ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

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E2 - 5748

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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W PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

1971

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Submitted to "Nuclear Physics"

1. Introduction

We are concerned with the properties of the cross section for the production of the W -boson in nucleon-nucleon collisions as $M \to \infty$, $s \to \infty$ and s/M^2 fixed, where s is the c.m. energy squared and M is the mass of the W. Although the behaviour of the cross section in this limit cannot be measured to know an asymptotic behaviour like that might be fruitful. In designing the corresponding experiment /1,2/ we must know something about the variation of the cross section with the mass of the W -boson which is searched for. The experimental data on the closely related μ -pair production /3/ and the lower limit on the mass of the W ($M \gtrsim 2.5$ GeV) /4/ suggest that in the experiments this asymptotic region will be reached.

It is known that the total cross section is dominated by the light-cone singularities as $M^2 \to \infty$, $s \to \infty$ and s/M^2 fixed. In Section 2 it is shown that the differential cross section in energy and solid angle of the secondary muon in forward direction in the laboratory frame is also dominated by the light-cone singularities as $M^2 \to \infty$, $s \to \infty$, $E_\mu \to \infty$ and s/M^2, $s/2mE_\mu$ fixed. Here, E_μ is the muon energy and m is the nucleon mass.

All the calculations which are done for the μ -pair production can be repeated with slight modifications.

The asymptotic behaviour is discussed in the model proposed by Altarelli, Brandt and Preparata /5/; by use of the automodelity principle of the authors of ref. /6/ and the parton picture suggested by Drell and Yan /7/.

Section 3 describes calculations which are performed in the model proposed by Berman, Levy and Neff /8/. We predict an expression for the total cross section which is a smooth function of s and M (see fig. 3).

2. General Considerations

We investigate the production of W -boson in interaction of nucleons /1,2/,

$$p + \text{nucleon} \longrightarrow W^{+} + \text{hadrons} . \qquad W^{+} \to \mu^{+} + \nu_{\mu} \tag{1}$$

This process is analogous to that of the interaction of hadrons, where a muon pair is produced /3,5,6,7,8,9/:

$$p + \text{nucleon} \longrightarrow \gamma^{*} \ (\text{virtual}) + \text{hadrons} . \qquad \gamma^{*} \to \mu^{+} + \mu^{-} \tag{2}$$

From the point of view of the experimental observations there is some difference between the two processes. The observation of the ($\mu^{+}\mu^{-}$) pairs is possible, therefore the cross section for the process (2) in the squared mass of the muon pair of a measurable quantity. When producing a W -boson, however, only the μ -meson can be observed; the total cross section as function of the mass of the W-boson cannot be measured. Here, the differential cross section in energy and solid angle of the muon is measurable.

The cross section for the process (1) is given as

$$d\sigma^{+} = \frac{12\pi \; B \frac{G}{\sqrt{2}}}{\sqrt{s\,(s-4m^2)}}\,\delta(q^2 - M^2)\,\ell_{\mu\nu}\,W^{(+)\mu\nu}\,\frac{d^3k\; d^3k'}{(2\pi)^3\, 2k_0 (2\pi)^3\; 2k_0'}\,, \tag{3}$$

where B is the branching ratio of the decay $W^{+} \to \mu^{+}\nu_{\mu}$, G is the weak coupling constant $(\approx 10^{-5}\ m^{-2})$, $\ell_{\mu\nu} = k_{\mu}k'_{\nu} + k_{\nu}k'_{\mu} - g_{\mu\nu}(kk') - i\epsilon_{\mu\nu\rho\omega}k^{\rho}k'^{\omega}$ and

$$W^{(+)}_{\mu\nu}(p_1, p_2, q) = \frac{1}{4}\sum_{\text{spins}}\int d^4x\, e^{-iqx} \langle p_1 p_2 in \mid J^{(+)}_{\mu}(x) J_{\nu}(0) \mid p_1 p_2 in\rangle_{0} \tag{4}$$

p_1, p_2, k, k' denote the four-momenta of the participating particle (see fig. 1), M is the mass of the W-boson, m is the nucleon mass and $J^{(\pm)}_{\mu}(x)$ denotes the corresponding weak current of the hadrons.

The total cross section can be given as follows

$$\sigma^{+}(s, M) = \frac{BG}{\sqrt{2}\,(2\pi)^4}\;\frac{1}{\sqrt{s(s-4m^2)}}\int\frac{d^3q}{q_0}\left(-q^2 g^{\mu\nu} + q^{\mu}q^{\nu}\right) W^{+}_{\mu\nu}(p_1, p_2, q). \tag{5}$$

It is known, that as $s \to \infty$, $M^2 \to \infty$ and $s/M^2 = r$ fixed, $\sigma^{+}(s, M)$ is dominated by the contributions of the light-cone singularities of the $J^{(+)}_{\mu}(x)\,J^{(-)}_{\nu}(0)$ operator product. (e.g. /5/). In order to determine the strength of the light-cone singularities of the process (2) the authors of ref. /5/ have assumed canonical dimensions for the operators appearing in the light-cone-expansion /10/ of the operator product of the electromagnetic current. To extract the functional forms in the other variables (than q^2) they utilized a multi-Regge model (see also /9/). These calculations can be repeated for the process (1), and we obtained for the total cross section the following form, in the given limit

$$\sigma^+(s, M) \xrightarrow[\substack{s \to \infty \\ M^2 \to \infty \\ \tau \ \text{fixed}}]{} G M^4 F_1(\tau) + G M^2 F_2(\tau), \tag{6}$$

where $F_1(\tau)$ and $F_2(\tau)$ are smooth functions. R. Brandt made some efforts to generalize the content of the CVC hypotheses for the operator products of the currents, assuming U(12) symmetry for the light-cone expansion /11/. His result, however, cannot be used to relate the cross sections for the processes (1) and (2) without introducing any new parameter. (Using the CVC, a rough estimate of the matrix elements of the products of the axialvector currents is given in ref. /12/).

Contrary to the formula (6), the parton model /7/ and the principle of the automodelity /6/, predict for the cross section an asymptotic expression as follows

$$\sigma^+(s, M) \xrightarrow[\substack{s \to \infty \\ M^2 \to \infty \\ \tau \ \text{fixed}}]{} G F(s/M^2). \tag{7}$$

Both the formulae (6) and (7) claim the remarkable fact that the total cross section for the process (1) as function of the mass of the W-boson increases in the given limit or at least it is a smooth function in M.

In the experiments the differential cross section $\frac{d\sigma}{dE_\mu \, d\Omega}$ is measured, where E_μ is the energy, Ω is the solid angle of the muon in the laboratory frame. For muon scattering in forward direction in the laboratory frame we find

$$\frac{d\sigma}{dE_\mu\, d\Omega}(\Omega=0, s, M, E_\mu) = \frac{3BG}{\sqrt{2}\,(2\pi)^4}\,\frac{E_\mu}{\sqrt{s(s-4m^2)}}\int dr\,\ell_{\mu\nu} W^{\mu\nu(+)} \,, \tag{8}$$

where $r = \frac{E_\nu}{E_\mu}$, E_ν is the energy of the muon-neutrino in the laboratory frame, and the range of the integration in r is given as

$$\frac{M^2}{4E_\mu m} \leq r \leq \frac{s+M^2-4m^2}{2mE_\mu} - \frac{M^2(s-m^2)}{4m^2 E_\mu^2} - 1 \,. \tag{9}$$

The phase corresponding to r is as follows

$$\Delta r = r_{max} - r_{min} = \left(\frac{s}{2mE_\mu} - 1\right)\left(1 - \frac{M^2}{2mE_\mu}\right) - \frac{2m}{E_\mu}$$

and becomes very small if $\tau = s/M^2$ is near to unity and if very energetic muons are observed. Therefore we shall restrict our considerations to the region $\tau \geq 1.5$.

Contributions to the differential cross section (8) come from the axialvector-vector parts of the product of the weak currents as well as from the vector-vector and axialvector-axialvector parts. On this account it is not clear whether the contributions of the light-cone singularities in the given limit dominate the cross section (8) or not. Using some plausible assumptions given below however, it can be shown, that the differential cross section (8) obtains the main contributions from the light-cone singularities as well and that the interference between the axialvector-vector parts can be neglected.

The Fourier transform of the corresponding matrix element of the current commutator is

$$W_{\mu\nu}(p_1, p_2, q) = \frac{1}{4}\sum_{spin}\int d^4x\, e^{-iqx} \langle p_1\, p_2\, in\, |[J_\mu^{(+)}(x), J_\nu^{(-)}(0)]|\, p_1\, p_2\, in \rangle_c \qquad (10a)$$

and

$$W_{\mu\nu}(p_1, p_2, q) = W_{\mu\nu}^{(+)}(p_1, p_2, q) - W_{\mu\nu}^{(-)}(p_1, p_2, -q). \qquad (10b)$$

$W^{(\pm)}(p_1, p_2, q)$ can be split up into a connected and into semi-connected parts /6/. To the physical region of the process (1) the semi-connected parts of $W^{(+)}(p_1, p_2, q)$ and the connected part of $W^{(-)}(p_1, p_2, -q)$ do not contribute. The semi-connected parts of $W^{(-)}(p_1, p_2, -q)$however, may give contributions to the commutator matrix element in the physical region. It is known /6/, at the same time, that they can be neglected in the limit as $s \to \infty$ and $M_N^2/s \to 0$. If we assume that the contributions of these semi-connected parts can be neglected as $M^2 \to \infty$, $s \to \infty$ and M^2/s fixed, as well, then we can use commutator of currents instead of their product. Using this assumption the formula (8) can be rewritten as follows

$$\frac{d\sigma^+}{dE_\mu\, d\Omega}(\Omega = 0, M^2, s, E_\mu) = \text{const} \int dr \int d^4x\, e^{-iqx} f(x^2, p_1x, p_2x, s), \qquad (11)$$

where

$$f(x^2, \ldots) = 0 \quad \text{if} \quad x^2 < 0. \qquad (11')$$

Introducing new variables $t = \lambda + \sigma$, $z = \sigma - \lambda$, $R = x^2 + y^2$, $tg\,\phi = y/x$ and performing the integration over ϕ we find

$$\frac{d\sigma^+}{dE_\mu\, d\Omega}(\Omega=0, M^2, s, E_\mu) \approx \text{const} \int dr \int d\sigma\, d\lambda \int_0^\infty dR^2 f(4\sigma\lambda - R^2, \dots) J_0(RM) \times$$

$$\times\, e^{2iE_\mu(1+r)\lambda}\; e^{-i(\sigma-\lambda)\frac{M^2}{2E_\mu}}, \qquad (12)$$

where $J_0(RM)$ is the zero-order Bessel function. From eqs. (11') and (12) it follows that as $s \to \infty$, $M^2 \to \infty$, $E_\mu \to \infty$ and $u = s/2mE_\mu$, $v = M^2/2mE_\mu$ fixed, the differential cross section gets the main contributions from the light-cone singularities, namely $4\lambda\sigma > x^2$ where

$$|\lambda| < \frac{1}{2E_\mu(1+r)} \qquad (13a)$$

$$|(\sigma-\lambda)| < \frac{2E_\mu}{M^2} \qquad (13b)$$

$$|\lambda\sigma| < \frac{1}{s} + \frac{1}{M^2}\,. \qquad (13c)$$

Performing similar calculations to those of ref. /5/, for the $\ell_{\mu\nu} W^{\mu\nu}$ part of the expression (8) we obtain the following asymptotic form

$$\ell_{\mu\nu} W^{\mu\nu} \xrightarrow[\substack{E_\mu \to \infty \\ M^2 \to \infty \\ s \to \infty \\ u,v \text{ fixed}}]{} q^2 s\left[a_1(\rho,\rho') + \left(u^2 + \frac{1}{v^2}\right) a_2(\rho,\rho') + \frac{1+r}{q^2} a_3(\rho,\rho')\right], \qquad (14)$$

where $\rho = \nu/M^2$, $\rho' = \nu'/M^2$, $\nu = p_1 q$, $\nu' = p_2 q$.
Neglecting the dependence on the variable r, we find the following expression

$$\frac{d\sigma^+}{dE_\mu\, d\Omega}(\Omega = 0, s, M^2, E_\mu) \xrightarrow[\substack{E_\mu \to \infty \\ s \to \infty \\ M^2 \to \infty \\ u,\, v \text{ fixed}}]{} \text{const } M^4 (u-1)(1-v) F(u,v). \qquad (15)$$

3. A Model Calculation

Finally we discuss a model proposed by Berman, Levy and Neff /8/ to predict the cross section for the process (2). We perform a similar model calculation for the process (1) as well. The model consists of the evaluation of the Feynmann diagrams given in fig. 2. The purely strong high-energy interactions are taken into account by the exchange of a single neutral vector meson with its propagator modified so that it reproduces the high-energy elastic cross sections. This neutral vector meson has no weak interactions either. The weak interactions are introduced by pointlike vector-axialvector coupling (pnW). (We use spin - ½ kinematics). The main assumption of the model is that the contributions of the multiparticle final states are taken into account by utilizing pointlike (nWp) vertex. Some arguments supporting this hypothesis are given in ref. /8/.

Performing the integrations over the momenta of the final state hadrons, we find the cross section for the process

$$P + N \to W^+_{\to \mu^+ \nu_\mu} + \dot{N} + N' \qquad \text{as}$$

(16)

$$d\sigma = \frac{BG}{4\sqrt{2}\,\pi^3}\,\sigma_{np}\langle t_{av}\rangle \frac{M^2}{s}\,\frac{d^3q}{(2\pi)2q_0}\left\{\frac{s}{2v^2}+\frac{s+q^2}{v+\ell}+\frac{v-q^2+\frac{1}{2}\ell}{s\ell}+\right.$$

$$\left.+\frac{v^2+s^2+q^4}{s\ell^2}+\frac{2q^2}{s\ell^4}\left[\ell^2 s+\ell v\,(s+q^2-\ell-v)+v^2q^2\right]\right\} e^{-\frac{m^2\ell^2}{\langle t_{av}\rangle v(\ell+v)}}, \qquad (16)$$

where $\ell = q^2-2p_2q$, $v = s-2p_1q$

$\sigma_{np} \approx 10$ mbarn , $\langle t_{av}\rangle \approx (300\ \mathrm{MeV})^2$ is the average momentum transfer in purely hadronic interactions.

Neglecting the exponential factor as $s \to \infty$, $M^2 \to \infty$ $\tau = s/M^2$ fixed, the integration over q can be performed. We find the total cross section

$$\sigma^{+}(s, M^2) \approx \frac{BG}{48\sqrt{2}\,\pi^3}\,F(\tau, \ln s/4m^2), \qquad (17a)$$

where

$$\begin{aligned} F(\tau,\ln s/4m^2) = {} & \left[2+\frac{3}{2}\tau^{-1}-\frac{3}{2}\tau^{-2}-2\tau^{-3}-3(\tau^{-1}+\tau^{-2})\ln\tau\right]\ln s/4m^2 + \\ & +\frac{1}{2}(3\tau^{-1}-6\tau^{-2}+8\tau^{-3})\ln\tau + \\ & +4\left(1+\frac{3}{4}\tau^{-1}-\frac{3}{4}\tau^{-2}-\tau^{-3}\right)\ln(1-\tau^{-1}) - \\ & -6\left[(\tau^{-1}+\tau^{-2})\ln\tau\right]\ln(1-\tau^{-1/2}) - \\ & -\frac{1}{12}(2+15\tau^{-1}-78\tau^{-2}-11\tau^{-3}+36\tau^{-3/2}+36\tau^{-5/2}). \end{aligned} \qquad (17b)$$

These asymptotic values of the cross sections are drown in figure 3. We can see that $\sigma^{+}(s, M^2)$ varies very slowly in a wide range of the variables s and M . (It increases and decreases logarithmically in s and M^2, respectively). Its average value is $\approx 3.10^{-36}\ \mathrm{cm}^2$ if $B = 0.4$.

We notice that we have found $\ln s/4m^2$ terms in addition to the $GF(\tau)$ scale term. These terms represent infra-red divergences as $m \to \infty$. Repeating the calculations of ref. /6/ we have found that for the process (2) $\ln s/4m^2$ terms must also be present, which break the scale-invariant form $d\sigma/dq^2 \approx q^{-4}F(s/q^2)$.

Considering that the differential cross section for muons, scattered in forward direction in the laboratory frame, is dominated by the light-cone singularities, this model predicts for the differential cross section smooth behaviour in E_μ, M and s, as well.

The author is indebted to Professor R.M. Muradyan for suggesting these considerations and to Professors A.N. Tavkhelidze, S.M. Bilenky, S.B. Gerasimov, A.V. Efremov and V.A. Matveev for helpful comments and discussions.

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Received by Publishing Department
on April 9, 1971.

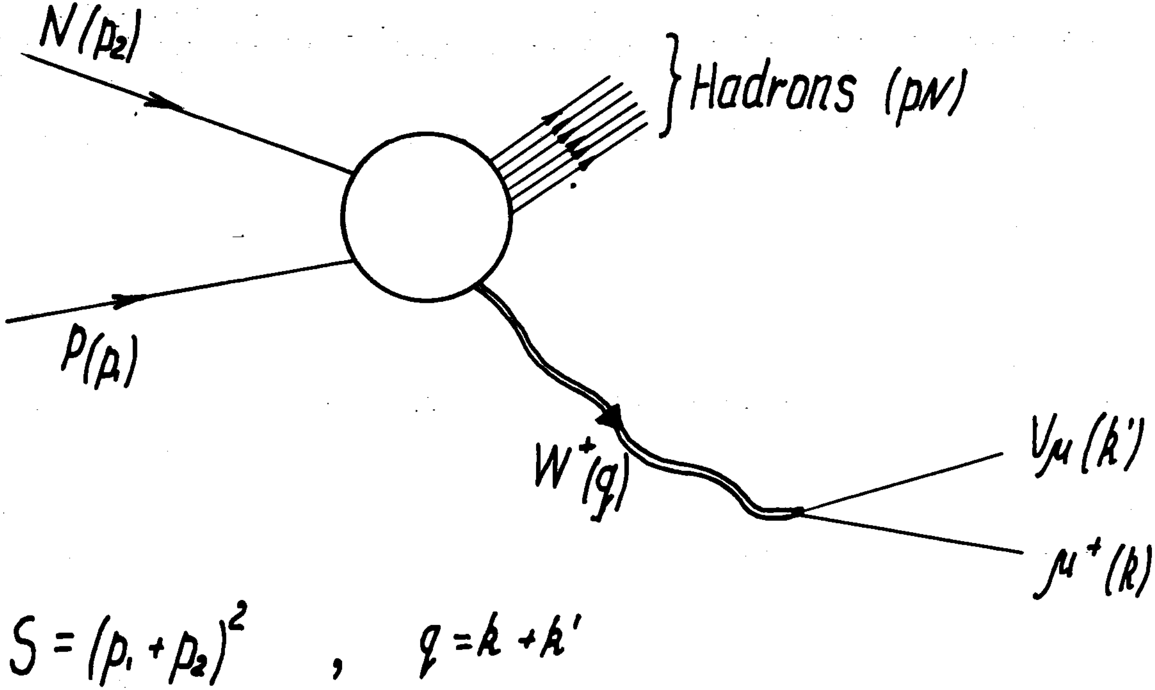

Fig. 1. Production of W^+-boson in PN collision.

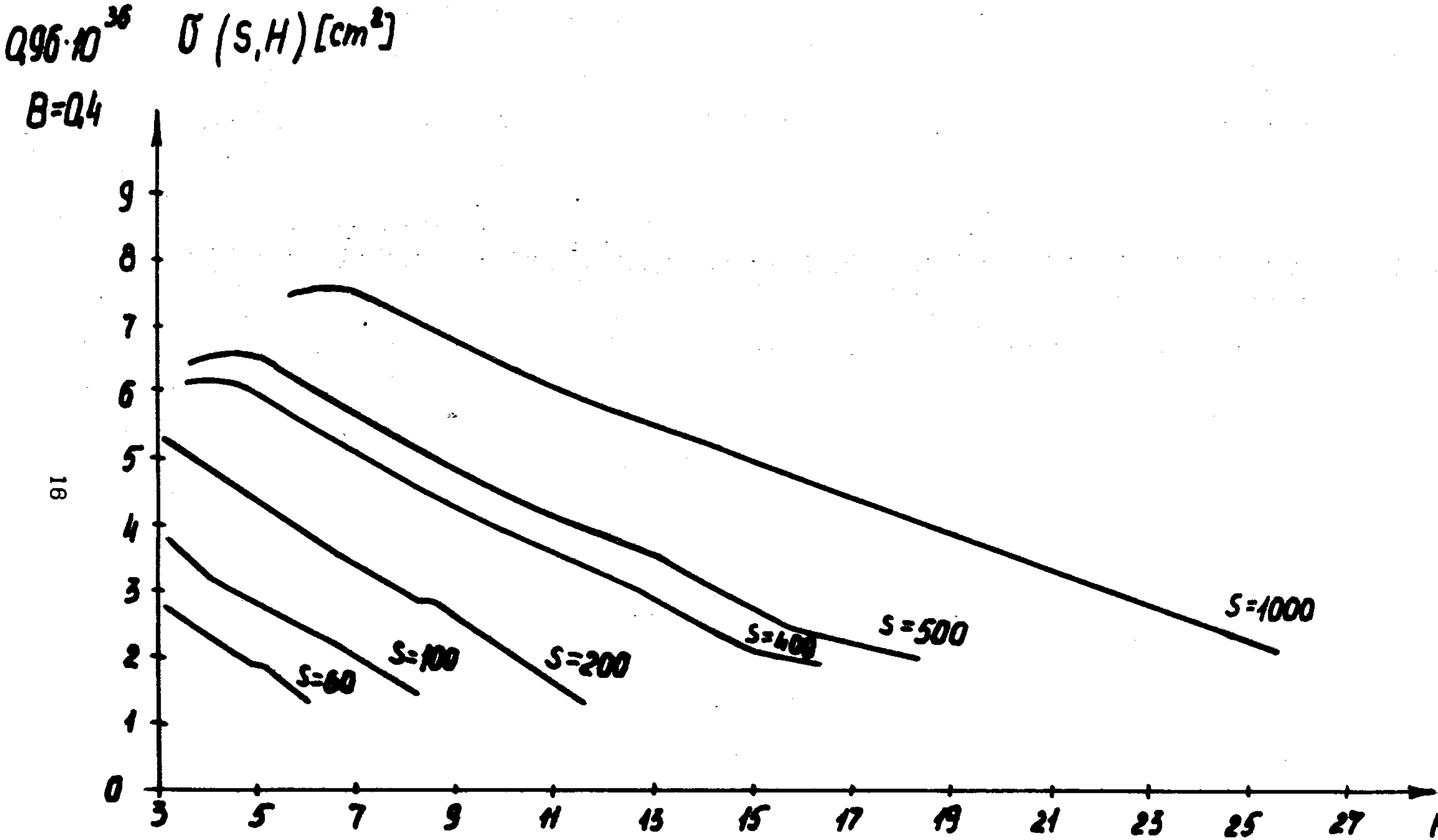

Fig. 3. Total cross section for the process N + N′ → W + hadrons as function of M and s, assuming for the branching ratio of the decay $W^+ \to \mu^+ + \nu_\mu$ B = 40%.