phi=0} \tag{4.1}$$

has in general case different position of a pole and a value of a residue than the Green function of free nucleons, i.e.

$$G(p) = \frac{1}{m^2 - p^2 + \Sigma(p^2)}\Big|_{p^2 \approx m^2_{ph}} \approx \frac{Z^{-1}}{(m^2_{ph} - p^2)}, \tag{4.2}$$

where

$$\begin{aligned} m^2_{ph} &= m^2 + \Sigma(m^2_{ph}) = m^2 + \delta m^2 \\ Z &= 1 - \frac{\partial \Sigma}{\partial p^2}(m^2_{ph}). \end{aligned} \tag{4.3}$$

For this reason in the definition of the scattering amplitude (3.1) as the residue of the two-particle Green function at the poles associated with the external nucleon tails we should write, for example, instead of the factor $(p_1^2 - m^2)$ the following one $Z(p_1^2 - m^2_{ph})$.

Moreover, it can be shown that due to the mass and wave functions renormalization the functional integrals in eq. (3.7) diverge or more precisely

$$\begin{aligned} \int [\delta\nu_1]^{\tau_1}_{-\xi_1} [\delta\nu_2]^{\tau_2}_{-\xi_2} F \xrightarrow[\substack{\tau_1, \xi_1 \to \infty \\ p_1^2,\ q_1^2 \approx m^2_{ph}}]{} & e^{-i \sum_{k=1,2} \tau_k [\delta m^2 + (1 - Z)(p_k^2 - m^2_{ph})]} \cdot \\ & \cdot e^{-i \sum_{k=1,2} \xi_k [\delta m^2 + (1-z)(p_k^2 - m^2_{ph})]} \cdot \int_R [\delta\nu_1]^{\infty}_{-\infty} \cdot [\delta\nu_2]^{\infty}_{-\infty} F, \end{aligned} \tag{4.4}$$

where the symbol $\int_R$ denotes the renormalized value of the functional integral, which is finite after extracting the divergent exponential factors.

Thus, we obtain for the scattering amplitude defined as a residue of the two-particle Green function at the physical poles the same expressions (3.9) and (3.13) with only difference that instead of $\int_{\infty} [\delta\nu_1]^{\infty}_{-\infty} \cdot [\delta\nu_2]^{\infty}_{-\infty}$ we should write more correctly $\int_R [\delta\nu_1]^{\infty}_{-\infty} \cdot [\delta\nu_2]^{\infty}_{-\infty}$.

The procedure of regularization of the functional integrals in general investigation of the structure of scattering amplitudes can be considerably simplified if one assumes that the following limits exist:

$$\frac{E^{(1)}(pq;\tau\xi|\nu)}{\int [\delta\nu]^{\tau}_{-\xi} E^{(1)}(pq;\tau\xi|\nu)} \xrightarrow[\tau, \xi \to \infty]{} e^{(1)}(pq|\nu), \tag{4.5}$$

where the momenta p and q are on the mass shell and the quantities $E^{(1)}$ are determined by eq.(2.26) with nucleon current given by (3.8).

These limits exist in that sense that the following "unproper" functional integrals exist

$$\int [\delta\nu]^{\infty}_{-\infty} e^{(1)}(pq|\nu) = 1 \tag{4.6}$$

and

$$\int [\delta\nu]^{\infty}_{-\infty} \cdot e^{(1)}(pq|\nu) A(\nu) = [A]_{e^{(1)}_{pq}} \tag{4.7}$$

for appropriate functionals $A(\nu)$.

Using eqs. (4.5) and (4.6-7) we get for the two-nucleon scattering amplitude (3.9) the expression

$$F(p_1 p_2 ; q_1 q_2) = r^{(1)}(t) \cdot r^{(2)}(t) \cdot f(p_1 p_2 : q_1 q_2), \tag{4.8}$$

where

$$f(p_1 p_2; q_1 q_2) = \int [\delta \nu_1]_{-\infty}^{\infty} e^{(1)}(p_1 q_1 | \nu_1) \int [\delta \nu_2]_{-\infty}^{\infty} \cdot e^{(2)} (p_2 q_2 | \nu_2) \cdot$$

$$\cdot \int dx \, e^{ix\Delta} g^2 D_{12}^* (x; p_i q_i | \nu_i) \int_0^1 d\gamma \, e^{i - i\gamma g^2 \int D_{12}^* l_1 l_2} \tag{4.9}$$

and

$$r_{(t)}^{(i)} = \int_R [\delta \nu_i]_{-\infty}^{\infty} \cdot E^{(i)} (p_i, q_i | \nu_i); \quad t = (p_i - q_i)^2 . \tag{4.10}$$

It can be shown that $r^{(i)}(t=0) = 1$.

One can see from the eq. (4.8) that a part of radiative corrections is factorized in scattering amplitude in form of the terms which depend only on the square of momentum transfers.

These radiative factors have a simple physical sense: they describe an interaction of the asymptotically free nucleons in the initial and the final states with the fluctuations of the meson vacuum.

The representation (4.8) may be useful in studying asymptotic behaviour of scattering amplitudes at high energies.

5. Generalization to the Inelastic Processes

Here we consider the generalization of the methods described above to the construction of amplitudes of the inelastic processes. We shall consider such inelastic processes when some number of secondary mesons is produced in two-nucleon collisions.

These processes can be described by means of the two-particle Green function of nucleons in the presence of external meson field $\phi^{ext}(x)$:

$$G(p_1 p_2; q_1 q_2 | \phi^{ext}) = i^2 \int_0^{\infty} d\tau_1 d\tau_2 \, e^{i\tau_1 (p_1^2 - m^2) + i\tau_2 (p_2^2 - m^2)} \cdot \int dx_1 dx_2 \, e^{ix_1 (p_1 - q_1) + ix_2 (p_2 - q_2)} \cdot$$

$$\cdot \int [\delta \nu_1]_0^{\tau_1} \cdot [\delta \nu_2]_0^{\tau_2} \cdot E(\phi^{ext}), \tag{5.1}$$

where

$$E(\phi^{ext}) \equiv \left[\exp \frac{i}{2} \int D \frac{\delta^2}{\delta \phi^2}\right] e^{ig \int \phi (l_1 + l_2)} \; S_0(\phi) \Big|_{\phi = \phi^{ext}} =$$

$$= e^{-ig^2/2 \int D (l_1 + l_2)^2 + ig \int \phi^{ext} (l_1 + l_2) + i \Pi (\phi^{ext} - g \int D (l_1 + l_2))} . \tag{5.2}$$

It is convenient to rewrite eq. (5.2) as follows

$$E(\phi^{ext}) = e^{i\Pi(\phi^{ext})} E \cdot R(\phi^{ext}), \tag{5.3}$$

where

$$R(\phi^{ext}) = e^{ig \int \phi^{ext} (l_1 + l_2) + i\Pi(\phi^{ext} - g \int D (l_1 + l_2)) - i\Pi(-g \int D (l_1 + l_2)) - i\Pi(\phi^{ext})} \tag{5.4}$$

and $E \equiv E(\phi^{ext} = 0)$ is defined by eq. (2.18) and corresponds to the pure elastic scattering process.

Introduce now the quantities

$$\Gamma_{i=1,2}(z | \phi^{ext}) = j_i + \int_0^1 d\sigma \int_0^1 d\lambda \int dy (D j_i)_y P(zy | \sigma\phi^{ext} - \lambda g \int D j_i) \tag{5.5}$$

and

$$\Gamma_{12}(z|\phi^{ext})=\int_0^1 d\sigma \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dy_1 dy_2 (Dj_1)_{y_1}(Dj_2)_{y_2}\Gamma(zy_1\, y_2|\sigma\phi^{ext}- -g\int D\cdot(\lambda_1 j_1+\lambda_2 j_2)), \tag{5.6}$$

where

$$\Gamma(zy_1\, y_2|\phi)=\frac{\delta^3}{\delta\phi_z\delta\phi_{y_1},\delta\phi_{y_2}}\Pi(\phi)\ - \tag{5.7}$$

is the generalized vertex operator of three-meson coupling in the presence of an external field, and the polarization operator $P(zy|\phi)$ is determined by eq. (2.23). The functional $R(\phi^{ext})$ in terms of the quantities (5.5–6) has the following form

$$R(\phi^{ext})=e^{i\Pi(0)}\cdot e^{ig\int\phi^{ext}(\Gamma_1+\Gamma_2)+ig^2\int\phi^{ext}\cdot\Gamma_{12}}\ . \tag{5.8}$$

The generating function for amplitudes of the inelastic processes is determined as the residue of the two-particle Green function of nucleons in the presence of an external field (5.1):

$$F(p_1p_2;q_1q_2|\phi^{ext})=\lim_{p_i^2,q_i^2\to m^2}\prod_{i=1,2}(p_i^2-m)(q_i^2-m)G(p_1p_2;q_1q_2|\phi^{ext})\ . \tag{5.9}$$

Dropping in the right hand side (5.9) the factor $\exp(i\Pi(\phi^{ext}))$ which does not contribute to the particle interaction, and performing changes of the variables (3.6), we get the following expression for the generating functional (5.9):

$$F(p_1p_2;q_1q_2|\phi^{ext})=\int dx_1dx_2\, e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)}\cdot \tag{5.10}$$

$$\cdot\int[\delta\nu_1]_{-\infty}^{\infty}\cdot E^{(1)}(p_1q_1|\nu_1)\,e^{ig\int\phi^{ext}\cdot\Gamma_1(p_1q_1x_1|\nu_1\phi^{ext})}\cdot$$

$$\cdot\int[\delta\nu_2]_{-\infty}^{\infty}\cdot E^{(2)}(p_2q_2|\nu_2)\,e^{ig\int\phi^{ext}\Gamma_2(p_2q_2x_2|\nu_2\phi^{ext})}\tilde{F}(x_1x_2|j_1j_2;\phi^{ext}),$$

where

$$\tilde{F}(x_1x_2|j_1\,j_2;\phi^{ext})=g^2\tilde{D}^*_{12}\int_0^1 d\gamma\, e^{-i\gamma g^2\int\tilde{D}^*_{12}\ j_1 j_2}\ . \tag{5.11}$$

Here the quantity $\tilde{D}^*_{12}$ is defined by

$$\tilde{D}^*_{12}\equiv\tilde{D}^*_{12}(x_1x_2;p_1p_2;q_1q_2|\nu_1\nu_2\phi^{ext})=D^*_{12}(x_1x_2;p_iq_i|\nu_i)- -\int_0^1 d\sigma\int_0^1 d\lambda_1\int_0^1 d\lambda_2\int dz\,dy_1dy_2\,\phi^{ext}(z)D(x_1-y_1)\Gamma_{12}(zy_1y_2|\sigma\phi^{ext}- -g\int D(\lambda_1 j_1+\lambda_2 j_2))\cdot D(y_2-x_2) \tag{5.12}$$

and corresponds to the full Green function of the meson field interacting simultaneously with an external field ϕ^{ext} and the sources j_1 and j_2 .

The amplitudes of the processes when N secondary mesons are produced in two-nucleon collision, are determined by the functional derivatives of the generating functional (5.10):

$$(2\pi)^4\delta(q_1+q_2-p_1-p_2-\sum_{l=1}^{N}k_l)F(p_1p_2;q_1q_2;k_1k_2\dots k_N)= =\prod_{l=1}^{N}\int dy_l e^{iy_lk_l}\frac{\delta}{\delta\phi^{ext}_{y_l}}\cdot F(p_1p_2;q_1q_2|\phi^{ext})|_{\phi^{ext}=0}\ . \tag{5.13}$$

For example, the amplitude of production of one secondary meson with the momentum k has the form

$$F(p_1 p_2; q_1 q_2; k) = ig \int[\delta\nu_1]_{-\infty}^{\infty} E^{(1)}(p_1 q_1|\nu_1) \int[\delta\nu_2]_{-\infty}^{\infty} E^{(2)}(p_2 q_2|\nu_2)\cdot$$

$$\cdot[\Gamma_1(p_1q_1;k_1|\nu_1)+\Gamma_2(p_2q_2;k_2|\nu_2)]\cdot\int dx\, e^{ix\Delta} g^2 D^*_{12}\cdot\int_0^1 d\gamma\, e^{-i\gamma g^2\int D^*_{12} j_1 j_2} + \tag{5.14}$$

$$+ig\int[\delta\nu_1]_{-\infty}^{\infty} E^{(1)}(p_1q_1|\nu_1)\int[\delta\nu_2]_{-\infty}^{\infty} E^{(2)}(p_2q_2|\nu_2)\int dx\, e^{ix\Delta}\tilde\Gamma_{12}(x; p_i q_i k|\nu_i)\cdot e^{-ig^2\int D^*_{12} j_1 j_2}.$$

Here we used the notation

$$\Gamma_i(p_i q_i; k_i|\nu_i) = \int dz\, e^{izk}\, \Gamma_i(z|\phi^{ext})\Big|_{\substack{x_i=0\\ \phi^{ext}=0}}, \quad i=1,2 \tag{5.15}$$

and

$$\tilde\Gamma_{12}(x; p_i q_i; k|\nu_i) = -g\int_0^1 d\lambda_1\int_0^1 d\lambda_2\int dz\, dy_1\, dy_2\, e^{izk}\cdot \tag{5.16}$$

$$\cdot D\left(y_1-\frac{x}{2}\right)D\left(y_2+\frac{x}{2}\right)\Gamma(y_1y_2z|-g\int D(\lambda_1 j_1+\lambda_2 j_2)).$$

The similar expressions can be obtained for the production amplitudes of two and more secondary mesons.

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Appendix

Consider the structure of the quantity Π_{12}, determined by eq. (2.22). Using Fourier-transformation of the functional $\Pi(\phi)$ to the functional space of the virtual nucleon current $\eta(x)$:

$$\Pi(\phi) = \int\delta\eta\,\Pi(\eta)\,e^{-i\int\phi\eta} \tag{A1}$$

and the definition of the polarization operator of the meson field (2.23) we find the following expression for the quantity Π_{12}:

$$\Pi_{12} = \int\delta\eta\;\Pi(\eta)(e^{ig\int D\eta j_1}-1)\cdot(e^{ig\int D\eta j_2}-1) =$$

$$= -g^2\int_0^1 d\lambda_1\int_0^1 d\lambda_2\int d\eta\;\Pi(\eta)\,e^{ig\int D(\lambda_1 j_1+\lambda_2 j_2)\eta}\int D\eta j_1\cdot\int D\eta j_2 = \tag{A2}$$

$$= -g^2\int_0^1 d\lambda_1\int_0^1 d\lambda_2\int dx_1\, dx_2 (Dj_1)_{x_1}(Dj_2)_{x_2}\, P(x_1x_2|-g\lambda_1\int Dj_1 - g\lambda_2\int Dj_2).$$

The similar expressions can be found for the quantities Π_i, determined by eq. (2.21). Really, taking into account the condition

$$\frac{\delta\Pi}{\delta\phi}\Big|_{\phi=0} = 0 \tag{A3}$$

we find

$$\Pi_i = \int d\eta\,\Pi(\eta)(e^{ig\int D\eta j_i} - ig\int D\eta j_i - 1) =$$

$$= -g^2\int_0^1 d\sigma\int_0^\sigma d\lambda\int d\eta\,\Pi(\eta)\,e^{ig\lambda\int D\eta j_i}\int D\eta j_i\int D\eta j_i = \tag{A4}$$

$$= -g^2\int_0^1 d\sigma\int_0^\sigma d\lambda\int dx_1 dx_2 (Dj_i)_{x_1}(Dj_2)_{x_2}\, P(x_1x_2) - \lambda g\int Dj_i).$$

Using formulas (A2) and (A 4) and the definition of the full Green function of the meson field in the presence of external sources, one can easily obtain the expressions (2.26-29).

References

1. A.N. Tavkhelidze. Rapporteur talk at the XV-th International Conference on High Energy Physics, Kiev, August-September, 1970; JINR, 5454, Dubna (1971).
2. A.A. Logunov, A.N. Tavkhelidze. Nuovo Cim., 29 380 (1963).
3. V.G. Garsevanishvili, V.A. Matveev, L.A. Slepchenko and A.N. Tavkhelidze. Talk at the Coral Gables Conference, Miami, 1969; Phys. Letters, 29B, 191 (1969); see also "Particles and Nucleus", vol. 1, Part I, pp. 93-130, Dubna (1970).
4. S.P. Alliluyev, S.S. Gerstein and A.A. Logunov. Phys. Letters, 18, 196 (1965).
5. A.A. Logunov, O.A. Khrustalev. Preprint IPHE, STP-69-20, 21 (1969). See also "Particles and Nucleus", vol. 1, Part I, pp. 73-90, Dubna (1970).
6. V.A. Matveev, A.N. Tavkhelidze. JINR, E2-5141, Dubna (1970). See also the invited talk at the XV-th International Conference on High Energy Physics, Kiev, August-September, 1970. "Fundamental Problems of the Elementary Particles Theory". Preprint ITP-70-99, Kiev (1970).
7. P.N. Bogolubov. JINR, P2-5684, Dubna (1971).
8. H.D.I. Abarbanel, C. Itzykson. Phys. Rev. Letters, 23, 53 (1969).
9. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, A.N. Sissakian. JINR, E2-4692, Dubna (1969); TMP, 3, 342 (1970).
10. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, V.N. Pervushin, A.N. Sissakian, A.N. Tavkhelidze. JINR, E2-5217, Dubna (1970), TMP, 5 330 (1970).

11. I.V.Andreev. Zh. Eksp. i Teor. Fiz. 58, 257 (1970).
I.V.Andreev, A.I.Batalin. Preprint FIAN, 60, Moscow (1970).
12. M.Levy, J.Sucher. Phys. Rev., 186, 1656 (1969).
13. H.Cheng, T.Wu. Phys. Rev., 186 , 1611 (1969); 182 , 1852 (1969).
14. S.J.Chang, S.K.Ma. Phys. Rev., 188 , 2378 (1969).
S.J. Chang, P.M.Fishbane. Preprint of Illinois University (1970).
15. F.Englert, P.Nicoletopoulos, R. Brout, C.Traffin. Nuovo Cim., 64 , 561 (1969).
16. R.Torgerson. Phys. Rev., 143, 1194 (1966).
17. R.C.Arnold. Phys. Rev., 153, 1523 (1967).
18. G.Tiktopoulos, S.B.Treiman. Preprint of Princeton University (1970). B.M.Barbashov, V.V.Nesterenko, TMP, 4, 293 (1970).
19. D.R.Yennie, S.C.Frautschi, H.Suuka. Annals of Physics 13, 379 (1961).
20. B.M.Barbashov. Zh. Eksp. i Teor. Fiz., 48, 607 (1965).
21. E.S.Fradkin. Nuclear Phys., 76 , 588 (1966).
22. N.N.Bogolubov, D.V.Shirkov. "Introduction to the Theory of Quantum Fields". GITTL, Moscow (1957).
23. F.Bloch, A.Nordsiech. Phys. Rev., 52 54 (1937).
24. B.M.Barbashov, S.P.Kuleshov, V.A.Matveev, A.N.Sissakian. JINR, E2-4883, Dubna (1970).
25. B.M.Barbashov, S.P.Kuleshov, V.A.Matveev, A.N.Sissakian, V.N.Pervushin. JINR, E2-5329, Dubna (1970). Preprint CNRS Marseille University (1970). Nuovo Cim., (1971).
26. Y.P.Yao. Preprint University of Michigan (1969).
27. V.N.Pervushin. JINR, P2-4866, Dubna (1970). TMP, 4 , 28 (1970).
28. B.M.Barbashov, M.K.Volkov, Zh.Eksp. i Teor. Fiz., 50, 660 (1966).
29. G.A.Milehin, E.C.Fradkin. Zh.Eksp. i Teor. Fiz., 45 , 1926 (1963).

30. H.M. Fried. Preprint University of Brown (1970).

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