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OSCILLATORY LEVELS OF A PARTICLE AS A RESULT OF STRONG INTERACTION WITH THE FIELD

Submitted to *ТМФ*

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1. Introduction

Nowadays a tendency exists to treat hadrons as some complicated structures of either the true elementary particles or certain quasielementary excitations. These may be: Quarks, if speaking about static features of a particle and the simplest properties of a scattering; coherent complecces[1], partons[2] or droplets, if more complicated characteristics of high-energy interaction should be considered. All these concepts usually are introduced purely operationally, and, as a rule, the interaction dynamical picture itself is not even touched. Recently very interesting work[4] has appeared in which a connection between the coherent state method and quark model has been established. So, in this work there firstly has been created the dynamical model, which explains in what way the quasielementary excitations of the coherent state type occur. In the present paper we wish to show that the above type structures are a natural result of the strong interaction of a particle with boson field.

The ideas on the complicated structure of the particle and on the presence of excited states are permanently inherent in any strong-coupling problem[5]. Here it should be recalled that the main difficulties of the above theory are due to the urgent necessity to work from the very beginning with the notions different from those of free field theory. The only well-known consistent model of strong coupling in field theory is the model of fixed source of the boson field. This model, nevertheless, is rather far from real interactions conserving the total momentum.

The method of a separation of the particle motion in the field which takes into account the translation degeneracy due to the total momentum conservation, has been introduced by N.N. Bogolubov[6]. This method makes it possible to construct a scheme of successive approximations for the energy and wave functions of a system, which allows for the conservation of the total momentum explicitly.

In this paper by using the Bogolubov method one of the strong-coupling problem, viz. an interaction of the nonrelativistic particle with the scalar field, is discussed. It is shown that stationary states of the above system are just the oscillatory states of the particle surrounded by a cloud of the scalar quanta, which to a certain degree are an analog of the states considered in refs.[1,4].

2. Interaction of a Particle with the Scalar Field in the Case of Weak and Intermediate Coupling

The problem of interaction of a nonrelativistic particle with scalar field was found to be useful in modelling rather wide class of more interesting physical problems. Here one can mention, first of all, such problems as: (i) Interaction of low-energy fermions with the scalar meson field, (ii) motion of an electron in polar crystal or semiconductor[6,7]. On writting the Hamiltonian of the system under consideration as

$$H = \frac{1}{2\mu} \vec{p}^{\,2} + g \sum a_f e^{i\vec{f}\vec{r}} b_f + a_f^* e^{-i\vec{f}\vec{r}} b_f^+ + \varepsilon^2 \sum \nu_f b_f^+ b_f , \qquad (1)$$
$$a_f^* = a_{-f} ,$$

where g and ε^2 are dimensionless constants, one can predict all the possible relations between the energy of a free scalar field (further its quanta will be called phonons) and that of the particle-field interaction. The case $\varepsilon^2 \approx 1$ and $g \ll 1$ is just the weak-coupling limit, and here the ordinary perturbation theory is applicable*. In this case it is possible, in the approximation, to distinguish rather well the state of the particle and look for the wave function of the state "particle + n phonons" in the form of the product of functions, depending only on the particle radius vector and the phonon variables:

$$\Psi(\vec{r}, n_f) = \varphi(\vec{r})\,\Theta(n_f)\,. \tag{2}$$

If this function is an eigenfunction of the total momentum operator

$$\vec{P} = \vec{p} + \hbar \sum \vec{f}\, b_f^+ b_f\,, \tag{3}$$

then it has the form

$$\Psi(\vec{r}, n_f) = \frac{1}{\sqrt{V}}\, \exp\left[i\, \frac{\vec{P}-\vec{P}'}{\hbar}\, \vec{r} \right] \Theta(n_f)\,, \tag{4}$$

where $\vec{P}'$ is the total momentum of the phonons. These functions at the same time are the eigenfunctions of the operator

$$\frac{1}{2\mu}\, \vec{p}^{\,2} + \varepsilon^2 \sum \nu_f\, b_f^+ b_f \tag{5}$$

which represents an unperturbed energy operator in the weak-coupling limit. Thus, within the theory of weak interactions

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*A possibility for an application of the Hamiltonian (1) under the assumption of weak coupling for problems of the motion of an electron in the polar crystal was studied in detail in ref.[8]

one can get quite simple expansions in the eigenfunctions of energy and momentum for the wave functions of a system.

However, the case $g \sim \varepsilon^2$ or $g \gg \varepsilon^2$ (i.e. as basic effects are to be considered those due to an equivalent account of the particle kinetic energy and that of the particle-field interaction) requires another approach. Here the main difficulty is: In what way one can correctly give account of the conservation law of the total momentum for the system.

The wave functions (4) cannot be the eigenfunctions of the operator

$$\frac{1}{2\mu}\vec{p}^{\,2} + g\sum a_f e^{i\vec{f}\vec{z}} b_f + a_f^* e^{-i\vec{f}\vec{z}} b_f^+ \tag{6}$$

which is an initial Hamiltonian in the strong-coupling limit. Therefore in the theory of strong coupling the problem arises of the correct choice of stationary wave functions, which make it possible to preserve the particle individuality as well as the total momentum.

To this end we do employ an adiabatic variant of perturbation theory developed in ref.[6]. The above work just dealt with the special case of adiabatic interactions when $g = \varepsilon \ll 1$ 1, i.e. the interaction is treated as small one though the interaction energy is much higher than that of a free field. However, the method of separating of the particle coordinate developed in this work, is so general that it can be immediately extended to the strong-coupling case $\varepsilon^2 = 1$, $g \gg 1$, considered here.

Before describing the method itself, let us try less general (or less rigorous) means for taking into account the momentum and particle individuality conservation. These may serve to illustrate once more the essence of the method of ref.[6].

One of the possibilities to take into account the momentum conservation is to pass to a representation in which the total momentum and energy operators become C-numbers. This possibility was indicated in ref.[6] and successively studied in ref.[9]. On using an appropriate canonical transformation

$$b_f \to e^{-i\vec{f}\vec{r}} a_f \quad , \qquad \vec{p} \to \vec{P} - \hbar \sum \vec{f} a_f^+ a_f \tag{7}$$

the Hamiltonian (1) takes the form (we put $\varepsilon^2=1$)

$$H = \frac{1}{2\mu}\left(\vec{P} - \hbar \sum \vec{f} a_f^+ a_f\right)^2 + g \sum a_f a_f + a_f^* a_f^+ + \sum \omega_f a_f^+ a_f \quad , \tag{8}$$

where $\vec{P}$ is a C-number.

The ground state in ref.[9] was searched by use of the minimum energy condition for trial functions corresponding to the system state which consists of a particle with the cloud of noncorrelating phonons around the particle. The mathematical expression of this is the canonical transformation which transforms the operators a_f and a_f^+ to operators ξ_f, ξ_f^+

$$a_f = u_f + \xi_f \; , \qquad a_f^+ = u_f^* + \xi_f^+ \; . \tag{9}$$

Under such a transformation a vacuum becomes the coherent phonon state. The assumption on statistical independence of phonons simplifies calculations but at the same time restricts rather strongly the domain of validity of the method. In employing the assumption (9) the particle must be considered without a recoil from the phonons emitted, because the recoil introduces a correlation between successively emitted phonons. The use of the transformation (9) makes the method of ref.[9] closely related to that of intermediate coupling introduced earlier by Tomonaga [10] for solving the problem of interaction of a fixed nucleon with charged mesons. The basic assumption of the method is that all the mesons forming the ground state of a system are described by the same wave functions though the number of virtual mesons is not limited. For the problems with the fixed nucleon the intermediate coupling method gives the correct relations in both strong and weak coupling limits. However, to problems concerning the motion of a particle this method wittingly cannot be applied in the strong coupling limit, in which the neglect of the recoil of a particle due to the emission of the huge amount of phonons cannot be justified. Now let us turn back to ref.[9]. The minimum energy condition gives the expression

$$g a_f + u_f^* \left\{ \nu_f - \frac{\hbar}{\mu} \vec{f}\vec{P} + \frac{\hbar^2}{2\mu} \vec{f}^{\,2} + \frac{\hbar^2}{2\mu} \vec{f} \cdot \sum \vec{f}' |u_{f'}|^2 \right\} = 0 . \qquad (10)$$

Symmetry properties of the numbers make it possible to represent the sum in Eq.(10) as

$$\sum \vec{f}\, |u_f|^2 = \eta \vec{P} \qquad (11)$$

and the equation for η follows

$$\eta \vec{P} = g^2 \sum \hbar \vec{f} \, \frac{|a_f|^2}{\nu_f - \frac{\hbar}{\mu} \vec{f}\vec{P}(1-\eta) + \frac{\hbar^2}{2\mu} \vec{f}^{\,2}} . \qquad (12)$$

On solving Eq.(12) it is possible by using Eq.(10) to obtain the numbers η_f and the energy of the ground state

$$E = \langle H \rangle = \frac{\vec{P}^2}{2\mu} + g \sum a_f u_f + a_f^* u_f^* + \frac{\hbar^2}{2\mu} \left(\sum \vec{f} |u_f|^2 \right)^2 + \sum |u_f|^2 \left(\nu_f - \hbar \frac{\vec{P}\vec{f}}{\mu} + \frac{\hbar^2 \vec{f}^{\,2}}{2\mu} \right). \qquad (13)$$

In ref.[9] Eq.(12) has been solved exactly for the special choice of frequencies ν_f and coefficients a_f, which corresponds to the problem of motion of an electron through a polar crystal.

As to our purposes, it is sufficient to note that for small g the sum of absolute squares of $|u_f|^2$ Eq.(10) as well as next to last term of (13) can be neglected. Then Eq.(13) gives the expression for the energy and effective mass of the particle just the same as those of the weak coupling theory[8]. Thus, an account of terms of higher orders in g by means of Eqs.(11), (12) leads only to corrections to the weak coupling theory, as it should be expected. In the subsequent paper[11] an attempt was made to allow for the correlations between virtual phonons by introducing more complicated trial functions on which one seeks to minimize the energy. Such an approach severely complicates calculations and that is more important, gives rise to the loss of clear physical criterion which permits to make a choice between the trial functions.

3. The Variational Principle in the Strong-Coupling Limit

The above considerations being quite rigorous once more manifest the complexity of the problem in strong-coupling limit. We find therefore it convenient to present some other arguments not so rigorous but more tightly related to the method of ref.[6]. So, if in the strong coupling limit the unperturbed Hamiltonian of the system (6) is linear in the operators b_f and b_f^+, then the Heisenberg equation of motion for the operators does not permit to identify, in this approximation, these operators with the creation and annihilation operators of the real phonons capable to transfer the energy and momenta.

The simplest way to be oonvinced of this is to change the operators b_f, b_f^+ by the complex coordinates

$$q_f = \frac{b_f + b_{-f}^+}{\sqrt{2}} , \qquad p_f = i\,\frac{b_f^+ - b_{-f}}{\sqrt{2}} . \tag{14}$$

In this case the Hamiltonian (6) depends on the variables q_f only and the Heisenberg equation of motion results in the solution

$$q_f(t) = const \tag{15}$$

In the system with the Hamiltonian (6) the phonons are viewed as a certain passive mass, which adheres to the particle and moves together and creates for the particle something like a potential well. So, to the first approximation one can picture the particle-field interaction as follows: The particle has dug the potential well in the field and then moves through the field, the motion being composite and equal to the sum of uniform motion of a velocity $\vec{C}$ and vibratory motion inside the well. The uniform motion (i.e. the momentum conservation, too) can be allowed for by inserting into the Hamiltonian an appropriate energy, i.e. going over to the Hamiltonian

$$H = \frac{1}{2\mu}\vec{p}^{\,2} + g\sum a_f e^{i\vec{f}\vec{r}} b_f + a_f^* e^{-i\vec{f}\vec{r}} b_f^+ + \sum \nu_f b_f^+ b_f - \vec{C}\left(\vec{p} + \hbar\sum \vec{f}\, b_f^+ b_f\right) \tag{16}$$

The ground state of the system will be searched by means of the variational principle, with the trial functions of the type Eq.(2). Before it is necessary to make the canonical transformation (9) of phonon field and choose the phonon states in the form of states with fixed number of phonons. To the first approximation, the choice of the trial functions in the form (2) is justified by the classical behaviour of phonons and by possible separation of the motion of the centre of mass of the system. The minimum enegry condition $\frac{\partial}{\partial u_f}\langle H\rangle=0,\ \frac{\partial}{\partial u_f^*}\langle H\rangle=0$ provides the following values for the numbers u_f:

$$u_f = -g\ \frac{a_f^* \langle e^{-i\vec{f}\vec{r}}\rangle}{\nu_f - \hbar\vec{C}\vec{f}}\ , \tag{17}$$

where symbol $\langle e^{-i\vec{f}\vec{r}}\rangle$ means the averaging of the exponential over the wave function of the particle ground state, and the Hamiltonian (15) takes the form

$$\begin{aligned} H = {} & \frac{1}{2\mu}\vec{p}^{\,2} + g\sum a_f u_f e^{i\vec{f}\vec{r}} + a_f^* u_f^* e^{-i\vec{f}\vec{r}} + \sum |u_f|^2(\nu_f - \hbar\vec{C}\vec{f}) - \vec{C}\vec{p}\ + \\ & + \sum\{g a_f e^{i\vec{f}\vec{r}} + u_f^*(\nu_f - \hbar\vec{C}\vec{f})\}b_f + \{g a_f^* e^{-i\vec{f}\vec{r}} + u_f(\nu_f - \hbar\vec{C}\vec{f})\}b_f^+ \\ & + \sum(\nu_f - \hbar\vec{C}\vec{f})b_f^+ b_f\ . \end{aligned} \tag{18}$$

An appearance of $-\vec{C}\vec{p}$ in Eq.(18) can be treated as passing to the coordinate system moving together with the particle, or in the Heisenberg operator language, the replacement

$$\vec{r} - \vec{r} - \vec{C}t\ , \tag{19}$$

i.e. the term $-\vec{C}\vec{p}$ can be excluded from the Hamiltonian (18) by a simple transformation of the wave function.

Next, representing the wave function of the system in the zeroth approximation in the form of product

$$\Psi(\vec{r}, n_f) = \varphi_0(\vec{r})\,\Phi_0(n_f) \tag{20}$$

and varying φ_0 and Φ_0 independently, we find that the variational principle

$$(\delta\Psi, (H-E)\Psi) = 0 \tag{21}$$

results in the equations

$$\left(\Phi_0, (H-E)\Phi_0\right)\varphi_0(\vec{z})=0. \tag{22}$$

$$\left(\varphi_0, (H-E)\varphi_0\right)\Phi_0(n_f)=0. \tag{23}$$

Because of (17) the Hamiltonian (18) naturally breaks down into the terms proportional to various powers of g , viz.: The first line of Eq.(18) is proportional to g^2 (the problem how to increase the order of kinetic energy will be discussed somewhat later), second - to g , third to zeroth-order of g. Expanding the energy in power series in g and putting $W_0=g^2E_0+\sum|u_f|^2(\nu_f-\hbar\vec{c}\vec{f})$ we find that Eq.(22) reduces to the Schrödinger equation for a particle

$$\left(\frac{1}{2\mu}\vec{p}^{\,2}+V(\vec{z})-W_0\right)\varphi_0(\vec{z})=0 \tag{24}$$

with the potential

$$V(\vec{z})=g\sum Q_f u_f e^{i\vec{f}\vec{z}}+Q_f^* u_f^* e^{-i\vec{f}\vec{z}}. \tag{25}$$

In virtue of the identity

$$\left(\varphi_0, (H_0-g^2E_0)\varphi_0\right)=0 \tag{26}$$

Eq.(23) reduces to the relation

$$\left[(\varphi_0, H_1\varphi_0)-gE_1\right]\Phi_0(n_f)=0, \tag{27}$$

where in H_1 there are involved the terms of Eq.(18) linear in g. Due to the condition (17) the average value of H_1 over the wave function specifying the ground state of a particle is equal to zero, and Eq.(27) can be satisfied provided we put E_1 =0. The wave function Φ_0 there remains arbitrary. In the given approximation, by fixing the term of the highest order in g in the exact expression for the momentum operator, it is possible to put the total momentum of the system equal to

$$\vec{P} = \hbar \sum \vec{f}\, |u_f|^2 \tag{28}$$

reducing the momentum operator to C-number.

Further, it is convenient to introduce the coefficients $\tilde{u}_f$ expressed through u_f as

$$\tilde{u}_f = \frac{\sqrt{2}\,\nu_f}{\nu_f + \hbar \vec{C}\vec{f}}\, u_f = -g \frac{A_f^* \nu_f}{\nu_f^2 - \hbar^2 (\vec{C}\vec{f})^2} < e^{-i\vec{f}\vec{z}} >, \quad A_f = \sqrt{2}\, a_f \,. \tag{29}$$

Then there may be indicated the more direct relationship of the momentum with the vector $\vec{C}$:

$$\vec{P} = \hbar^2 \sum \frac{\vec{f}(\vec{f}\vec{C})}{\nu_f} |\tilde{u}_f|^2. \tag{30}$$

The energy, without that specifying the motion of a system as a whole, can be expressed through the coefficients $\tilde{u}_f$ as

$$g^2 E_0 = W_c + \frac{1}{2} \sum |\tilde{u}_f|^2 \left(\nu_f + \frac{\hbar^2 (\vec{C}\vec{f})^2}{\nu_f}\right) . \tag{31}$$

Using the Schrödinger equation (23) which results in the relation $\frac{\partial W}{\partial \vec{C}} = < \frac{\partial V}{\partial \vec{C}} >$, one can show that

$$\frac{\partial g^2 E_0}{\partial C^\alpha} = \sum_\beta C^\beta \frac{\partial P^\beta}{\partial C^\alpha} \tag{32}$$

from which

$$\vec{C} = \frac{\partial g^2 E_0}{\partial \vec{P}}, \tag{33}$$

follows, i.e. the vector $\vec{C}$ represents the mean velocity of the particle. Note that if $g^2 E_0$ and $\vec{P}$ are quantities of order of g^2 then the vector $\vec{C}$ is of the zeroth order in g, i.e. the momentum transferred by the particle is significantly less than the total momentum of the system*).

Thus, the calculations based on the above variational principle indicate that as the first approximation to describing strong field-particle interactions, one can really employ simple assumptions given at the beginning of the section. These consist in that the main effect of interaction is just the preparation by the particle of the potential well.

Proceeding from Eq.(30) for the total energy it is easy to get the value of the particle effective mass

$$\mu_{эфф} = \frac{1}{3}\sum \frac{\hbar^2 \vec{f}^2}{\nu_f} |\tilde{u}_f^{(0)}|^2, \tag{34}$$

where $\tilde{u}_f^{(0)}$ are the values of u_f computed at $\vec{C}=0$. The expressions for u_f and the effective mass differ from those derived by use of the canonical transformation which reduces the total momentum to C-number. The expressions (17) for u_f now contain a form factor of the particle taking into account the recoil in the phonon emission. Thus the new version of the variational principle reflects, to a degree, the true picture of interactions. One of the spesific features of nonweak interactions of a particle with the field is just the pronounced nonlinearity of the equation for the particle ground state, in which the effective potential is expressed by the form factor of the particle in the ground state. Therefore to determine in fact the wave function of the ground state it is more convenient

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*) Later it will be shown that the coordinate transformation which increases the order of the kinetic energy, does not influence this conclusion.

to employ the variational principle

$$\frac{\hbar^2}{\mu}\int \frac{\partial\varphi^*(\vec{z})}{\partial\vec{z}}\,\frac{\partial\varphi(\vec{z})}{\partial\vec{z}}\,d\vec{z} - \sum \frac{g^2\nu_f|A_f|^2}{\nu_f^2-\hbar^2(\vec{C}\vec{f})^2}\left|\int e^{-i\vec{f}\vec{z}}\varphi^*(\vec{z})\varphi(\vec{z})\right|^2 = min \quad (35)$$

with the condition

$$\int |\varphi(\vec{z})|^2 d\vec{z} = 1\ . \quad (36)$$

On cosidering at the same time both the equation in variations and Eq.(24) which follow from (35), the linear integro-differential equation

$$\left(-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\vec{z}^2} + V(\vec{z}) - W\right)\varphi(\vec{z}) = \int K(\vec{z},\vec{z}\,')\varphi(\vec{z}\,')\,d\vec{z}\,' \quad (37)$$

is obtained with the kernel

$$K(\vec{z},\vec{z}\,') = \sum g^2 \frac{\nu_f|A_f|^2}{\nu_f^2-\hbar^2(\vec{C}\vec{f})^2}\, e^{i\vec{f}(\vec{z}-\vec{z}\,')}\varphi_0(\vec{z})\varphi^*(\vec{z}\,') \quad (38)$$

specifying the excited states of the particle.

Thus, the variational principle described above reproduces correctly specific nature of strong particle-field interactions. It is not very hard, however, to learn the nonstrictness and insufficiency inherent in this principle: The exact account of the momentum conservation is replaced here by the approximate separation of the principal part of the total momentum. Although this trick makes it possible to separate the gross effect of interaction, it provides no hints concerning the details of interaction. In formulating the variational principle the quantum properties of the phonon field appeared to be out of the consideration. For, if we take into account the energy and momentum transfer by phonons, we at once lose a chance to represent the wave function of the system in the form of product Eq.(20) and identify Eq.(30) with the total momentum of the system This makes unknown the degree of an accuracy to which the particle state can be described by the Schrödinger equation with the potential (25). Later on it will be shown that this equation being modified a little, may serve, in fact, for describing of quasistationary states of a particle in the field, but to do this the more detailed analysis is necessary of the effects caused by the translation degeneracy due to the total momentum conservation.

4. The Bogolubov Transformation

An attempt to allow for the translation degeneracy due to the momentum conservation, transforming to the moving coordinate system by means of (19) is, of course, very naive. In ref.[6] the total momentum conservation as well as the translation degeneracy were taken into account by the transformation

$$\vec{r} \to \vec{\lambda} + \vec{q} \tag{39}$$

which introduces instead of one variable , two independent variables $\vec{q}$ and $\vec{\lambda}$; $\vec{q}$ being associated with the uniform motion of the particle and $\vec{\lambda}$ - with the oscillatory motion inside the potential well. An explicit form of the canonical transformation of $\vec{r}$ and b_f to the new operators can be found from the condition

$$-i\hbar \frac{\partial}{\partial \vec{q}} = \vec{P} + \hbar \sum \vec{f} b_f^+ b_f \tag{40}$$

according to which the derivative with respect to $\vec{q}$ must be the total momentum operator.

For further consideration it is more convenient to change the operators b_f , b_f^+ by the complex coordinates $q_f, P_f = -i\frac{\partial}{\partial q_f}$ which break the energy of the free phonon field into the kinetic and potential energy of field oscillators. Let us perform this transformation in such a way that both the potential energy of the field oscillators and the interaction energy of the particle with the field become quantities of the same order. To this end let us go back to the Hamiltonian (1) adding to it the energy of the field zeroth oscillations

$$H=\frac{1}{2\mu}\vec{P}^2+g\sum_f\left(a_f e^{i\vec{f}\vec{r}}b_f+a_f^* e^{-i\vec{f}\vec{r}}b_f^+\right)+\frac{\varepsilon^2}{2}\sum_f\frac{1}{2}\left(b_f^+b_f+b_f b_f^+\right). \tag{41}$$

It is useful to introduce the following combinations of the constants g and ε^2:

$$\gamma=\frac{\varepsilon^2}{g}\;,\quad \varkappa^4=\frac{g^2}{\varepsilon^2}\;. \tag{42}$$

The quantity γ is a small parameter in both cases of strong coupling ($\varepsilon=1$, $g\gg 1$) and adiabatic one ($\varepsilon\ll 1$, $g=\varepsilon$). For the adiabatic coupling $\varkappa=1$, but in the strong-coupling limit there arises new small parameter $\varkappa^{-1}\ll 1$ which still more simplifies the problem. The constants g and ε^2 are expressed through the new ones in the following way

$$g=\varkappa^4\gamma,\qquad \varepsilon^2=\varkappa^4\gamma^2\;. \tag{43}$$

Now let us define the operators

$$q_f=\gamma\,\frac{b_f+b_{-f}^+}{\sqrt{2}}\;,\quad P_f=\frac{i}{\gamma}\,\frac{b_f^+-b_{-f}}{\sqrt{2}} \tag{44}$$

which obey the commutation relations

$$[\,q_f\,,\,P_{f'}]=i\delta_{ff'}. \tag{45}$$

and the reality conditions

$$q_f^+=q_{-f}\;,\quad P_f^+=P_{-f}\;. \tag{46}$$

Then the Hamiltonian (22) takes on the form

$$H=\frac{1}{2\mu}\vec{p}^{2}+\varkappa^{4}\left\{\sum A_f e^{i\vec{f}\vec{z}}q_f+\frac{1}{2}\sum \nu_f q_{-f}q_f\right\}+\frac{\varkappa^4\gamma^4}{2}\sum \nu_f P_{-f}P_f \qquad (47)$$

and the total momentum operator

$$\vec{P}=\vec{p}-i\hbar\sum \vec{f}\,q_f P_f \quad . \qquad (48)$$

Inserting the variable $\vec{q}$ satisfying the relation (40) in the representation in which the operator q_f reduces to multiplication by a number, it should be put

$$\frac{\partial q_f}{\partial \vec{q}}=-i\vec{f}q_f\,, \qquad (49)$$

$$\frac{\partial z_\alpha}{\partial q_\beta}=\delta_{\alpha\beta} \quad . \qquad (50)$$

Bearing in mind that now the potential energy, generally speaking, is rather large quantity, we break up $\vec{z}$ into components in such a way that in the following it would be possible to take into account the kinetic enegry of oscillatory motion inside the well even in the first order

$$\vec{z}=\vec{q}+\frac{1}{\varkappa}\vec{\lambda} \quad . \qquad (51)$$

In this case the operator of the particle kinetic energy will be as follows

$$-\varkappa^2\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\vec{\lambda}^2} \quad . \qquad (52)$$

Eq.(30) indicates that the operators q_f have to be expressed in terms of new variables as

$$q_f=e^{-i(\vec{f}\vec{q})}B_f \quad . \qquad (53)$$

Note that the transformations (51), (53) introduce instead of , $\vec{z},\ldots,q_f,\ldots$ the variables $\vec{\lambda}$, $\vec{q},\ldots,B_f,\ldots$, the number of which is by 3 greater than earlier. As $\vec{\lambda}$ and $\vec{q}$ have been introduced to be independent variables, the number of the independent variables B_f should be constrained.

We notice that in the Hamiltonian (47) the kinetic energy represents a small perturbation, but, on the other hand just this energy depends on the variable $\vec{q}$. If this one is neglected then q_f will commute with the Hamiltonian and energy levels will be functions of q_f. In particular, the ground state of the system will be determined by some set of the numbers u_f. No dependence can arise between them because an additional variable has not yet appeared in this approximation. An account of the kinetic energy would produce small deviations of q_f from u_f, and the variables describing this deviation must obey three additional conditions. So, the transformation (53) reduces to the replacement

$$q_f = (u_f + \gamma Q_f)\, e^{-i\vec{f}\vec{q}}. \tag{54}$$

Variables Q_f can be subjected to the simplest linear additional conditions

$$\sum_f \vec{f}\, v_f^* Q_f = 0 . \tag{55}$$

All the quantities introduced anew satisfy the reality conditions

$$u_f^* = u_{-f}\ ,\quad v_f^* = v_{-f}\ ,\quad Q_f^+ = Q_{-f}\ , \tag{56}$$

and the numbers v_f without restricting the generality, may be chosen that the relation

$$\sum_f f_\alpha f_\beta u_f v_f^* = \delta_{\alpha\beta} \tag{57}$$

holds. Note that by the numbers u_f and v_f it is possible to construct a projection matrix

$$A_{fe} = \delta_{fe} - u_f (\vec{f}\vec{e}) v_e^* \tag{58}$$

and the variables Q_f can be represented as a linear combination of the independent variables Z_f :

$$Q_f = \sum_e A_{fe} Z_e \tag{59}$$

satisfying the reality condition. In this case the additional conditions (36) are fulfilled automatically. Now derivatives with respect to q_f reduce to the expressions

$$-i\frac{\partial}{\partial q_f} = \frac{1}{\gamma} e^{i\vec{f}\vec{q}} P_f' + \frac{\partial \vec{q}}{\partial q_f}\left[-i\frac{\partial}{\partial \vec{q}} + i\varkappa\frac{\partial}{\partial \lambda} + i\sum_e \vec{e}\, Q_e\, P_e'\right], \tag{60}$$

where

$$P_f' = P_f - v_f^* \sum_e (\vec{f}\vec{e}) u_e P_e \quad , \quad P_f = -i\frac{\partial}{\partial Q_f} \quad . \tag{61}$$

The derivative $\dfrac{\partial \vec{q}}{\partial q_f}$ can be easily found by differentiating the relation

$$\sum \vec{f} v_f^* \left(q_f e^{i\vec{f}\vec{q}} - u_f\right) = 0 \tag{62}$$

which follows from (55), and using the orthogonality condition Eq.(57). As a result we get the relation

$$\frac{\partial \vec{q}}{\partial q_f} = i\vec{f} v_f^* e^{i\vec{f}\vec{q}} - \gamma \sum \vec{e} \cdot \frac{\partial \vec{q}}{\partial q_f} v_e^* Q_e \,, \tag{63}$$

which gives

$$\frac{\partial \vec{q}}{\partial q_f} = e^{i\vec{f}\vec{q}} \vec{B}_f \quad . \tag{64}$$

Vector $\vec{B}_f$ can be explicitly written as a power series in small parameter γ Now for us it is sufficient to note that $\vec{B}_f$ depends only on Q_f , and all the dependence of $\dfrac{\partial \vec{q}}{\partial q_f}$ on $\vec{q}$ is concentrated in the exponential $e^{i\vec{f}\vec{q}}$. So, the derivative with respect to q_f becomes

$$-i\frac{\partial}{\partial q_f} = e^{i\vec{f}\vec{q}}\left[\frac{1}{\gamma} P_f' + \vec{B}_f\left(-i\frac{\partial}{\partial \vec{q}} + i\varkappa\frac{\partial}{\partial \lambda} + i\sum \vec{e}\, Q_e\, P_e'\right)\right] \tag{65}$$

and the kinetic energy is rewritten in the form

$$\frac{1}{2}\varkappa^4\gamma^4\sum \nu_f\left[\frac{1}{\gamma}P'_{-f}+\vec{B}_{-f}\left(-i\frac{\partial}{\partial\vec{q}}+i\varkappa\frac{\partial}{\partial\vec{\lambda}}+\vec{f}+i\sum\vec{e}\,Q_e P'_e\right)\right]\left[\frac{1}{\gamma}P'_f+\vec{B}_f\left(-i\frac{\partial}{\partial\vec{q}}+i\varkappa\frac{\partial}{\partial\vec{\lambda}}+i\sum\vec{e}\,Q_e P'_e\right)\right] . \quad (66)$$

Now let us make two remarks: First, the expression (66) reveals that the transformed Hamiltonian (47) does not depend explicitly on $\vec{q}$, and this results in the conservation of the total momentum. Second, on writing the Heisenberg equations for $\vec{q}(t)$ one can see that the time dependence of $\vec{q}(t)$ is rather complicated, so the naive picture described earlier, of the uniform motion of the potential well with a particle, may be regarded only as the first approximation to the true behaviour of the particle-field system.

The Bogolubov transformation (51), (54) has a simple group meaning[6]. The conservation of the total momentum is a result of the invariance of (47) with respect to the group of transformations

$$\vec{z}\to\vec{z}+\vec{q}\ , \quad q_f\to q_f e^{i\vec{f}\vec{q}}\ , \quad \vec{q}=const\ .$$

If $\vec{q}$ is treated as an operator then the canonically conjugate operator of derivative should be identified with the total momentum operator. On the other hand, supposing in (51), (54) the vector $\vec{q}$ being independent variable, we break up the vector $\vec{z}$ into two parts: First one, $\vec{x}$ is invariant under translations. Second, $\vec{q}$ changes with the phase of the operator q_f .

5. The Equation for the Ground State of a Particle

The derivatives with respect to $\vec{q}$ and Q_f in (66) contain small parameters γ^2 and γ respectively, as factors. Before proceeding to expand the energy and wave function in power series in γ, the wave function, therefore, should be transformed as

$$\Psi(\vec{q},\vec{\lambda},Q_f)=\exp\left[i\frac{\vec{J}\vec{q}}{\hbar}\cdot\frac{1}{\delta^2}\right]\exp\left[\frac{1}{\gamma}\sum S_f Q_f\right]\Psi'(\vec{\lambda},Q_f)\ . \tag{67}$$

The numbers S must satisfy the reality condition $S_f^*=S_{-f}$ and, besides, due to (55) they may be chosen in such a way that the conditions

$$\sum \vec{f} u_f S_f = 0 \tag{68}$$

hold. On this transformation the derivative with respect to $\vec{q}$ replaces by $\frac{1}{\delta^2}\frac{\vec{J}}{\hbar}$, and the momenta P_f' by $\frac{1}{\gamma}S_f+P_f'$. Now it is easy to get an expansion of the transformed Hamiltonian (47) in powers of the small parameter γ. A scheme for obtaining this expansion is given in detail in ref.[6]. Here it seems sufficient to confine our consideration to the first two terms

$$H=H_0+\gamma H_1\ , \tag{69}$$

where

$$H_0=-\varkappa^2\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\vec{\lambda}^2}-\varkappa^4\sum A_f u_f e^{i\frac{\vec{f}\vec{\lambda}}{\varkappa}}+\frac{\varkappa^4}{2}\sum\nu_f|u_f|^2+ \\ +\frac{\varkappa^4}{2}\sum\nu_f|\alpha_f|^2, \tag{70}$$

$$H_1=\sum\nu_f\alpha_f^* P_f'+\sum\left\{A_f e^{i\frac{\vec{f}\vec{\lambda}}{\varkappa}}+\nu_f u_f^*-\alpha_f\sum_e\nu_e\alpha_e^*(\vec{e}\vec{f})v_f^*\right\}Q_f\ , \tag{71}$$

$$\alpha_f=S_f+iv_f^*\frac{\vec{J}\vec{f}}{\hbar}\ . \tag{72}$$

Expanding the wave function and energy in powers of :

$$E=E_0+\delta E_1+\ldots\ ,\quad \Psi'=\Phi_0+\gamma\Phi_1+\ldots \tag{73}$$

the equation

$$(H-E)\Psi'=0 \tag{74}$$

can be reduced to the system

$$(H_0 - E_0)\Phi_1 + (H_1 - E_1)\Phi_0 = 0 \ , . \tag{75}$$

The operator H_0 does not act on the variables Q_f , so the wave function Φ_0 becomes the product

$$\Phi_0 = \varphi_0(\vec{\lambda})\Theta_0(Q_f), \tag{76}$$

where $\Theta_0(Q_f)$ is an arbitrary function of Q_f , and $\varphi_0(\vec{\lambda})$ obeys the equation

$$\left(-\varkappa^2 \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \vec{\lambda}^2} + \varkappa^4 \sum A_f u_f e^{i \frac{\vec{f}\vec{\lambda}}{\varkappa}} - W_0\right)\varphi_0(\vec{\lambda}) = 0 \, , \tag{77}$$

where

$$W_0 = E_0 - \frac{\varkappa^4}{2}\sum \nu_f |u_f|^2 - \frac{\varkappa^4}{2}\sum \nu_f |\alpha_f|^2 \ . \tag{78}$$

Taking the second of Eqs.(75) we find that because of the identity

$$\int \varphi_0^*(\vec{\lambda})(H_0 - E_0)\Phi_1 \, d\vec{\lambda} = 0 \tag{79}$$

the wave function $\Theta_0(Q_f)$ must obey the equation

$$\left[\int \varphi_0^*(\vec{\lambda}) H_1 \varphi_0(\vec{\lambda}) d\vec{\lambda} - E_1\right]\Theta_0(Q_f) = 0 \ . \tag{80}$$

The operator H_1 averaged over the wave functions $\varphi_0(\vec{\lambda})$ is linear in Q_f , P_f' , therefore Eq.(80) cannot have a regular solution except $\langle H_1 \rangle = 0$. Thus, second of (75), though not eliminating the arbitrariness in the choice of function $\Theta_0(Q_f)$, results in the conditions $E_1 = 0$,

$$\sum_f \left\{A_f \int e^{i\frac{\vec{f}\vec{\lambda}}{\varkappa}} |\varphi_0(\vec{\lambda})|^2 d\vec{\lambda} + \nu_f u_f^* - \alpha_f \sum \nu_e \alpha_e^* (\vec{f}\vec{e}) v_e^* \right\} Q_f = 0, \tag{81}$$

$$\sum \nu_f \alpha_f^* P_f' = 0 \ . \tag{82}$$

To satisfy (81), (82) one may put all coefficients of Q_f in (81) equal to zero and make use of the relation

$$\sum \vec{f} u_f P_f' = 0 \tag{83}$$

which immediately follows from the definition (61). The last equality indicates that (82) would be satisfied if α_f would be chosen so that

$$\nu_f \alpha_f^* = -i u_f \hbar (\vec{f}\vec{C}), \tag{84}$$

where $\vec{C}$ is some vector, which has to be taken in such a way that the additional conditions on S_f Eq.(68) remains valid. Taking into account all the conditions we get the relation for $\vec{C}$ to the total momentum

$$\vec{J} = \hbar^2 \sum \frac{\vec{f}(\vec{f}\vec{C})}{\nu_f} |u_f|^2 . \tag{85}$$

Substituting (84) in the expressions for the coefficients of the series (81) we find the numbers u_f:

$$u_f = -\frac{A_f^* \nu_f}{\nu_f^2 - \hbar^2(\vec{f}\vec{C})^2} \int e^{-i\frac{\vec{f}\vec{\lambda}}{\varkappa}} |\varphi_0(\vec{\lambda})|^2 d\vec{\lambda} . \tag{86}$$

Thus, considering the Hamiltonian in the first order in γ one can derive the expression for the potential in Eq.(77)

$$\varkappa^4 V(\vec{\lambda}) = \varkappa^4 \sum_f A_f u_f e^{i\frac{\vec{f}\vec{\lambda}}{\varkappa}} = -\varkappa^4 \sum_f \frac{\nu_f |A_f|^2 \langle e^{-i\frac{\vec{f}\vec{\lambda}}{\varkappa}} \rangle}{\nu_f^2 - \hbar^2(\vec{C}\vec{f})^2} e^{i\frac{\vec{f}\vec{\lambda}}{\varkappa}} . \tag{87}$$

The energy of the ground state is

$$E_0 = W_0 + \frac{1}{2} \sum |u_f|^2 \left(\nu_f + \frac{\hbar^2(\vec{C}\vec{f})^2}{\nu_f} \right) . \tag{88}$$

The expressions (87) and (88) for the potential of the Schrödinger equation and for the total energy as well as formula (85) for the total momentum coincide formally with the corresponding expressions derived in the previous section. Nevertheless there should be emphasized once more the different physical interpretation of these expressions. The relation (85), which connects the total momentum, the mean velocity of the particle and the numbers u_f , is the exact expression not depending on the accuracy of calculations. Therefore the transformation (51), (53) enables one to say with certainty that the main effect of the particle-field interactions in the non-weak coupling case is just the appearence of the potential well for the particle. In ref.[6] there was pointed out the general method in what way the higher-order approximations to the energy and wave functions can be taken into account if the quantum properties of the phonon field are included into consideration. We leave this question be open and proceed to study the strong coupling of a particle with the field.

6. Strong Coupling of a Particle with the Field

Up to now we have kept ourselves within the framework of adiabatic or intermediate coupling, especially dealing only with the small parameter γ . Now let us make use of the chance that in the strong-coupling limit there arises one more small parameter $\varkappa^{-1}$. In this case to describe the particle motion in the field and transitions between the excited states it is sufficient to retain in all expressions for exponential only such terms as

$$e^{i\frac{\vec{f}\vec{\Lambda}}{\varkappa}} = 1 + i\frac{\vec{f}\vec{\Lambda}}{\varkappa} - \frac{1}{2}\frac{(\vec{f}\vec{\Lambda})^2}{\varkappa^2}. \qquad (89)$$

Remembering the results of previous section one can guess that the potential well in such a case should be just the oscillatory well. This is quite natural approximation for the potential in problems with very deep and precipitous well and not too strongly excited state. Nevertheless, this limit is of a special interest from the physical viewpoint even if the perturbation series, which has a different structure than in the adiabatic coupling case, is not considered. If the strong coupling limit is only discussed ($\varepsilon^2=1$) we get the following expression for the coefficients γ and $\varkappa$

$$\gamma = \frac{1}{g} , \quad \varkappa = \sqrt{g} \quad . \tag{90}$$

In this case the Hamiltonian, wave function and energy are expanded in fractional powers of the coupling constant, so it is suitable to rewrite anew the corresponding expressions for the transformed Hamiltonian (47). In the accepted approximation the terms including only Q_f and $\vec{\lambda}$, provide the following contribution to the Hamiltonian

$$\begin{aligned} & g^2\sum A_f e^{i\vec{f}\vec{z}} q_f + \frac{1}{2} g^2 \sum \nu_f q_{-f} q_f = g^2 \sum A_f u_f + \frac{1}{2} g^2 \sum \nu_f |u_f|^2 + \\ & + i g^{3/2} \sum A_f (\vec{f}\vec{\lambda}) u_f - \frac{1}{2} g \sum A_f u_f (\vec{f}\vec{\lambda})^2 + g\sum A_f Q_f + g \sum \nu_f u_f^* Q_f + \\ & \quad + i g^{1/2} \sum A_f (\vec{f}\vec{\lambda}) Q_f + \frac{1}{2} \sum \nu_f Q_{-f} Q_f - \frac{1}{2} \sum A_f (\vec{f}\vec{\lambda})^2 Q_f \quad . \end{aligned} \tag{91}$$

On replacing the wave function analogously with (67) we get

$$\Psi(\vec{q},\vec{\lambda},Q) = \exp\left(i g \frac{\vec{J}\vec{q}}{\hbar}\right) \exp\left(i g \sum s_f Q_f\right) \Phi(\vec{\lambda}, Q) \quad . \tag{92}$$

The part of Hamiltonian associated with the kinetic energy of phonons takes the form

$$
\begin{aligned}
\frac{1}{2g^2}\sum \nu_f P_{-f} P_f = {} & \frac{1}{2} g^2 \sum \nu_f |\alpha_f|^2 + g \sum \nu_f \alpha_f^* P_f' - g \sum_f \alpha_f Q_f \sum_e (\vec{f}\vec{e}) \nu_e \alpha_e^* v_e^* \\
& - g^{1/2} \sum \nu_f \alpha_f^* v_f^* \left(\vec{f}\frac{\partial}{\partial \vec{\lambda}}\right) + \frac{1}{2} \sum \nu_f \left(P_{-f}' + v_f \sum_e (\vec{f}\vec{e}) \alpha_e Q_e\right)\left(P_f' - v_f^* \sum_e (\vec{f}\vec{e}) \alpha_e Q_e\right) \\
& + \sum \nu_k \alpha_k^* (\vec{k}\vec{f})(\vec{f}\vec{e}) \alpha_e v_k^* v_f^* Q_f Q_e - \sum \nu_k \alpha_k^* v_k^* (\vec{k}\vec{f}) Q_f P_f' - \\
& - \frac{i}{2} \sum \nu_f v_f \vec{f}^2 \alpha_f
\end{aligned}
\tag{93}
$$

The kinetic energy in the strong coupling limit is proportional to g. Thus all the terms of transformed Hamiltonian (47) do not contain the variables Q_f and $\vec{\lambda}$ simultaneously, and Q_f and P_f' are only in a linear combination in the Hamiltonian. On taking the initial Hamiltonian in the form of the sum of terms of the order of g and higher, we find that the wave function in the zeroth approximation, as before, is represented by the product of functions depending on $\vec{\lambda}$ and Q_f only, and to ensure the regularity for the function depending only on Q, it should be required that the conditions analogous with (81), (82)

$$\sum \nu_f \alpha_f^* P_f' = 0 , \tag{94}$$

$$A_f + \nu_f u_f^* - \alpha_f \sum (\vec{f}\vec{e}) \alpha_e^* \nu_e v_e^* = 0 \tag{95}$$

must hold. Therefore it is necessary to put again $\nu_f \alpha_f^* = -i u_f k (\vec{f}\vec{c})$ and

$$u_f = - \frac{\nu_f A_f^*}{\nu_f^2 - k^2 (\vec{f}\vec{c})} . \tag{96}$$

On performing an additional transformation of the wave function

$$\Phi(\vec{\lambda}, Q) = \exp\left(-i g^{-1/2} \frac{\mu \vec{c}\vec{\lambda}}{k}\right) \Phi'(\vec{\lambda}, Q) \tag{97}$$

the initial Hamiltonian becomes

$$H = g^2 \sum A_f u_f + \frac{1}{2} g^2 \sum |u_f|^2 \left(\nu_f + \frac{\hbar^2 (\vec{f}\vec{c})^2}{\nu_f} \right) - \frac{1}{2} \mu \vec{c}^{\,2}$$
$$- g \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \vec{\lambda}^2} - \frac{1}{2} g \sum A_f u_f (\vec{f}\vec{\lambda})^2 - i g^{1/2} \sum A_f (\vec{f}\vec{\lambda}) Q_f \tag{98}$$
$$+ \frac{1}{2} \sum \nu_f P''_{-f} P''_f - \frac{i}{2} \hbar \sum (\vec{f}\vec{c}) (Q_f P''_f + P''_f Q_f) + \frac{1}{2} \sum \nu_f Q_{-f} Q_f \, ,$$

where

$$P''_f = P'_f - i \hbar \, \zeta_f^* \sum_e \frac{(\vec{f}\vec{e})(\vec{e}\vec{c})}{\nu_e} Q_e \, . \tag{99}$$

In the first-order approximation the particle wave function obeys the equations

$$\left(- \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial \vec{\lambda}^2} - \frac{1}{2} \sum A_f u_f (\vec{f}\vec{\lambda})^2 - W \right) \varphi(\vec{\lambda}) = 0 \, , \tag{100}$$

$$W = \frac{1}{g} \left[E_0 - g^2 \sum A_f u_f - \frac{1}{2} \sum \nu_f |u_f|^2 \left(\nu_f + \frac{\hbar^2 (\vec{f}\vec{c})^2}{\nu_f} \right) + \frac{1}{2} \mu \vec{c}^{\,2} \right. \, . \tag{101}$$

It is easy to show that

$$- \frac{1}{2} \sum A_f u_f (\vec{f}\vec{\lambda})^2 = A \vec{\lambda}^2 + B (\vec{c}\vec{\lambda})^2 , \tag{102}$$

where

$$A = - \frac{1}{4} \sum A_f u_f \left[\vec{f}^{\,2} - \frac{(\vec{f}\vec{c})^2}{\vec{c}^{\,2}} \right] \, , \tag{103}$$

$$B\vec{C}^2 = -\frac{1}{4}\sum A_f u_f\left[3\frac{(\vec{f}\vec{C})^2}{\vec{C}^2} - \vec{f}^2\right] . \qquad (104)$$

For simplicity choosing $\vec{C}$ along z-axis we obtain that Eq.(100) describes the motion of the anisotropic oscillator with frequencies

$$\omega_1 = \omega_2 = \sqrt{\frac{2A}{\mu}} \quad , \quad \omega_3 = \sqrt{\frac{2(A+B\vec{C}^2)}{\mu}} . \qquad (105)$$

The particle energy levels are given by

$$E_{n_1 n_2 n_3} = g\hbar\left[\omega_1\left(n_1+\frac{1}{2}\right) + \omega_2\left(n_2+\frac{1}{2}\right) + \omega_3\left(n_3+\frac{1}{2}\right)\right] + g^2\sum A_f u_f + \frac{1}{2}g^2\sum |u_f|^2\left(\nu_f + \frac{\hbar^2(\vec{f}\vec{C})^2}{\nu_f}\right) - \frac{1}{2}\mu\vec{C}^2 \qquad (106)$$

from which it follows that a distance between the energy levels is proportional to g. Note that in the strong coupling limit the levels of the ground and excited states are determined by the same Eq.(100). This is due to the fact that in the strong coupling limit the relations (96) which determine u_f , do not contain the form factor of the particle. In this case the potential well is so precipitous that the particle may only oscillate with a large frequency but negligible amplitude around its equilibrium point. Therefore Eq.(100) becomes linear in the wave function of the ground state and the resultant equation in the variations coincides with the initial one. The same result can be derived from Eq.(37) by expanding the kernel Eq.(38) in power series in $\varkappa^{-1}$ and putting the wave functions of the excited states being orthogonal to that of the ground state. It should be emphasized that though the expressions for the kinetic energy and effective mass now include only the absolute square of A_f , as in the weak coupling limit, nevertheless the corresponding formulae have very different structure and are not reduced to those of the weak coupling theory. All the effects due to the recoil in phonon emission have been taken into account in splitting the radius vector into two parts by the transformation (51), (54), and now the total momentum conservation is allowed for in a different way than by the canonical transformation (7) used in the weak coupling limit. This can be understood if recall that in the weak coupling limit the phonon field is represented by a superposition of independent free phonon states, of which the operators obey the canonical commutation relations, while in the strong coupling case the phonon field states are determined by the variables Q_f , P_f' satisfying the additional conditions Eqs.(55),(89) and treated as some kind of collective coordinates

The states corresponding to the levels of the energy (106) are not, of course, the stationary states of the system. However, before we will look for transitions between those states, note that we have not yet had the equations of motion for the operators Q_f , P_f' . To get these equations and the above stationary states one needs to diagonalize a certain quadratic form composed by Q_f and P_f' . This problem was discussed in detail in ref.[6]. Here one may stay in the simpler qualitative consideration. It is seen from Eq.(98) that the phonon field energy is small compared to that of the particle and dipole interaction between the particle and phonons, of which the density is proportional to $g^{1/2}$. Therefore in the first approximation the quadratic form is

$$\sum \nu_f Z_f^+ Z_f \quad , \tag{107}$$

where the operators Z_f , Z_f^+ obey the canonical commutation relations, may be associated with the energy of free phonon field. This form can be picked out from the Hamiltonian (98) by use of Eqs. (59) and (61) which relate the variables Q_f , P_f' to the corresponding independent variables. Considering the transitions caused by the dipole interaction

$$i g^{1/2} \sum A_f (\vec{f}\vec{\lambda}) Q_f \tag{108}$$

it is easy to see that the probability of the above transitions and, consequently, the width of the levels (106) are proportional to the coupling constant g ,i.e. the width Eq. (106) is of the order of a distance between levels. Therefore the states of harmonic oscillator only very hardly may be regarded as the stationary states of the system. Nevertheless, it is not difficult to get more stable states. To this end, the insertion of bilinear form in $\vec{\lambda}$ and Q_f Eq.(108) as well as quadratic form Eq. (107) into the initial Hamiltonian seems to be sufficient

For higher symmetry let us express the coordinates and momenta of the oscillator through the appropriate creation and annihilation operators.

$$\begin{aligned} a_f &= \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{\mu\omega_\alpha}{\hbar}}\,\lambda_\alpha + i\sqrt{\frac{1}{\mu\hbar\omega_\alpha}}\,P_\alpha\right\}, \\ a_f^+ &= \frac{1}{\sqrt{2}}\left\{\sqrt{\frac{\mu\omega_\alpha}{\hbar}}\,\lambda_\alpha - i\sqrt{\frac{1}{\mu\hbar\omega_\alpha}}\,P_\alpha\right\} \end{aligned} \tag{109}$$

and Q_f through the operators Z_f by means of Eqs.(59),(61), Then, on extracting the factor g the initial Hamiltonian may be put as

$$\sum_{\alpha} \hbar \omega_{\alpha} a_{\alpha}^{+} a_{\alpha} + \sum_{\alpha, f} (a_{\alpha} + a_{\alpha}^{+})(\mathcal{D}_{\alpha f} Z_f + \mathcal{D}_{\alpha f}^{*} Z_f^{+}) + \sum \tilde{\nu}_f Z_f^{+} Z_f \, , \tag{110}$$

where

$$\mathcal{D}_{\alpha f} = -i g^{-1/2} \cdot \frac{1}{2} \sqrt{\frac{\hbar}{\mu \omega_{\alpha}}} \sum_{e} A_e \, e_{\alpha} \left(\delta_{fe} - u_e (\vec{e} \vec{f}) v_f^{*} \right), \quad \tilde{\nu}_f = \frac{\nu_f}{g} \, . \tag{111}$$

Introducing uniform notations for the operators a_{α} and Z_f

$$b_{\alpha} = \begin{pmatrix} a \\ Z \end{pmatrix} \tag{112}$$

and defining matrices

$$B = \begin{pmatrix} \omega & \mathcal{D} \\ \mathcal{D}^{+} & \tilde{\nu} \end{pmatrix}, \quad A = \begin{pmatrix} 0 & \mathcal{D}^{*} \\ \mathcal{D}^{+} & 0 \end{pmatrix} \tag{113}$$

the quadratic form (110) takes on the form

$$\frac{1}{2} \sum A_{\alpha\beta} b_{\alpha}^{+} b_{\beta}^{+} + \frac{1}{2} \sum A_{\alpha\beta}^{*} b_{\alpha} b_{\beta} + \sum B_{\alpha\beta} b_{\alpha}^{+} b_{\beta} \, , \tag{114}$$

which is diagonalized by the well-known canonical transformation

$$\xi_{\mu} = \sum \left(u_{\alpha\mu}^{*} b_{\alpha} - v_{\alpha\mu}^{*} b_{\alpha}^{+} \right), \quad \xi_{\mu}^{+} = \sum \left(u_{\alpha\mu} b_{\alpha}^{+} - v_{\alpha\mu} b_{\alpha} \right) , \tag{115}$$

where set of eigenfunctions $u_{\alpha\mu}$, $v_{\alpha\mu}$ obey the equations

$$E_{\mu} u_{\alpha\mu} = \sum A_{\alpha\beta} v_{\beta\mu} + B_{\alpha\beta} u_{\beta\mu}, \quad -E_{\mu} v_{\alpha\mu} = \sum A_{\alpha\beta}^{*} u_{\beta\mu} + B_{\alpha\beta}^{*} v_{\beta\mu} \tag{116}$$

and the orthonormality conditions

$$\sum_{\mu} u_{\alpha\mu} u^{*}_{\alpha\mu'} - v_{\alpha\mu} v^{*}_{\alpha\mu'} = \delta_{\mu\mu'} \quad , \quad \sum_{\mu} u_{\beta\mu} u^{*}_{\alpha\mu} - v_{\beta\mu} v^{*}_{\alpha\mu} = \delta_{\alpha\beta} \; . \tag{117}$$

The quadratic form (110) by this transformation is reduced to the form

$$\sum_{\mu} E_{\mu} \xi^{+}_{\mu} \xi_{\mu} - \sum_{\alpha\mu} E_{\mu} v^{*}_{\alpha\mu} v_{\alpha\mu} \; . \tag{118}$$

From the definition of the matrix $\mathcal{D}$ (111) it follows that the elements of A -matrix producing a deviation of the quadratic form (114) from the diagonal shape, is proportional to a small quantity $g^{-1/2}$. Hence the eigenfunctions $v_{\alpha\mu}$ which determine the deviation of (115) from the identical transformation are proportional to $g^{-\frac{1}{2}}$, as well. Thus the account of dipole interaction (108) results in a small shift of levels (106), and distances between the new stationary levels are proportional to g. The matrix elements responsible for the transitions between these states are of zeroth order in g. Therefore the width of levels is considerably less that the distance between those.

7. Conclusion

Thus, we can conclude that the application of the Bogolubov method to the strong-coupling problem discussed above makes it possible to separate the motion of a particle in the field taking into account the conservation law of the total momentum. A picture of preparing the potential well by particle which then moves together with the particle has been constructed by this transformation and found to be successive.

In the strong-coupling limit this potential well becomes oscillatory one, and the ground state of the system is described by a set of shifted oscillators formed by the particle and field variables.

A simple problem discussed here may be a convenient fool for modelling the high-energy oscillatory interaction considered in Refs./2,4,13/.

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