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ААБОРАТОРНЯ TEOPETИUELKOИ் О МЗМКИ

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## THE ACCOUNT OF ClOSED LOOPS

 AND RELATIVISTIC EIKONAL REPRESENTATION1971

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THE ACCOUNT OF CLOSED LOOPS<br>AND RELATIVISTIC EIKONAL REPRESENTATION

1. Owing to the fact that the eikonal representation for the scattering amplitude is found to be a convenient tool for theoretical analysis of experimental data on high-energy hadron scattering /1/, numerous attempts have been made to derive an eikonal formula from a set of Feynman graphs of a certain class.

The first step towards this are some papers devoted to the summation of the set of s -channel ladder diagrams for multy-meson exchange
$s$

in electrodynamics $/ 2 /$ as well as in certain simplest field theory models /3,4/.

In investigating this problems a rich "tool kit", from direct summation of leading perturbation series terms to methods of continuous integration, has been used. All these methods are, to a certain extent, approximate and imply the supposition that "high-energy individuality" of incident particles is kept in the process of their interaction. In spite of a large number of attempts to give grounds for this supposition (egg., refs. 5) it is
not well-founded. Nevertheless, it is easy to see that in this case the eikonal formula reproduces correctly the asymptotic at high energies and fixed momentum transfers, at least, in some first perturbation orders.

It would be very valuable to obtain a rigorous proof for the eikonal methods of summing the Feynman graphs. At the same time, it is also interesting to attempt to extend these methods to a wider class of graphs and to obtain on their basis various predictions on the high-energy behaviour for the amplitudes of elastic collisions of hadrons. The necessary condition is that the approximations and assumptions leading to an eikonal picture of the highenergy particle scattering should be formulated as clear as possible.

Thus, the second step towars studying the relativistic eikonal approximation is a large series of papers in which the mentioned methods were used for summing different $s$-channel diagrams taking into account radiational corrections/6/, vacuum polarization effects and closed nucleon loops/7/. The predictions made on this basis concern both elastic and inelastic processes.

To study the behaviour of the high-energy hadron interaction amplitude the Dubna group has used methods of functional integration in quantum field theory. The aduantage of these methods consists in that they make it possible to explore the structure of the scattering amplitude in a closed form/8/.

The straight-1ine path approximation developed on the basis of functional integration methods makes it possible to take into account the contribution of Feynman paths, which are located most closely to the classic (straight-1ine for $s \rightarrow \infty$ and fixed $t$, particle trajectory, to the
continuous integral for the scattering amplitude. Note that the assumption on the dominance of the contribution of these paths together with other results (egg., refs./3,9/) leads simply to an eikonal representation for the scattering amplitude which is defined by the sum of the multi-meson exchange diagrams including the radiative corrections. In the present note we have attempted to apply a simplified version of the straight-line path approximation to the case of a wider class of Feynman graphs for which this approximation can not, generally speaking, be considered a prior to be valid. It is shown that in the framework of the approximation used the scattering amplitude has the form of the eikonal representation

II. We consider the scattering of scalar "nucleons" (field $\psi$, mass $m$ ) exchanging of vector "mesons" (field $\overline{A_{a}}$, mass $\mu$ ) in a model with the interaction Lagrangian

$$
\begin{equation*}
L_{t \mathrm{nt}}=g: \psi^{*}(x) i \partial_{a} \psi(x) A_{a}(x):+\frac{g}{2}^{2}: A_{a}^{2}(x) \psi^{*}(x) \psi(x): \tag{3}
\end{equation*}
$$

The two-particle scattering amplitude is determined by the formula

$$
\begin{equation*}
i(2 \pi)^{4} \delta^{4}\left(p_{1}+p_{2}-q_{1}-q_{2}\right) f\left(p_{1}, p_{2} \mid q_{1}, q_{2}\right)=\underset{p_{\ell}^{2}, q_{\ell}^{2} \rightarrow m^{2} \prod_{=1}^{2}}{\lim _{l}}{\left(p_{l}^{2}-m^{2}\right)\left(q_{l}^{2}-m^{2}\right)}^{2} \tag{4}
\end{equation*}
$$

$C_{A} \int \delta^{4} A_{a} e^{-\frac{1}{2} A_{a} \times D_{a}^{-1} \times A^{A} \beta^{+\Pi(A)}} G\left(p_{1}, q_{1} \mid A\right) G\left(p_{2}, q_{2} \mid A\right)$.
In eq. (4) the following notations are introduced: $e^{\Pi(A)}$ is the vacuum expectation value of the $S$-matrix in a field $A$ which takes into account the contribution
of closed nucleon loops to the scattering amplitudes, $C_{A}$ is the normalization factor

$$
\begin{equation*}
C_{A}^{-1}=\int \delta^{4} A_{a} e^{-\frac{1}{2} A_{a} \times D_{a}^{-1} \times A^{A}+\Pi_{(A)}} \tag{5}
\end{equation*}
$$

and the symbol $\times$ means the operation

$$
\begin{equation*}
A \times B=\int d^{4} x A(x) \cdot B(x) \tag{6}
\end{equation*}
$$

The one-particle nucleon Green function $G(x, y \mid A)$ in the model under consideration obeys the equation

$$
\begin{equation*}
\left\{\left[i \partial_{\alpha}+g A_{a}(x)\right]^{2}-m^{2}\right\} G(x, y \mid A)=-\delta^{4}(x-y) \tag{7}
\end{equation*}
$$

and for its Fourier transform the following representation in the form of a functional integral over trajectories/9/

$$
\begin{gathered}
G(p, q \mid A)=i \int_{0}^{\infty} d s e^{i s\left(p^{2}-m^{2}\right)} \int \frac{d y}{(2 \pi)^{4}} e^{t(p-q) y} \cdot \\
\cdot \int\left[\delta^{4} \nu\right]_{0}^{s} \exp \left\{2 i g_{0} \int_{0}^{s} d \xi\left[\nu_{a}(\xi)+p_{a}\right] A_{a}\left[y+2 p \xi+2 \int_{0}^{\zeta} \nu(\eta) d \eta\right]\right.
\end{gathered}
$$

holds, where $\left[\delta^{4} \nu\right]_{\tau_{1}}^{T_{2}^{2}}$ denotes the functional averaging with Gaussian weight

$$
\begin{equation*}
\left[\delta^{4} \nu\right]_{\tau_{1}}^{\tau_{2}}=\frac{\delta^{4} \nu e^{-1 \int_{1}^{2} \nu^{2}(\eta) d \eta}}{\int \delta^{4} \nu e^{-1} \int_{1}^{2} \nu^{2}(\eta) d \eta} . \tag{9}
\end{equation*}
$$

Taking into account (8) the scattering amplitude takes the form

$$
\begin{aligned}
& i(2 \pi)^{4} \delta\left(p_{1}+p_{2}-q_{1}-q_{2}\right) f\left(p_{1}, p_{2} \mid q_{1}, q_{2}\right)=\operatorname{pim}_{p_{\ell}^{2} \cdot q_{l}^{2} m^{2}} \prod_{\ell=1}^{2}\left(p_{l}^{2}-m^{2}\right)\left(q_{l}^{2}-m^{2}\right)
\end{aligned}
$$

where the functional

$$
\begin{equation*}
\Pi(j) \equiv C_{A} \int \delta{ }^{4} A_{a} e^{-\frac{1}{2} A_{a} \times D_{a \beta}^{-1} \times A^{A} \beta^{+\Pi(A)+1]_{a}} \times A_{a}} \tag{11}
\end{equation*}
$$

depends upon the sum of the currents of two nucleons

$$
\begin{equation*}
j_{a}=j_{1 a}^{\left(0, a_{2}\right)}\left(x \mid y_{1}\right)+j_{2 a}^{\left(0, s_{2}\right)}\left(x \mid y_{2}\right), \tag{12}
\end{equation*}
$$

in this case
$j_{\ell a}^{\left(0, Q_{\ell}\right)}\left(x \mid y_{\ell}\right)=2 g \int_{0}^{\Omega \ell} d \xi_{\ell}\left[\nu_{\ell a}+p_{\ell a}\right] \delta^{4}\left[y_{\ell}+2 p_{\ell} \xi_{\ell}+2 \int_{0}^{\xi_{\ell}} v_{\ell}(\eta) d \eta-x\right]$.
In order to go over to the mass shell in expression (10) one should make a substraction corresponding to the propagation of two particles without interaction/10,8/. It is obvious that this procedure reduces to the replacement in expression (10) of $\pi\left(j_{1}^{\left(0, s_{1}\right)}+j_{2}^{\left(0, \theta_{2}\right)}\right.$ )
by
$\left.\left.\pi\left(j_{1}^{\left(0, a_{1}\right)}+j_{2}^{\left(0, \theta_{2}\right)}\right)-\pi j_{i}^{\left(0, s_{1}\right)}\right) \pi\left(j_{2}^{(0,8}\right)_{2}\right)=$
$\left.=\pi\left(j_{1}^{\left(0, s_{1}\right)}\right) \pi\left(_{j_{2}}^{\left(0, A_{2}\right)}\right)\left[\Gamma\left(j_{1}^{\left(0, \theta_{1}\right)}, j_{2}^{(0,8}\right)_{2}\right)-1\right]$.

Further it is convenient to employ the following representation
$\left.\Gamma\left(j_{1}^{\left(0, s_{1}\right)}, j_{2}^{\left(0, s_{2}\right)}\right)-l=j_{1}^{(0, s)} \mathcal{Z}_{1_{1}}\right) \times j_{2}^{\left(0, s_{2}\right)}\left(x_{2}\right) \times F\left(x_{1}, x_{2} \mid j_{1}^{\left(0, A_{1}\right)}, j_{2}^{\left(0, \theta_{2}\right)}\right)$
the validity of which follows from expansion of $\Gamma\left(j_{2}^{(0,0,1)}, j_{2}^{\left(0, \theta_{2}\right)}\right)-1 \quad$ in a power series in $j_{1}^{(0,0,1)}$ and $j_{2}^{\left(0, \theta_{2}\right)}$ $\Gamma(j_{2}^{\left(0, \theta_{1}\right)}, j_{2}^{\left(0, s_{2}\right)}-1=\sum_{n_{1}, n_{2} Z^{2}}^{j_{1}^{\left(0, s_{2}\right)} \times \ldots \times j_{1}^{\left(0, \theta_{1}\right)}} \times \underbrace{j_{2}^{\left(0, s_{2}\right)} \times \ldots \times j_{2}^{(0,82)}}_{n_{2}} \times \underbrace{\prime}_{n_{1}, n_{2}}$
and which, as is easily seen, corresponds to the expansion of the scattering amplitude in a series in the coupling constant.

It should be noted that, due to the energy-momentum conservation law the functional $F$ possesses the property of translation invariance with respect to $x$.

When eq. (13) is taken into account, eq. (15) takes the following form
$\Gamma\left(j_{1}^{\left(0, s_{1}\right)}, j_{2}^{\left(0, s_{2}\right)}\right)-1=\int_{0}^{s_{1}} d \eta_{1} \int_{0}^{s_{2}} d \eta_{2}\left[2 p_{1 a}+2 \nu_{1 \alpha}\left(\eta_{1}\right)\right]\left[2 p_{2} \beta^{+2 \nu_{2 \beta}}\left(\eta_{2}\right)\right]$.

- $F_{a \beta}\left[y_{1}+2 p_{1} \eta_{1}+2 \int_{0}^{\eta_{1}} \nu_{1}(\eta) d \eta ; y_{2}+2 p_{2} \eta_{2}+\left.2 \int_{0}^{\eta_{2}} \nu_{2}(\eta) d \eta\right|_{j_{1}}{ }^{\left(0, s, s_{1}\right)} j_{j_{2}}^{(0, s}{ }^{(0\}}\right]$.

Inserting eq. (17) in expression for the scattering amplitude (10) we make the following replacements of variables

$$
\begin{align*}
& \text { a) } y_{\ell} \rightarrow y_{\ell}-2 p_{\ell} \eta_{\ell}-2 \int_{0}^{\eta_{\ell}} \nu_{\ell}(\eta) d \eta ; \\
& \text { b) } \quad \nu_{\ell}\left(\xi_{\ell}\right) \rightarrow \nu_{\ell}-\left(p_{\ell}-q_{\ell}\right) \theta\left(\eta_{\ell}-\xi_{\ell}\right) ; \\
& \text { c) } \quad s_{\ell} \rightarrow s_{\ell}+\eta_{\ell} ; \\
& \text { d) } \quad \xi_{\ell} \rightarrow \xi_{\ell}+\eta_{\ell} ; \\
& \text { e) } \quad \nu_{\ell}\left(\eta_{\ell}+\xi_{\ell}\right) \rightarrow \nu_{\ell}\left(\xi_{\ell}\right) ;  \tag{18}\\
& \text { f) } \quad x \rightarrow x+\frac{y_{1}+y_{2}}{2} .
\end{align*}
$$

Then taking into account translation invariance the twoparticle scattering amplitude reads

$$
\begin{align*}
& i(2 \pi)^{4} \delta^{4}\left(p_{1}+p_{2}-q_{1}-q_{2}\right) f\left(p_{1}, p_{2} \mid q_{1}, q_{2}\right)=\lim _{\substack{2 \\
p_{\ell}^{2}, q_{\ell}^{2} \rightarrow m}}^{\prod_{R=1}^{2}\left(p_{\ell}^{2}-m^{2}\right)\left(q_{\ell^{2}}^{2} m^{2}\right) .} \\
& \cdot \iint d^{4} y_{\ell} e^{i y_{\ell}\left(p_{\ell}-p_{\ell}\right.} \int_{0}^{\infty} \int_{0}^{\infty} d s_{\ell} e^{1\left(p_{\ell}^{2}-q_{\ell}^{2}\right)_{\ell}} \iint\left[\delta^{4} \nu_{\ell}\right]_{-\eta_{\ell}}^{s} . \\
& \text {. } \pi\left(j_{\ell}^{\left(-\eta_{\ell},{ }_{\ell}\right)}\right) \int_{0}^{\infty} \int_{0}^{\infty} d \eta_{\ell}\left[2 p_{\ell a}+2 \nu_{\ell a}\left(\eta_{\ell}\right)\right] . \tag{19}
\end{align*}
$$

where

$$
\begin{align*}
& j^{\left(-\eta_{\ell}, s \ell\right.}\left\{x,(-1)^{\ell-1}\left(y_{1}-y_{2}\right) / 2\right]=g \int_{-\eta_{\ell}}^{s} d \xi\left[2 \nu_{\ell}(\xi)+a_{\ell}(\xi)\right] \\
& \cdot \delta^{4}\left[(-1)^{\ell-1}\left(y_{1}-y_{2}\right) / 2+a_{\ell}(\xi)+2 \int_{0}^{\xi_{\ell}} \nu_{\ell}(\eta) d \eta-x\right]  \tag{20}\\
& a_{\ell}(\xi)=2 p_{\ell} \theta(\xi)+2 q_{\ell} \theta(-\xi) .
\end{align*}
$$

Making a translation to the mass shell $p_{\ell}^{2}, q_{\ell}^{2} \rightarrow m^{2}\left(s_{\ell}, \eta_{\ell} \rightarrow \infty\right)$ as it was done in refs. $/ 3,8 /$ and integrating the expression over $\bar{y}\left(\bar{y}=\frac{y_{2}+y_{2}}{2}, y=\frac{y_{1}-y_{2}}{2}\right)$ we finally get $f\left(p_{1}, p_{2} \mid q_{1}, q_{2}\right)=\frac{i}{(2 \pi)^{4}} \int d^{4} y e^{i y\left(p_{1}-q_{1}\right)} \int\left[\delta^{4} \nu_{1}\right]_{-\infty}^{\infty}\left[\delta \nu_{2}\right]_{-\infty}^{\infty} \pi\left(j_{1}^{(-\infty, \infty)}\right)$.
$\left.\cdot \pi\left(j_{2}^{(-\infty, \infty)}\right)\left[2 \nu_{2}(0)+a_{1}(0)\right]\left[2 \nu_{2}(0)+a_{2}(0)\right] F_{a \beta^{\prime}} \frac{y}{2}, \left.-\frac{y}{2} \right\rvert\, y_{1}^{(-\infty, \infty)}, y_{2}^{(-\infty, \infty)}\right)$.
III. In what follows we shall be interested in expression (21) in the asymptotic domain $s \rightarrow \infty$ and for fixed $t$. Since an exact calculation of the functional integrals over $\nu_{1}, \nu_{2}$ in eq. (21) seems to be impossible, we use an approximation according to which in the functional $F_{a \beta}$ one neglects the dependence upon the variables $\nu_{1}$ and $\nu_{2}$,
which describe the deflection of the particle path from the classical one, as well as the dependence upon the momentum transfer $p_{i}-q_{i}$.

Making the replacement of the variables

$$
\begin{equation*}
y=y_{L}+2 p_{2} \gamma_{1}-2 p_{2} \gamma_{2} \tag{22}
\end{equation*}
$$

and considering the domain in which, the condition/3/

$$
\begin{equation*}
\left(p_{1}-q_{1}\right) y \approx\left(p_{1}-q_{2}\right) y_{L} \tag{23}
\end{equation*}
$$

holds, we obtain for the scattering amplitude the following expression

$$
\begin{align*}
& f\left(P_{1}, P_{2} \mid q_{1}, q_{2}\right)=\frac{i \overline{\pi\left(j_{1}^{(-\infty, \infty)}\right) \pi\left(j_{2}^{(-\infty, \infty)}\right)}}{(2 \pi)^{4}} \int d^{4} y e^{t y\left(p_{1}-q_{1}\right)} P_{1 \alpha} P_{2} \beta . \\
& \text { - } F_{a \beta}\left(\frac{y}{2}, \left.-\frac{y}{2} \right\rvert\, \tilde{j}_{1}, j_{2}\right)=\frac{\overline{i \pi\left(j_{1}^{(-\infty, \infty)}\right) \pi\left(j_{2}^{(-\infty, \infty)}\right)}}{(2 \pi)^{4}} . \\
& \cdot s \int d^{2} \gamma_{\perp} e^{i y_{1}\left(p_{1}-q_{2}\right)_{1}} \int d \gamma_{1} d \gamma_{2} P_{1 a} P_{2 \beta} F_{a \beta}\left(\frac{y_{1}}{2}+P_{1} \gamma_{1}-P_{2} \gamma_{2},-\frac{y_{\perp}}{2}-\right.  \tag{24}\\
& \left.-p_{1} \gamma_{1}+p_{2} \gamma_{2} \mid \overline{j_{1}, j_{2}}\right)=\frac{\overline{i \pi\left(j_{1}^{(-\infty, \infty)}\right) \pi\left(j_{2}^{(-\infty, \infty)}\right)}}{(2 \pi)^{4}} s \int d^{2} y_{1} e^{t y_{1}\left(p_{1}-q_{1}\right)} \int d x_{i} .
\end{align*}
$$

$\cdot I x_{2} j_{1 a}\left(x_{1}+y_{\perp}\right) j_{2 \beta}\left(x_{2}-y_{\perp}\right) \cdot F_{a \beta}\left(\frac{x_{1}-x_{2}}{2}, \left.-\frac{x_{1}-x_{2}}{2} \right\rvert\, \tilde{i_{1}}, \tilde{j_{2}}\right)$
where
$\tilde{j}_{\ell_{a}}(x)=2 g \int_{-\infty}^{\infty} d \xi_{P_{\ell a}} \delta^{4}\left(x-2 P_{\ell} \xi\right)=g \frac{P_{\ell_{a}}}{P_{\ell_{0}}} \delta^{3}\left(\frac{P_{\ell a}}{P_{\ell_{0}}} x_{0}-\vec{x}\right)$,
is the classical current of the particle.
In expression (24) the factor $\overline{\pi\left(j_{1}\right) \pi\left(j_{2}\right)}$
corresponds to the contribution of the radiative corrections and is independent of $s$, and the stroke means the averaging over the functional variables $\nu_{1}$ and $\nu_{2}$. Owing to translation invariance of the functional $F$ we have

$$
\begin{equation*}
F\left(\frac{x_{1}-x_{2}}{2}, \left.-\frac{x_{1}-x_{2}}{2} \right\rvert\, \tilde{j_{2}, j_{2}}\right) \rightarrow F\left(x_{1}, x_{2} \mid \tilde{j_{2}}, \tilde{j}_{2}\right) . \tag{26}
\end{equation*}
$$

Now taking into account (17) we obtain for the twoparticle scattering amplitude an expression in the eikonal form

$$
\begin{equation*}
f\left(p_{1}, p_{2} \mid q_{1}, q_{2}\right)=\frac{\sqrt{\pi_{2} M_{2}}}{(2 \pi)^{4}} s \int d^{2} y_{\perp} e^{t y_{L}\left(p_{1}-q_{1}\right)_{\perp}}\left(e^{\left(X X\left(y_{\perp}\right)\right.}-1\right) \tag{27}
\end{equation*}
$$

where

$$
\begin{equation*}
i \chi=\ln \Gamma\left(y_{\perp} \mid \dot{j}_{1}, j_{2}\right)=\ln \pi\left(\overline{j_{2}}+\hat{j}_{2}\right) . \tag{28}
\end{equation*}
$$

In the representation derived the phase contains the functional over the field $A$ (see (II)) which can be performed in an explicit form after expanding the polarization operator $\Pi(A)$ in a power series in $A$ up to terms of an order not higher than $A^{2}$. An analysis of the eikonal phase structure (28) is given in Appendix.

On the basis of expression (27) one can study the asymptotic behaviour of the scattering amplitudes corresponding to a certain class of Feynman graphs withclosed nucleon loops. The choice of the class is realized by the choice of the appropriate terms in the expression for the phase written in the form of a series. An example of the so-called "exchange of tower" in the framework of this approach

is considered in the paper by Barbashov and Nesterenko/7/.
In conclusion we would like, however, to note that the problem of validity of the used approximations leading to the eikonal representation for the scattering amplitude, taking into account the so complicated classes of Feynman graphs, remains still open. In spite of numerous attempts to advance along these lines it is not clear yet what is the range of application of the eikonal formulas obtained in the framework of field theory models.

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## Appendix

Here we present a method of approximate calculation of the phase (28) which allows to take into account, in addition to multi-meson exchange between nucleon lines and closed loops (or "blocks" containing such loops), the connetting blocks internal meson lines the maximum of which is defined by the number of blocks.


We write down the expression for the phase

$$
\begin{equation*}
i \tilde{\chi}=\ln \pi\left(\tilde{j_{1}}+\tilde{j_{2}}\right) \tag{II}
\end{equation*}
$$

where
$\Pi(j)=C_{A} \int \delta^{4} A_{a} e^{-\frac{1}{2} A_{a} \times D_{a \beta}^{-1} \times A_{\beta}+\Pi(A)+1 I_{a} \times A_{a}}$.

Performing in (III) the replacement of variables

$$
\begin{equation*}
A \rightarrow A+j \times D \tag{IV}
\end{equation*}
$$

$\pi(j)=e^{\frac{1}{2}, \times{ }_{D} \times 1} C_{A} \int \delta^{4} A_{a} e^{\left.-\frac{1}{2} A_{a} \times D_{a}^{-1} \beta \times{ }^{A} \beta^{+} \Pi_{(A+1} \times D\right)}$

To calculate the functional integral in (III) it is necessary to expand $\Pi(A+j \times D) \quad$ in a power series in $A$

$$
\begin{equation*}
\Pi(A+j \times D)=\Pi(j \times D)+A \times \Pi_{2}(j \times D)+A \times \Pi_{2}(j \times D) \times A+\ldots \tag{VI}
\end{equation*}
$$

where the index for $\Pi$ denotes the number of integral meson lines going out from each nucleon loop (block). Taking into account, for example, the first two terms of expansion in (VI) we get an expression for $\chi_{(3)}$ which allows to take into account the set of diagrams with a maximum number of internal meson lines equal to four

$$
\begin{align*}
x_{(3)} & =\frac{i}{2} j \times D \times j+\Pi(j \times D)+\Pi_{1}(j \times D) \\
& \cdot\left[D^{-1}+2 i \Pi_{2}(j \times D)^{-1} \Pi_{1}(j \times D)-\right.  \tag{VII}\\
& -\frac{i}{2} \ln \operatorname{det}\left\{I+2 i\left[\Pi_{2}(j \times D)-\Pi_{2}^{0}\right]\left[D^{-1}+\Pi_{2}^{0}\right]^{-1}\right\},
\end{align*}
$$

where $\quad l$ is the unit matrix, $\Pi_{2}^{o}$ is the first term of the expansion $\Pi(A)=A \times \Pi_{2}^{0} \times A+\ldots$.

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