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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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The quasipotential method proposed by Logunov and Tavkhelidze/1,2/ is effectively applied to the investigation of bound-state spectra and also to study of analytic and assymptotic proporties of the scattering amplitude/2-7/. The quasipotential approach based on the equal-time description of the two particle system does not include a relative time and this is one of its merits. As is known the relative time in the Bethe-Salpeter equation gives rise to essential difficulties connected with mathematics and with physical interpretation. In the simplest case of scattering of two spinless particles of equal masses the quasipotential equation for the scattering amplitude reads /1,2/

$$T(\vec{p},\vec{k};E)=V(\vec{p},\vec{k};E)+\int\frac{d\vec{q}\,V(\vec{p},\vec{q};E)\,T(\vec{q},\vec{k};E)}{\sqrt{m^2+\vec{q}^{\,2}}\left[\vec{q}^{\,2}+m^2-E^2-i0\right]}, \qquad (1.1)$$

where $\vec{p}$, $\vec{k}$ are the center of mass system relative momenta of the initial and final states, respectively. The physical relativistically invariant scattering amplitude $T(s,t)$ is defined by the condition

$$T(s,t)=32\pi^3\,T(\vec{p},\vec{k};E)\Big|_{\substack{s=4E^2=4(m^2+\vec{p}^{\,2})=4(m^2+\vec{k}^{\,2})\\ t=-(\vec{p}-\vec{k})^2}} . \qquad (1.2)$$

Some possible versions of the quasipotential approach to the more general cases are investigated in papers/8-10/ .

On the basis of the three-dimensional formulation of quantum field theory an equation similar to (I,I) has been obtained/2,11/as a generalization of the non-relativistic Lippmann-Sohwinger equation on the spirit of the Lobachevsky geometry. It can be written in the form

$$T(\vec{p},\vec{k};E)=V(\vec{p},\vec{k};E)+\frac{1}{(4\pi)^3}\int\frac{d\vec{q}\,V(\vec{p},\vec{q};E)T(\vec{q},\vec{k};E)}{\sqrt{m^2+\vec{q}^2}\,E_q\,(E_q-E-i0)}\,, \tag{1.3}$$

where $E_q=\sqrt{\vec{q}^{\,2}+m^2}$.

Note, that (1.3) differs from (1.1) only by the form of the free propagators.

In the present paper the equations of a quasipotential type are proposed to describe a system of two relativistic partioles. These are derived by using the retarted part of the 2-time Green's function. It will be shown that this part contains the full information about the physical relativistic scattering amplitude. The derivation of the quasipotential equations proposed in the paper may be extremely useful in considerations of the three and more partiole problem.

2. Two-Time Green Functions.

Consider the Green function for the two particle system with masses m_1 and m_2 and arbitrary spins

$$\hat{G}(x_1,x_2;y_1,y_2)=\langle 0|T\{\Psi_1(x_1)\Psi_2(x_2)\,\Psi_1^+(y_1)\,\Psi_2^+(y_2)\}|0\rangle\,. \tag{2.1}$$

Now write its Fourier transform taking into account the translation invariance

$$G(p_1,p_2;q_1,q_2)=\int \hat{G}(x_1,x_2;y_1,y_2)\,d^{(4)}x_1\,d^{(4)}x_2\,d^{(4)}y_1\,d^{(4)}y_2\,e^{\sum_{j=1}^{2} i(p_j x_j - q_j y_j)}$$

$$G(p_1,p_2;q_1,q_2)=(2\pi)^8\,\delta(P-Q)\,G(P;p,q). \tag{2.2}$$

Here the following notations are used

$$P=p_1+p_2 \qquad Q=q_1+q_2$$

$$p=\mu_2 p_1-\mu_1 p_2 \qquad q=\mu_2 q_1-\mu_1 q_2$$

$$p_1=\mu_1 P+p \qquad q_1=\mu_1 Q+q \qquad \mu_i=\frac{m_i}{m_1+m_2} \tag{2.3}$$

$$p_2=\mu_2 P-p \qquad q_2=\mu_2 Q-q$$

The Bethe-Salpeter equation for Green function in the momentum representation is of the form

$$G(P;p,q)=G_0(P,p)\delta(p-q)+G_0(P,p)\int K(P;p,k)\,d^{(4)}k\,G(P;k,q), \tag{2.4}$$

where $G_0(P,p)=S_1(p_1)\,S_2(p_2)$ is the propagator of two free particles and, for instance, in the scalar particle case

$$S_1=\frac{i}{p_1^2-m_1^2+i\varepsilon}$$

in the spinor particle case (2.5)

$$S_1(p_1)=\frac{i(\hat{p}_1+m_1)}{p_1^2-m_1^2+i\varepsilon}.$$

$K(P_{;}p,q)$ is the kernel corresponding to the sum of the two-particle irreducible graphs.

Let us now introduce a 2-time Green function

$$\tilde{\hat{G}}_1(t-t';\vec{x}_1,\vec{x}_2;\vec{y}_1,\vec{y}_2)=\langle 0|T\{\Psi_1(t,\vec{x}_1)\Psi_2(t,\vec{x}_2)\Psi_1^+(t',\vec{y}_1)\Psi_2^+(t',\vec{y}_2)\}|0\rangle \tag{2.6}$$

and determine its Fourier transform as

$$(2\pi)^5\tilde{G}(P_{;}\vec{p},\vec{q})\,\delta(P-Q)=$$
$$=\int dt\,d\vec{x}_1 d\vec{x}_2 d\vec{y}_1 d\vec{y}_2\,\tilde{\hat{G}}(t;\vec{x}_1,\vec{x}_2;\vec{y}_1,\vec{y}_2)\exp i[P_0t-\vec{p}_1\vec{x}_1-\vec{p}_2\vec{x}_2+\vec{q}_1\vec{y}_1+\vec{q}_2\vec{y}_2]. \tag{2.7}$$

Then expressing the time-ordered product in (2.6) in terms of the Θ-functions it is not difficult to derive the spectral representation /1/

$$\tilde{G}(P_{;}\vec{p},\vec{q})=\int_0^\infty\frac{I_z(E,\vec{P},\vec{p},\vec{q})}{P_0-E+i\varepsilon}\pm\int_a^\infty\frac{I_a(E,\vec{P},\vec{p},\vec{q})}{P_0+E-i\varepsilon}, \tag{2.8}$$

where

$$I_z(E,\vec{P},\vec{p},\vec{q})=\frac{i\pi}{2E}\sum_n\delta(E-E_n)\delta(\vec{P}-\vec{K}_n)\chi_{on}(\vec{p})\otimes\chi^+_{on}(\vec{q})$$

$$I_a(E,\vec{P},\vec{p},\vec{q})=\frac{i\pi}{2E}\sum_h\delta(E-E_n)\delta(\vec{P}+\vec{K}_n)\chi_{no}(\vec{p})\otimes\chi^+_{no}(\vec{q}). \tag{2.9}$$

Here $\chi^+_{on}(\vec{p})$ and $\chi^+_{on}(\vec{p})$ have the meaning of the relativistic wave functions of a relative motion in momentum space. Indeed, it can be shown, that

$$\delta(\vec{P}-\vec{K}_n)\chi_{on}(\vec{p})=\frac{2\sqrt{E_n}}{(2\pi)^3}\int d\vec{x}_1 d\vec{x}_2\, e^{-i\vec{p}_1\vec{x}_1-i\vec{p}_2\vec{x}_2}\langle o|\psi_1(o,\vec{x}_1)\psi_2(o,\vec{x}_2)|n\rangle$$

$$\delta(\vec{P}-\vec{K}_n)\chi^+_{on}(\vec{p})=\frac{2\sqrt{E_n}}{(2\pi)^3}\int d\vec{x}_1 d\vec{x}_2\, e^{i\vec{p}_1\vec{x}_1+i\vec{p}_2\vec{x}_2}\langle n|\psi^+_1(o,\vec{x}_1)\psi^+_2(o,\vec{x}_2)|o\rangle . \tag{2.10}$$

The plus sign in (2.8) is for the case of odd number of fermions, in other cases there is the minus sign. The first and second terms in (2.8) represent the retarded and advanced parts of 2-time Green function, respectively. We denote them by $\tilde{G}^{r}$ and $\tilde{G}^{a}$.

Now let us derive explicitely the division (2.8) for the 2-time Green function of two free particles using the relation

$$\tilde{G}(P,\vec{p},\vec{q})=\int_{-\infty}^{\infty} dp_0\, G(P,p,q)\, dq_0 \tag{2.11}$$

which follows from the definition (2.7).

To this end we write down the propagator of one particle with an arbitrary spin in the form

$$S(p)=\frac{i}{2\omega(\vec{p})}\left[\frac{\sum_k U^k(\vec{p})\otimes\bar{U}^k(\vec{p})}{p_0-\omega(\vec{p})+i\varepsilon}\pm\frac{\sum_k V^k(\vec{p})\otimes\bar{V}^k(\vec{p})}{p_0+\omega(\vec{p})-i\varepsilon}\right], \tag{2.12}$$

where $\omega(\vec{p})=\sqrt{\vec{p}^2+m^2}$.

$U^k(\vec{p})$ is the wave function of positive energy free particle with k-th - spin direction and $V^k(\vec{p})$ is the wave function of negative energy free particle.

$$\left(\bar{U}^{k}(\vec{p})\, U^{i}(\vec{p})\right)=\begin{cases} 2m\,\delta_{ik} & \text{in the fermion case} \\ \delta_{ik} & \text{in the boson case} \end{cases}.$$

The plus sign in (2.12) is in the fermion case and the minus sign in the boson case. It is easy to check (2.12) in the spinor case taking into account the following relations

$$\sum_{k} U^{k}(\vec{p}) \otimes \bar{U}^{k}(\vec{p}) = \omega(\vec{p})\gamma_0 - \vec{p}\vec{\gamma} + m$$

$$\sum_{k} V^{k}(\vec{p}) \otimes \bar{V}^{k}(\vec{p}) = \omega(\vec{p})\gamma_0 - \vec{p}\vec{\gamma} - m \, .$$

Inserting (2.12) into (2,II) we have /I2,13/

$$\tilde{G}_0(P_0;\vec{p},\vec{q}) = \delta(\vec{p}-\vec{q})\frac{\pi i}{2\omega_1\omega_2}\left[\frac{\Lambda_1^{(+)}\otimes\Lambda_2^{(+)}}{P_0-\omega_1-\omega_2+i\varepsilon} \pm \frac{\Lambda_1^{(-)}\otimes\Lambda_2^{(-)}}{P_0+\omega_1+\omega_2-i\varepsilon}\right], \quad (2.13)$$

where $\omega_i = \sqrt{\vec{p}_i^{\,2}+m_i^2}$

$$\Lambda_i^{(+)} = U_i(\vec{p}_i) \otimes \bar{U}_i(\vec{p}_i)$$

$$\Lambda_i^{(-)} = V_i(\vec{p}_i) \otimes \bar{V}_i(\vec{p}_i) \, .$$

The choice of the sign is the same as in (2.8).

3. The Equations for a System of Two Particles.

For the sake of simplicity in this section we consider the case of scalar particles. Then

$$\tilde{G}_0(P;\vec{p},\vec{q})=\tilde{G}_0(P,\vec{p})\,\delta(\vec{p}-\vec{q})=$$
$$=\delta(\vec{p}-\vec{q})\,\frac{\pi i\left[\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)\right]}{\omega_1(\vec{p}_1)\,\omega_2(\vec{p}_2)\left[P_0^2-\left(\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)\right)^2+i\varepsilon\right]} \tag{3.1}$$

$$\tilde{G}_0^r(P;\vec{p},\vec{q})=\tilde{G}_0^r(P,\vec{p})\,\delta(\vec{p}-\vec{q})=\frac{\pi i\,\delta(\vec{p}-\vec{q})}{2\omega_1(\vec{p}_1)\,\omega_2(\vec{p}_2)\left[P_0-\omega_1(\vec{p}_1)-\omega_2(\vec{p}_2)+i\varepsilon\right]}\,. \tag{3.2}$$

First of all we note that the spectral representation (2.8) indicates that all the information about the physical scattering amplitude is contained in the retarded part of the 2-time Green function.

Indeed, define function $T^r(P;\vec{p},\vec{q})$ in the following manner

$$\tilde{G}^r(P;\vec{p},\vec{q})=\tilde{G}_0^r(P;\vec{p},\vec{q})+\frac{i}{\pi}\,\tilde{G}_0^r(P,\vec{p})\,T^r(P;\vec{p},\vec{q})\,\tilde{G}_0^r(P,\vec{q})\,. \tag{3.3}$$

Then using the connection of the relativistic scattering amplitude $T(s,t)$ with the usual 4-time Green function

$$G(P;p,q)=G_0(P;p,q)+G_0(P,p)\,T(P;p,q)\,G_0(P,q)$$

(3.4)

$$T(s,t)=-16\pi^4 i\; T(P;p,q)\Big|_{p_i^2=q_i^2=m_i^2}$$

we obtain

$$\int G_0(P,p)\,T(P;p,q)\,G_0(P,q)\,dp_0\,dq_0=$$

(3.5)

$$=\frac{i}{\pi}\tilde{G}_0^{\,z}(P,\vec{p})\,T^z(P;\vec{p},\vec{q})\,\tilde{G}_0^{\,z}(P,\vec{q})+\tilde{G}^a(P;\vec{p},\vec{q})-\tilde{G}_0^{\,a}(P;\vec{p},\vec{q})\,.$$

According to (2.8) $\tilde{G}^a$ and $\tilde{G}_0^a$ have no poles at positive values of P_0. So, from (3.5) it follows that

$$\frac{i}{\pi}T^z(P;\vec{p},\vec{q})\Big|_{\substack{P_0=\omega_1(\vec{q}_1)+\omega_2(\vec{q}_2)\\ P_0=\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)}} = T(P;p,q)\Big|_{p_i^2=q_i^2=m_i^2} = \frac{i}{16\pi^4}T(s,t)\,. \quad (3.6)$$

The latter relation proves the assertion given at the beginning of this section and makes reasonable to use the function $T^z(P;\vec{p},\vec{q})$ as the off-shell relativistic scattering amplitude. Writing down the 4-time Green function (2.4) in the form of iteration series by use of the Bethe-Salpeter equation putting relative times in the initial and final states equal to zero (in the momentum representation this means the operation (2, 11)) and picking out the retarded part one gets

$$\tilde{G}^{\tau}=\tilde{G}_0^{\tau}+\left(\widetilde{G_0 K G_0}\right)^{\tau}+\left(\widetilde{G_0 K G_0 K G_0}\right)^{\tau}. \tag{3.7}$$

Treating $\tilde{G}^{\tau}(P;\vec{p},\vec{q})$ as an operator in the Hilbert spaoe whioh consists of funotions of the three-dimensional veotor, we have for the inverse operator $\left(\tilde{G}^{\tau}\right)^{-1}$

$$\left(\tilde{G}^{\tau}\right)^{-1}=\left(\tilde{G}_0^{\tau}\right)^{-1}-\left(\tilde{G}_0^{\tau}\right)^{-1}\left(\widetilde{G_0 K G_0}\right)^{\tau}\left(\tilde{G}_0^{\tau}\right)^{-1}+\dots \tag{3.8}$$

Introduoing quasipotential $K^{\tau}(P;\vec{p},\vec{q})$

$$\frac{i}{\pi}K^{\tau}=\left(\tilde{G}_0^{\tau}\right)^{-1}\left(\widetilde{G_0 K G_0}\right)^{\tau}\left(\tilde{G}_0^{\tau}\right)^{-1}+\dots=\left(\tilde{G}_0^{\tau}\right)^{-1}-\left(\tilde{G}^{\tau}\right)^{-1} \tag{3.9}$$

one gets the following equation for $\tilde{G}^{\tau}$

$$\tilde{G}^{\tau}=\tilde{G}_0^{\tau}+\frac{i}{\pi}\tilde{G}^{\tau}K^{\tau}\tilde{G}_0^{\tau}. \tag{3.10}$$

Eq.(3.10) is the symbolio notation of the following integral equation

$$\tilde{G}^{\tau}(P;\vec{p},\vec{q})=\frac{\pi i\,\delta(\vec{p}-\vec{q})}{2\omega_1\omega_2[P_0-\omega_1-\omega_2+i\varepsilon]}-$$
$$-\frac{1}{2\omega_1\omega_2[P_0-\omega_1-\omega_2+i\varepsilon]}\int K^{\tau}(P;\vec{p},\vec{k})\,d\vec{k}\,\tilde{G}^{\tau}(P;\vec{k},\vec{q}). \tag{3.11}$$

From here and the definition (3.3) the equation for the off-shell scattering amplitude follows

$$T^{\tau}(P;\vec{p},\vec{q})=K^{\tau}(P;\vec{p},\vec{q})+\int\frac{T^{\tau}(P;\vec{p},\vec{k})\,d\vec{k}\,K^{\tau}(P;\vec{p},\vec{q})}{2\omega_1(\vec{k}_1)\omega_2(\vec{k}_2)\left[\omega_1(\vec{k}_1)+\omega_2(\vec{k}_2)-P_0-i\varepsilon\right]}\,. \tag{3.12}$$

Here it should be noted that the equation (3.12) up to unessential constants coincides formally with the Kadyshevsky equation[2] (1.3).

The part of the function $\tilde{G}^{\tau}$ corresponding to the contribution of one-particle state with the mass M_B to the sum (2.9) has the form

$$\tilde{G}^{\tau}(P;\vec{p},\vec{q})\approx\frac{i\pi}{2E_B}\,\frac{\chi_B(\vec{p})\otimes\chi_B^{+}(\vec{q})}{P_0-E_B+i\varepsilon}\qquad E_B=\sqrt{\vec{P}^2+M_B^2}\,. \tag{3.13}$$

Consequently the wave function $\chi_B(\vec{p})$ satisfies the following equation

$$\left(\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)-P_0\right)\chi_B(\vec{p})=\frac{1}{2\omega_1(\vec{p}_1)\omega_2(\vec{p}_2)}\int K^{\tau}(P;\vec{p},\vec{k})\,d\vec{k}\,\chi_B(\vec{k}), \tag{3.14}$$

where $P_0=E_B$

It is easy to see, that if the kernel $K^{\tau}(P;\vec{p},\vec{q})$ reproduces the correct spectral representation for $\tilde{G}^{\tau}$ by means of the equation (3.11) (see (2.8)), then the equation (3.14) will have only solution at the positive values of total energy P_0.

Using the identity[12]

$$\tilde{G}^{\tau}\left((\tilde{G}_0^{\tau})^{-1}-\frac{i}{\pi}K^{\tau}\right)\tilde{G}^{\tau}=\tilde{G}^{\tau} \tag{3.15}$$

we get the normalization condition for the bound-state wave functions

$$\int d\vec{p}\, 2\omega_1(\vec{p}_1)\omega_2(\vec{p}_2)\chi_B^{+}(\vec{p})\chi_B(\vec{p})+\int \chi_B^{+}(\vec{p})K^{\tau}(P;\vec{p},\vec{q})\chi_B(\vec{q})d\vec{p}\,d\vec{q}=2E_B\,. \tag{3.16}$$

4. Generalization to the Case of Particles with an Arbitrary Spin.

The results of the previous section cannot be extended directly to the case of higher spins because the operator $\tilde{G}_0^{\tau}$ is singular. Indeed, according to (2.13) we have

$$\tilde{G}_0^{\tau}(P;\vec{p},\vec{q})=\delta(\vec{p}-\vec{q})\frac{\pi i}{2\omega_1\omega_2}\,\frac{\Lambda_1^{(+)}(\vec{p}_1)\otimes\Lambda_2^{(+)}(\vec{p}_2)}{P_0-\omega_1-\omega_2+i\varepsilon} \tag{4.1}$$

and it is clear that $\tilde{G}_0^{\tau}$ has no the inverse operator as far as $\Lambda_i^{(+)}(\vec{p}_i)$ has the projection properties. This difficulty which is well - known in a quasipotential approach /12-16/ can be avoided if we try to construct the equation for the 2-time Green function projected onto the space of the states with positive energies :

$$G^{(+)}_{ii';kk'}(P;\vec{p},\vec{q}) = \frac{1}{16m_1^2m_2^2}\bar{U}_1^{i}(\vec{p}_1)\bar{U}_2^{i'}(\vec{p}_2)\tilde{G}^{2}(P;\vec{p},\vec{q})U_1^{k}(\vec{q}_1)U_2^{k'}(\vec{q}_2) \,. \qquad (4.2)$$

Indices i, i′, k, k′ denote the spin directions of each particle.

Using (4.1) we have

$$G^{(+)}_{0;i,i';k,k'}(P;\vec{p},\vec{q}) = \frac{\pi i\,\delta(\vec{p}-\vec{q})\,\delta_{ik}\,\delta_{i'k'}}{2\omega_1\omega_2\left[P_0-\omega_1-\omega_2+i\varepsilon\right]} \,. \qquad (4.3)$$

Now, considering $G^{(+)}$ and $G_0^{(+)}$ as the operators in the above-mentioned subspace we can introduce the quasipotential $K^{(+)}$ by the following relation

$$\frac{i}{\pi}K^{(+)} = \left(G_0^{(+)}\right)^{-1} - \left(G^{(+)}\right)^{-1}. \qquad (4.4)$$

From here the equation for $G^{(+)}$ follows

$$G^{(+)}_{ii';kk'}(P;\vec{p},\vec{q}) = \frac{\pi i\,\delta_{ik}\,\delta_{i'k'}\,\delta(\vec{p}-\vec{q})}{2\omega_1\omega_2\left[P_0-\omega_1-\omega_2+i\varepsilon\right]} -$$

$$-\frac{1}{2\omega_1\omega_2(P_0-\omega_1-\omega_2+i\varepsilon)}\sum_{m,n}\int K^{(+)}_{ii';m,n}(P;\vec{p},\vec{k})\,d\vec{k}\,G^{(+)}_{m,n;k,k'}(P;\vec{k},\vec{q}) \,. \qquad (4.5)$$

It is easy to see that by analogy with the scalar case the equation for the bound-state wave function and normalization condition have the form

$$\left(\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)-E_B\right)\chi^{(+)}_{Bii'}(\vec{p})=\frac{1}{2\omega_1(\vec{p}_1)\omega_2(\vec{p}_2)}\int\sum_{m,n}K^{(+)}_{ii';m,n}(E_B,\vec{P};\vec{p},\vec{k})\,d\vec{k}\,\chi^{(+)}_{Bmn}(\vec{k}) \tag{4.6}$$

$$\sum_{i,i'}\int\bar{\chi}^{(+)}_{Bii'}(\vec{p})\,2\omega_1(\vec{p}_1)\,\omega_2(\vec{p}_2)\,\chi^{(+)}_{Bii'}(\vec{p})\,d\vec{p}\,+$$

$$+\sum_{i,i',k,k'}\int\bar{\chi}^{(+)}_{Bii'}(\vec{p})\left[\frac{\partial}{\partial P_0}K^{(+)}_{ii';kk'}(P;\vec{p}\vec{q})\right]_{P_0=E_B}\chi^{(+)}_{Bk,k'}(\vec{q})\,d\vec{p}\,d\vec{q}=2E_B\,, \tag{4.7}$$

where $\chi^{(+)}_{Bii'}=\bar{U}^{i}_{1}\bar{U}^{i'}_{2}\chi_B\ ;\quad \bar{\chi}^{(+)}_{Bii'}=\chi^{+}_{B}U^{i}_{1}U^{i'}_{2}$.

It is not hard to check that the function $T^{(+)}_{ii';k,k'}(P;\vec{p},\vec{k})\cdot 16\pi^3$ defined by the operator relation

$$G^{(+)}=G^{(+)}_0+\frac{i}{\pi}G^{(+)}_0\,T^{(+)}\,G^{(+)}_0 \tag{4.8}$$

coincides with the relativistic scattering amplitude if the arguments satisfy the mass-shell condition

$$P_0=\omega_1(\vec{p}_1)+\omega_2(\vec{p}_2)=\omega_1(\vec{k}_1)+\omega_2(\vec{k}_2)\,.$$

According to (4.5) and (4.8) $T^{(+)}$ obeys the following equation

$$T^{(+)}_{i,i';k,k'}(P;\vec{p},\vec{q}) = K^{(+)}_{i,i';k,k'}(P;\vec{p},\vec{q}) +$$

$$+\sum_{m,n}\int \frac{K^{(+)}_{i,i';m,n}(P;\vec{p},\vec{k})\,d\vec{k}\,T^{(+)}_{m,n;k,k'}(P;\vec{k},\vec{q})}{2\omega_1(\vec{k}_1)\omega_2(\vec{k}_2)[\omega_1(\vec{k}_1)+\omega_2(\vec{k}_2)-P_0-i\varepsilon]} \quad (4.9)$$

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