ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Дубна

E2 - 5770

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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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In Refs.[1,2] an approach to particle scattering at high energies has been developed. The considerations are based on the Logunov-Tavkhelidze quasipotential equation for the scattering amplitude in quantum field theory[3,4] and on the assumption of a smooth behaviour of the local quasipotential at high energies[5] [x]. The behaviour of the scattering amplitude at small and large angles has been analysed in detail. For the small angle scattering a representation of the eikonal type has been found. The corrections to the eikonal approximation are discussed in Ref.[7]. A detailed analysis of the eikonal approximation and its applications to the description of high energy data can be found in Ref.[8].

It is interesting to study the spin effects in the high energy scattering. In the framework of the quasipotential formalism this problem reduces to solving of the quasipotential equation for particles with spin in the high energy limit.

It should be noted that recently a number of papers[9] has been devoted to the quasipotential approach for two particle system.

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[x] The problem of smoothness of the quasipotential in the quantum field theory models is considered in the papers[6].

In the present paper we study the scattering of the scalar particle on the spin ½ -particle.

Our considerations will be based on the quasipotential equation, which has been obtained in Ref./10/.

$$[E\gamma_0+(1+\frac{\omega(\vec{p})}{W(\vec{p})})(\vec{\gamma}\vec{p}-M)]\psi(p)=-\frac{1}{\omega(\vec{p})}\int V(E;\vec{p},\vec{k})\,\psi(\vec{k})d\vec{k}, \qquad (1)$$

where E is the total energy, $\vec{p}$ and $\vec{k}$ are the c.m.s. relative momenta of two particles in the initial and final states respectively.

$$\omega(\vec{p})=\sqrt{\mu^2+\vec{p}^{\,2}},\quad W(\vec{p})=\sqrt{M^2+\vec{p}^{\,2}}.$$

This equation in the Foldy-Wouthhuysen representation in the case of local quasipotential is as follows ($\vec{r}$ -space).

$$[E\gamma_0-\omega(-i\vec{\nabla})-W(-i\vec{\nabla})]\psi^F(\vec{r})=-\frac{1}{\omega(-i\vec{\nabla})}V^F(E;\vec{r})\psi^F(\vec{r}), \qquad (2)$$

where $\omega(\vec{x})=\sqrt{\mu^2+\vec{x}^2}$; $W(\vec{x})=\sqrt{M^2+\vec{x}^2}$.

In what follows we omit the symbol "F" for the quantities in Eq. (2). We will find the scattering amplitude retaining the terms of the relative order $1/p$ as compared to the leading contributions at high energies, supposing the quasipotential $V(E;\vec{r})$ in Eq. (2) to be a nonsingular function of r^2 increasing as p when $p\to\infty$.

Problems of such a kind have been studied in the framework of the Shroedinger and Dirac equations in Refs./11-15/. In particular, Ref./15/ contains a detailed qualitative analysis of the high

energy πN scattering on the basis of the Shroedinger equation with smooth potential.

We will seek the solution of Eq. (2) corresponding to the small-angle scattering at high energies in the form

$$\psi(\vec{r})=e^{ipz}\,F(\vec{r});\quad \vec{p}=(0,0,p). \qquad (3)$$

$F(\vec{r})$ is assumed to be a slowly varying function of its argument.

Inserting (3) into Eq. (2) and making use of the operator expansions:

$$e^{-ipz}\,\omega(-i\vec{\nabla})\,e^{ipz}\underset{p\to\infty}{\approx}e^{-ipz}W(-i\vec{\nabla})\,e^{ipz}\approx p\left(1-\frac{i\partial_z}{p}\right) \qquad (4)$$

$$e^{-ipz}\frac{1}{\omega(-i\vec{\nabla})}e^{ipz}\underset{p\to\infty}{\approx}\frac{1}{p}\left(1+\frac{i\partial_z}{p}\right) \qquad (5)$$

we have the following equation for $F(\vec{r})$:

$$[p(\gamma_0-1)+i\partial_z]F(\vec{r})=-\left(1+\frac{i\partial_z}{p}\right)\bar{V}(E;\vec{r})F(\vec{r}) \quad . \qquad (6)$$

The function $\bar{V}(E;\vec{r})$ is related to the initial quasipotential $V(E;\vec{r})$ by:

$$V(E;\vec{r})\,e^{ipz}=2p\,e^{ipz}\,\bar{V}(E;\vec{r}) \ . \qquad (7)$$

Equating terms which increase as p in Eq. (6) to zero, we obtain that the four-component spinor $F(\vec{r})$ is represented in the form:

$$F(\vec{r}) = \begin{pmatrix} f(\vec{r}) \\ 0 \end{pmatrix} \tag{8}$$

where $f(\vec{r})$ is the two-component spinor.

Let us now consider the quasipotential

$$V(E;\vec{r}) = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \tag{9}$$

The nondiagonal elements V_{12} and V_{21} are essential for the description of backward scattering and here we do not take them into account, i.e. hereafter we shall consider the quasipotential of the following quasidiagonal form:

$$V(E;\vec{r}) = \begin{pmatrix} V_{11} & 0 \\ 0 & V_{22} \end{pmatrix} \tag{10}$$

where V_{11} and V_{22} are the 2 x 2 matrices.

Define now the function $\Phi(\vec{r})$ by:

$$f(\vec{r}) = \Phi(\vec{r})\,\chi_{1/2,m_z}(\vec{p}) \,. \tag{11}$$

It is easy to see that the function $\Phi(\vec{r})$ obeys the equation

$$i\partial_z \Phi(\vec{r}) = -\left(1 + \frac{i\partial_z}{p}\right) \bar{V}_{11}(E;\vec{r})\,\Phi(\vec{r}) \tag{12}$$

with the boundary condition:

$$\Phi(\vec{r})\big|_{z\to-\infty} = 1 \,. \tag{13}$$

In what follows we will omit the indices of the function V_{11} and $\bar{V}_{11}$ and this will not cause any misunderstanding.

It can be shown that the scattering amplitude obtained in solving the above mentioned problem is of the form

$$T(\vec{p},\vec{k}) = \frac{p}{2\pi}\,\chi^{+}_{1/2,m_z'}(\vec{k}) \int d\vec{r}\, e^{i(\vec{p}-\vec{k})\vec{r}}\, \bar{V}(E;\vec{r})\,\Phi(\vec{r})\,\chi_{1/2,m_z}(\vec{p}) \cdot \tag{14}$$

$$\chi^{+}_{1/2,m_z'}(\vec{p})\,\chi_{1/2,m_z}(\vec{p}) = \delta_{m_z',m_z}$$

and

$$\frac{d\sigma}{d\Omega} = \sum_{\rm spin} |T|^2 \,.$$

Now we proceed to solving the Eq. (12) with the boundary condition (13). We choose the quasipotential $V(E;\vec{r})$ in the following form:

$$V(E;\vec{r}) = V_1(E;\vec{r}) + \frac{1}{2ip} V_2(E;\vec{r})(\vec{\sigma}\vec{L}); \quad \vec{L} = -i[\vec{r}\times\vec{\nabla}] \,, \tag{15}$$

where V_1 and V_2 are assumed to be smooth functions of r^2 increasing linearly when $p\to\infty$. Neglecting the terms of order $1/p^2$ in Eq. (12) we are led to the equation:

$$2ip\,\partial_z \Phi(\vec{r}) = \left[-U_1(E;\vec{r}) + \frac{1}{2ip} U_2(E;\vec{r})\right]\Phi(\vec{r}) \,, \tag{16}$$

where

$$U_1(E;\vec r) = V_1(E;\vec r) + \frac{1}{2i} V_2(E;\vec r)[\vec\sigma \times \vec r\,]_z \tag{17a}$$

$$U_2(E;\vec r) = -V_2(E;\vec r)(\vec\sigma\vec L) + 2\partial_z U_1(E;\vec r) + \frac{i}{p} U_1^2(E;\vec r)^{x/} \tag{17b}$$

We shall seek the solution of Eq. (16) in the form:

$$\Phi(\vec r) = \Phi^{(0)}(\vec r) + \frac{1}{2ip}\Phi^{(1)}(\vec r)\,. \tag{18}$$

The functions $\Phi^{(0)}(\vec r)$ and $\Phi^{(1)}(\vec r)$ obey the system of equations

$$2ip\,\partial_z \Phi^{(0)}(\vec r) = -U_1(E;\vec r)\Phi^{(0)}(\vec r) \tag{19a}$$

$$2ip\,\partial_z \Phi^{(1)}(\vec r) = -U_1(E;\vec r)\Phi^{(1)}(\vec r) + U_2(E;\vec r)\Phi^{(0)}(\vec r) \tag{19b}$$

with the boundary conditions:

$$\Phi^{(0)}(\vec r)\big|_{z\to-\infty} = 1\,; \quad \Phi^{(1)}(\vec r)\big|_{z\to-\infty} = 0\,. \tag{20}$$

It is easy to see that the solution of the system (19) with the boundary conditions (20) is of the form:

$$\Phi^{(0)}(\vec r) = \exp\Big[-\frac{1}{2ip}\int_{-\infty}^{z} U_1(E;\rho, z')\,dz'\Big]; \quad \vec r = (\vec\rho, z) \tag{21a}$$

$$\Phi^{(1)}(\vec r) = \frac{1}{2ip}\Phi^{(0)}(\vec r)\int_{-\infty}^{z}[\Phi^{(0)}(\vec\rho, z')]^{-1}U_2(E;\rho, z')\Phi^{(0)}(\vec\rho, z')\,dz'\,. \tag{21b}$$

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x/ The last two terms appear in Eq. (17b) due to the presence of factor $1/\omega(-i\vec\nabla)$ in Eq. (12).

Substituting (21) into (14) we have for the scattering amplitude:

$$T(\vec p, \vec k) = \chi^{+}_{1/2,m_z'}(\vec k)\Big[M^{(0)}(\vec p,\vec k) + \frac{1}{2ip}M^{(1)}(\vec p,\vec k)\Big]\chi_{1/2,m_z}(\vec p), \tag{22}$$

where

$$M^{(0)}(\vec p,\vec k) = \frac{1}{4\pi}\int d\vec r\, e^{i(\vec p-\vec k)\vec r}\; U_1(E;\vec r)\Phi^{(0)}(\vec r) \tag{23a}$$

$$M^{(1)}(\vec p,\vec k) = \frac{1}{4\pi}\int d\vec r\, e^{i(\vec p-\vec k)\vec r}\;[V_2(E;\vec r)(\vec\sigma\vec L)\Phi^{(0)}(\vec r) + \tag{23b}$$

$$+ U_1(E;\vec r)\Phi^{(1)}(\vec r)\,]\,.$$

Using formulas (21a) and (21b), the expressions for $M^{(0)}$ and $M^{(1)}$ can be rewritten as

$$M^{(0)}(\vec p,\vec k) = -\frac{ip}{2\pi}\int d^2\rho\; e^{i\vec\rho\vec\Delta_\perp}\Big[e^{-\frac{1}{2ip}\int_{-\infty}^{\infty}U_1(E;\vec r)dz} - 1\Big] \tag{24a}$$

$$M^{(1)}(\vec p,\vec k) = -\frac{1}{4\pi}\int d^2\rho\; e^{i\vec\rho\vec\Delta_\perp}\Big\{\exp\Big[-\frac{1}{2ip}\int_{-\infty}^{\infty}U_1(E;\vec r)dz\Big]\times\int_{-\infty}^{\infty}dz\Big[\frac{i}{p}U_1^2(E;\vec r) -$$

$$- (\Phi^{(0)}(\vec r))^{-1}V_2(E;\vec r)(\vec\sigma\vec L)\Phi^{(0)}(\vec r)\Big] + \vec\Delta_\perp^2\int_{-\infty}^{\infty}dz\; z\,U_1(E;\vec r)\exp\Big[-\frac{1}{2ip}\int_{-\infty}^{\infty}U_1(E;\rho, z')\,dz'\Big]\Big\}\,. \tag{24b}$$

Picking out explicitly the matrix structure of Eq. (24a) we get

$$M^{(0)}(\vec p,\vec k) = a^{(0)} + i\sigma_y\, b^{(0)}\,, \tag{25}$$

where

$$a^{(0)} = -ip \int_0^\infty \rho\, d\rho\, J_0(\rho\Delta_\perp)[e^{\chi_1(E;\rho)}\cos\chi_2(E;\rho) - 1] \tag{26a}$$

$$b^{(0)} = p \int_0^\infty \rho\, d\rho\, J_1(\rho\Delta_\perp)e^{\chi_1(E;\rho)}\sin\chi_2(E;\rho) \; . \tag{26b}$$

The eikonal functions $\chi_1(E;\rho)$ and $\chi_2(E;\rho)$ are connected with the quasipotential by the following relations:

$$\chi_1(E;\rho) = -\frac{1}{2ip}\int_{-\infty}^{\infty} V_1(E;\vec{r})\,dz \tag{27a}$$

$$\chi_2(E;\rho) = -\frac{1}{2ip}\frac{\rho}{2}\int_{-\infty}^{\infty} V\,(E;\vec{r})\,dz \tag{27b}$$

similar calculations give the following expression for $M^{(1)}$

$$M^{(1)}(\vec{p},\vec{k}) = a^{(1)} + i\,\sigma_y\, b^{(1)} \; , \tag{28}$$

where

$$a^{(1)} = -\frac{1}{2}\int_0^\infty \rho\, d\rho\, J_0(\rho\Delta_\perp)\{ e^{\chi_1(E;\rho)}[\cos\chi_2(E;\rho)A_1^{(1)}(E;\rho) + \sin\chi_2(E;\rho)A_2^{(1)}(E;\rho)] + \vec{\Delta}_\perp^{2} B_1^{(1)}(E;\rho)\} \tag{29a}$$

$$b^{(1)} = -\frac{i}{2}\int_0 \rho\, d\rho\, J_1(\rho\Delta_\perp)\{ e^{\chi_1(E;\rho)}[\sin\chi_2(E;\rho)A_1^{(1)}(E;\rho) - \cos\chi_2(E;\rho)A_2^{(1)}(E;\rho)] + \vec{\Delta}_\perp^{2} B_2^{(1)}(E;\rho)\} \; . \tag{29b}$$

The functions $A_1^{(1)}(E;\rho)$, $A_2^{(1)}(E;\rho)$, $B_1^{(1)}(E;\rho)$, $B_2^{(1)}(E;\rho)$ are as follows

$$A_1^{(1)}(E;\rho) = \int_{-\infty}^{\infty} dz\,\{\frac{i}{p}V_1^2(E;\rho,z) - V_2(E;\rho,z)[\frac{z}{\rho}\sin\chi_2(E;\rho,z)\cos\chi_2(E;\rho,z) - \sin^2\chi_2(E;\rho,z) + z\frac{\partial\chi_2(E;\rho,z)}{\partial\rho}]\} \tag{30a}$$

$$A_2^{(1)}(E;\rho) = \int_{-\infty}^{\infty} dz\{\frac{\rho}{2ip}V_1(E;\rho,z)V_2(E;\rho,z) - V_2(E;\rho,z)[\frac{z}{\rho}\sin^2\chi_2(E;\rho,z) + \sin\chi_2(E;\rho,z)\cos\chi_2(E;\rho,z) + z\frac{\partial\chi_1(E;\rho,z)}{\partial\rho}]\} \tag{30b}$$

$$B_1^{(1)}(E;\rho) = \int_{-\infty} dz\; z\, e^{\chi_1(E;\rho,z)}[V_1(E;\rho,z)\cos\chi_2(E;\rho,z) - \frac{\rho}{2}V_2(E;\rho,z)\sin\chi_2(E;\rho,z)] \tag{30c}$$

$$B_2^{(1)}(E;\rho) = \int_{-\infty}^{\infty} dz\, z\, e^{\chi_1(E;\rho,z)}[V_1(E;\rho,z)\sin\chi_2(E;\rho,z) + \frac{\rho}{2}V_2(E;\rho,z)\cos\chi_2(E;\rho,z)] . \tag{30d}$$

The quantities $\chi_1(E;\rho,z)$ and $\chi_2(E;\rho,z)$ are connected with the quasipotential by

$$\chi_1(E;\rho,z) = -\frac{1}{2ip}\int_{-\infty}^{z} V_1(E;\rho,z')\,dz' \tag{31a}$$

$$\chi_2(E;\rho,z) = -\frac{1}{2ip}\frac{\rho}{2}\int_{-\infty}^{z} V_2(E;\rho,z')\,dz' \; . \tag{31b}$$

The results obtained may be used, for instance, for the analysis of the experimental data on the elastic πN and kN -scattering at high energies.

The authors express their deep gratitude to N.N. Bogolubov, A.D. Donkov, V.G. Kadyshevsky, O.A. Khrustalev, A.N. Kvinikhidze, A.A. Logunov, M.A. Mestvirishvili, R.M. Mir-Kasimov, R.M. Muradyan, V.I. Savrin, V.P. Shelest, D.V. Shirkov, N.B. Skachkov, A.N. Tavkhelidze, N.E. Tyurin for interesting discussions.

Appendix

We derive here the formula (14) for the scattering amplitude. Quadrating Eq. (2) we have:

$$[\vec{\nabla}^2 + \vec{p}^{\,2} - \omega(-i\vec{\nabla})W(-i\vec{\nabla}) + \omega(\vec{p})W(\vec{p})]\psi(\vec{r}) = -K(E;\vec{r})\psi(\vec{r}), \qquad (A1)$$

where

$$K(E;\vec{r}) = [E\gamma_0 + \omega(-i\vec{\nabla}) + W(-i\vec{\nabla})]\frac{1}{\omega(-i\vec{\nabla})} V(E;\vec{r}) \; . \qquad (A2)$$

It can be easily seen that the Green function of the Eq. (A1) corresponding to the scattering problem is determined by:

$$G_{\vec{p}}(\vec{r}) = \frac{1}{(2\pi)^3}\int d\vec{k} \; \frac{e^{i\vec{k}\vec{r}}}{\epsilon(\vec{p}) - \epsilon(\vec{k}) + i0} \qquad (A3)$$

where

$$\epsilon(\vec{x}) = \vec{x}^2 + \omega(\vec{x})W(\vec{x}) \; . \qquad (A4)$$

We are interested in the asymptotic of $G_{\vec{p}}(\vec{r})$ when $r \to \infty$ and $p \to \infty$. Integrating (A3) we have:

$$G_{\vec{p}}(\vec{r})\Big|_{\substack{p\to\infty \\ r\to\infty}} \approx -\frac{e^{ipr}}{8\pi r} \; . \qquad (A5)$$

Now let us write the integral equation, corresponding to the Eq. (A1) with the boundary condition:

$$\psi(\vec{r})\Big|_{z\to-\infty} = e^{ipz}\,\chi_{1/2,m_z}(\vec{p}) \qquad (A6)$$

$$\psi(\vec{r}) = e^{ipz}\,\chi_{1/2,m_z}(\vec{p}) - \int G_{\vec{p}}(\vec{r}-\vec{r}\,')K(E;\vec{r}\,')\psi(\vec{r}\,')\,d\vec{r}\,' \; . \qquad (A7)$$

Defining the scattering amplitude:

$$\psi(r) = e^{ipz}\,\chi_{1/2,m_z}(\vec{p}) + T(\vec{p},\vec{k})\frac{e^{ikr}}{r}\,\chi_{1/2,m_z}(\vec{k}) \qquad (A8)$$

and using the formulas (5), (7) and (A5) we obtain the formula (14).

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Received by Publishing Department
on April 27, 1971.