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SU (3) SYMMETRY AND STATISTICAL MODEL FOR MULTIPLE PARTICLE PRODUCTION

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The characteristic feature of high energy inelastic interactions is the production of new particles. It is known [1] that the average multiplicity in these processes grows as some power of the incident energy and at an energy of a few TeV reaches large values. From this point of view the future theory of inelastic interactions must necessarily contain some elements of the statistics. Therefore a critical analysis and further development of the statistical models are of considerable interest.

Let us consider the SU(3) version of the statistical model of multiple particle production [2]. In this case, besides the basic statistical approach, an additional assumption about the SU(3) symmetry in imposed on the amplitudes and on the particles involved in this process. Such an assumption reduces considerably the number of different "particles"-multiplets, which form a "final state". The state observed in an experiment is generated by particles which are fixed representatives of the allowed multiplets. The probability of this state is determined by a quantity called statistical weight.

Unfortunately, the calculation of the statistical weights for many-particle states is a difficult problem in the $SU(3)$ approach. The difficulties sharply grow with the increase of the number of particles. They become almost unsolvable for states with six and more particles. This fact forced us to refuse the traditional methods of calculation and to adopt the method of Cerulus /3/ developed for the calculation of similar quantities in the $SU(2)$ approach.

To demonstrate the method suggested we consider, as an example, an arbitrary quark system. Such states are realized, e.g. in the additive quark model for inelastic interactions /4/ where hadrons of the final state are formed from a very large number of quarks and antiquarks.

A realistic way for solving the problem implies that a closed expression should be constructed for the statistical weight. The first step along this line is the parametrization of the considered group.

It is known /5/ that the typical element of the three-dimensional unimodular unitary group is an 8-parameter expression. From the physical point of view such matrix elements are of interest which are diagonal with respect to h_1 and h_2 (the quantum numbers of the representation of the group in the Gelfand's scheme /6/) and to the internal states of the multiplet. These quantities (like those of the $U(2)$ group) are called the diagonal $D^{[h_1\ h_2\ 0]}_{\ell,\ell}(g)$ -functions (the index ℓ running over all the multiplet states $[h_1\ h_2\ 0]$).

The technique of calculation developed by Chacon and Moshinsky /7/ allows us to write the diagonal triplet D -functions in the form

$$D^{[100]}_{1,1} = e^{i\delta_1} \operatorname{Cos} \theta_1 \operatorname{Cos} \phi_1$$

$$D^{[100]}_{2,2} = e^{i\delta_2} \operatorname{Cos} \theta_1 \operatorname{Cos} \phi_2$$

$$D^{[100]}_{3,3} = e^{-i(\delta_1+\delta_2)} [\operatorname{Cos}\phi_1 \operatorname{Cos}\phi_2 - e^{-i(\sigma_1-\sigma_2-\sigma_3)} \operatorname{Sin}\theta_1 \operatorname{Sin}\phi_1 \operatorname{Sin}\phi_2] , \tag{1}$$

where θ_1, ϕ_1, ϕ_2 have the meaning of longitudinal angles and δ_i, σ_j may be identified with the latitudinal angles.

The corresponding $D^{[110]}_{\ell,\ell}$ - functions for the antitriplet are equal to the conjugated quantities.

The sum of the D -functions over all the states of the triplet (antitriplet) defines the χ -character of the triplet (χ^* - antitriplet) representations. Knowing them and noting that the character of the sum of representations equals the sum of the characters, and the direct product of representations equals the product of the characters, it is easy to define the character of any representations we are interested in, for example:

$$\begin{aligned} \chi^{[200]} &= \chi^2 - \chi^* \\ \chi^{[220]} &= (\chi^*)^2 - \chi \\ \chi^{[210]} &= \chi \chi^* - 1 \ . \end{aligned} \tag{2}$$

Finally, using the Murnaghan method /5/ it is possible to calculate the volume element of the group

$$dg = \prod_{i=1}^{2} \prod_{j=1}^{3} \prod_{k=1}^{2} \operatorname{Sin} 2\theta_i \operatorname{Cos}^2 \theta_i \operatorname{Sin} 2\phi_i \, d\theta_i \, d\phi_i \, d\sigma_j \, d\delta_k \quad . \tag{3}$$

Eqs. (1)-(3) are the parametrization of the D -functions, the characters and the element of volume of the group.

The statistical weight is equal to the square of the projection of the vector A, which in this case is the direct product of the corresponding number of triplets and antitriplets on the subspace transforming according to the irreducible representation $[\,h_1 \; h_2 \; 0\,]$. This quantity equals

$$|<[\,h_1 \; h_2 \; 0\,] \,|\, A>|^2 = <A \,|\, \hat{P}^{[h_1 h_2 0]} \,|\, A> \quad , \tag{4}$$

where the operator $\hat{P}^{[h_1 h_2 0]}$ is of the form

$$\hat{P}^{[h_1 h_2 0]}(g) = N^{[h_1 h_2 0]} \int \chi^{*[h_1 h_2 0]}(g) \hat{D}(g) \, dg \tag{5}$$

and projects the vectors of the subspace in which the operator $\hat{D}(g)$ acts, onto the above mentioned subspace. For the volume element of the group normalized to unity $N^{[h_1 h_2 0]}$ is equal to the dimension of the $[\,h_1 \; h_2 \; 0\,]$ representation.

Inserting (5) into (4) we get the final result: the statistical weight of the state containing m quarks and (n-m) antiquarks is equal to

$$|<[\,h_1 \; h_2 \; 0\,] \,|\, A>| = \frac{2 N^{[h_1 h_2 0]}}{(2\pi)^5} \int dg \; \chi^{*[h_1 h_2 0]} \tag{6}$$

$$\prod_{\ell=m+1}^{n} \prod_{k=1}^{m} D^{[110]}_{i_\ell, i_\ell} \; D^{[100]}_{j_k, j_k} \quad ,$$

where $i = \bar{1}, \bar{2}, \bar{3}$, $j = 1, 2, 3$, $D^{[110]}_{i_\ell, i_\ell}$ is the i- th member of the ℓ- th antitriplet, and $D^{[100]}_{j_k, j_k}$ is the j-th member of the k -th triplet.

In the processes of multiple particle production each particle of the final state belongs to either an octet or a decuplet rather than a triplet and an antitriplet, as it has been considered earlier.

Let us now show, however, that any diagonal D -function of the octet or decuplet may be represented by the sum of the product corresponding to the number of triplet and antitriplet D -functions (not only diagonal). So far as the baryon charge is an external quantum number related to the multiplets of $SU(3)$, octets may be constructed from either three triplets, or a triplet and an antitriplet. The second possibility seems to be more preferable. Then taking into account the "quark content" of the wave functions of octet particles /8/ it is easy to get

$$D^{[210]}_{P,P} = D^{[100]}_{1,1} \; D^{[110]}_{\bar{3},\bar{3}} \tag{7}$$

$D^{[210]}_{N,N}$ from Eq. (7) by the substitution $1 \to 2$, $D^{[210]}_{\Xi^-,\Xi^-}$ and $D^{[210]}_{\Xi^0,\Xi^0}$ correspondingly from $D^{[210]}_{P,P}$ and $D^{[210]}_{N,N}$ by the replacement $\bar{3} \to 3$, $1 \to \bar{1}$, $2 \to \bar{2}$;

$$D^{[210]}_{\Sigma^+,\Sigma^+} = D^{[100]}_{1,1} \; D^{[110]}_{\bar{2},\bar{2}} \tag{8}$$

and $D^{[210]}_{\Sigma^-,\Sigma^-}$ is got from (8) by the substitutions $1 \to \bar{1}$, $2 \to \bar{2}$. The remaining octet functions are related to the triplet functions by

$$D^{[210]}_{\Sigma^0,\Sigma^0} = \frac{1}{2}\{ D^{[100]}_{1,1} D^{[110]}_{\bar{1},\bar{1}} + D^{[100]}_{2,2} D^{[110]}_{\bar{2},\bar{2}} - D^{[100]}_{1,2} D^{[110]}_{\bar{1},\bar{2}} - D^{[100]}_{2,1} D^{[110]}_{\bar{2},\bar{1}} \}, \tag{9}$$

$$\begin{aligned} D^{[210]}_{\Lambda,\Lambda} = \frac{1}{6}\{ & D^{[100]}_{1,1} D^{[110]}_{\bar{1},\bar{1}} + D^{[100]}_{2,2} D^{[110]}_{\bar{2},\bar{2}} + \\ & + D^{[100]}_{1,2} D^{[110]}_{\bar{1},\bar{2}} + D^{[100]}_{2,1} D^{[110]}_{\bar{2},\bar{1}} + \\ & + 4 D^{[100]}_{3,3} D^{[110]}_{\bar{3},\bar{3}} - 2 D^{[100]}_{1,3} D^{[110]}_{\bar{1},\bar{3}} - \\ & - 2 D^{[100]}_{2,3} D^{[110]}_{\bar{2},\bar{3}} - 2 D^{[100]}_{3,1} D^{[110]}_{\bar{3},\bar{1}} - \\ & - 2 D^{[100]}_{3,2} D^{[110]}_{\bar{3},\bar{2}} \}. \end{aligned} \tag{10}$$

* The upper indices of the D -functions correspond to the irreducible representations of $SU(3)$ in Gelfand's scheme. The lower indices of triplet D -functions mark the states between which the transitions are regarded inside a triplet or an antitriplet. In the octet D -functions listed here the lower indices correspond to the states of a baryon octet. For boson octets the lower indices of the D -functions are to be altered in an appropriate manner without changing the right-hand side of the equations. To construct the diagonal decuplet D -
-functions from the triplet ones let us consider a "quark content"

of the wave functions of the particles occupying a decuplet. In this case the orthonormal symmetrical three-quark composition /8/ must be connected with each particle of the decuplet. Using this fact it is easy to get the equations relating the diagonal decuplet D -functions with the triplet ones.

$$D^{[300]}_{N^{*++},N^{*++}} = D^{[100]}_{1,1}\, D^{[100]}_{1,1}\, D^{[100]}_{1,1} \,. \tag{11}$$

Functions $D^{[300]}_{N^{*-},N^{*-}}$ and $D^{[300]}_{\Omega,\Omega}$ are obtained from Eq.(11) by changing indices 1 $\to$ 2 and 1 $\to$ 3;

$$D^{[300]}_{N^{*+},N^{*+}} = D^{[100]}_{1,1}\, D^{[100]}_{1,1}\, D^{[100]}_{2,2} + 2D^{[100]}_{1,1}\, D^{[100]}_{1,2}\, D^{[100]}_{2,1} \,. \tag{12}$$

The expressions for $D^{[300]}_{\Sigma^{*+},\Sigma^{*+}}$; $D^{[300]}_{N^{*0},N^{*0}}$; $D^{[300]}_{\Sigma^{*-},\Sigma^{*-}}$ $D^{[300]}_{\Xi^{*0},\Xi^{*0}}$; $D^{[300]}_{\Xi^{*-},\Xi^{*-}}$ are obtained from Eq. (12) by replacing the indices: 2 $\to$ 3 for Σ^{*+}, 1 $\leftrightarrow$ 2 for N^{*0}, 1 $\to$ 2 and 2 $\to$ 3 for Σ^{*-}, 1 $\to$ 3 and 2 $\to$ 1 for Σ^{*0} and finally, 1 $\to$ 3 for the last function.

$$\begin{aligned} D^{[300]}_{\Sigma^{*0},\Sigma^{*0}} = {} & D^{[100]}_{1,1}\, D^{[100]}_{2,2}\, D^{[100]}_{3,3} + D^{[100]}_{1,2}\, D^{[100]}_{2,3}\, D^{[100]}_{3,1} + \\ & + D^{[100]}_{2,1}\, D^{[100]}_{1,3}\, D^{[100]}_{3,2} + D^{[100]}_{1,2}\, D^{[100]}_{2,1}\, D^{[100]}_{3,3} + \\ & + D^{[100]}_{1,3}\, D^{[100]}_{3,1}\, D^{[100]}_{2,2} + D^{[100]}_{2,3}\, D^{[100]}_{3,2}\, D^{[100]}_{1,1} \,. \end{aligned} \tag{13}$$

Eqs. (7)-(13) enable us to calculate the statistical weight of many-particle states on "quark level".

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