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TRANSFORMATION PROPERTIES OF MASSIVE PARTICLE STATES AT INFINITE MOMENTUM

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1. Introduction

Scattering process at high energies with Pomeron exchange are elastic scattering reactions and also for instance inelastic reaction of the type[1]

$$A + B \to A + C \qquad \text{with} \qquad \Delta P_{BC} = (-1)^{\Delta J_{BC}}.$$

On the other hand, in recent time in some of these processes $\pi N \to \pi N$, $\gamma p \to \rho^0 p$ helicity conservation has been observed and it was found a connection between Pomeron exchange and helicity conservation[2]. If it is possible to characterize the S-matrix element at high energies essentially with the help of the quantum numbers of the participating particles then the question arises: what is the meaning of the mass, the helicity and the spin at high energies?[3,4] We do not think that these are kinematical effects, but the first step in such a direction is the study of the transformation properties of massive particle states at infinite momentum. The states of a physical particle form a representation space of the Poincaré group[3,5]. These states may be obtained by the method of induced representations. In a very formal manner this means for massive particles[6] the following:

First define the particle states in the rest system $|\overset{\circ}{p} s \lambda\rangle$, second apply to these states a definite Lorentz transformation $U(L_p)$ (boost transformation) with the property $L_p \overset{\circ}{p} = p$, with $\overset{\circ}{p} = (m, 0, 0, 0)$ and obtain

$$|p, L_p, s, \lambda\rangle = U(L_p)\,|\overset{\circ}{p}, s, \lambda\rangle \quad . \tag{1}$$

Clearly there are degrees of freedom in the definition of L_p which distinguish the different basis systems. The application of a general Lorentz transformation to these states gives

$$U(\Lambda)|p, L_p, s, \lambda\rangle = |\Lambda p, L_{\Lambda p}, s, \lambda'\rangle\, \mathcal{D}^s_{\lambda'\lambda}(R_{W(\Lambda,p)}) \,, \tag{2}$$
$$R_{W(\Lambda,p)} = L^{-1}_{\Lambda p}\,\Lambda\, L_p \quad .$$

$\mathcal{D}^s_{\lambda'\lambda}$ is a representation matrix of the group $SU(2)$. We write down the known boosts L_p in a suitable form for helicity states/6,7,8/ and canonical states/9/. In the following sections we discuss Wigner rotations for helicity states and canonical states in the limit $p \to \infty$. For helicity states we obtain the well known result/3/ (eq. (10))

$$\lim_{p\to\infty} R^H_{W(\Lambda,p)} = \lim_{p\to\infty} L^{H-1}_{\Lambda p}\,\Lambda\, L^H_p = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3}$$

This means $\lim\limits_{p\to\infty} \mathcal{D}^s_{\lambda'\lambda}(R^H_W) = \delta_{\lambda\lambda'}\, e^{i\lambda\varphi}$ which agrees with the transformation properties of massless particles and includes the vanishing of transitions (induced by Lorentz transformations) between different helicity states of the massive particle in the limit $p \to \infty$. Furthermore it shows the importance of the helicity states. It is useful to note that the elastic scattering of massless particles (without discrete transformations) includes the conservation of the helicity. For canonical states at infinite momentum such a simple result can not be derived. But in the case $\Lambda = L^c_k$ (pure Lorentz transformation) the Wigner rotation is a rotation around the axis

$\vec{k} \times \vec{p}$ with the angle $\alpha = \Theta - \Theta'$, where Θ and Θ' are the angles occuring by the abberation of light. The angle φ apearing by the transformation of the helicity states is in a simple way geometrically related to the angle α. For a general Lorentz transfromation Λ it is also possible to calculate the value of the angle φ.

2. Boost Matrices

To get in a simple manner a determination of the boost it is customary to characterize L_p by its action on a complete system of basis vectors (t^0 , n_1^0 , n_2^0 , n_3^0)/6/ in the rest system

$$t = L_p t^0 , \; n_{1(p)} = L_p n_1^0 , \; n_{2(p)} = L_p n_2^0 , \; n_{3(p)} = L_p n_3^0 . \tag{4}$$

If we choose for t^0 , n_1^0 , n_2^0 , n_3^0 the normed vectors in the direction of the coordinate axis, then the vectors t , $n_{1(p)}$, $n_{2(p)}$, $n_{3(p)}$ correspond to the column vectors of the transformation matrix L_p . We consider the boosts for helicity states and canonical states separately.

a) Helicity states

For helicity states/7,8/ the quantization axis $\vec{n}_{3(p)}$ is directed along the three-dimensional momentum $\vec{p}$. $n_{1(p)}$ and $n_{2(p)}$ are perpendicular to p and $n_{3(p)}$. There remains an undetermined rotation around the n_3-axis. We fix the vectors $n_{1(p)}$ and $n_{2(p)}$ following/10/. This gives

$$L^{H\,\mu}_{p\;\;\nu} = \begin{pmatrix} p^0 m^{-1} & 0 & 0 & p\, m^{-1} \\ p^1 m^{-1} & (p_\perp^2 p)^{-1}(p^3 - p)(p^1)^2 + 1 & (p_\perp^2 p)^{-1}(p^3 - p)\, p^1 p^2 & p^1 p^0 (mp)^{-1} \\ p^2 m^{-1} & (p_\perp^2 p)^{-1}(p^3 - p)\, p^1 p^2 & (p_\perp^2 p)^{-1}(p^3 - p)(p^2)^2 + 1 & p^2 p^0 (mp)^{-1} \\ p^3 m^{-1} & -p^{-1} p^1 & -p^{-1} p^2 & p^3 p^0 (mp)^{-1} \end{pmatrix} , \tag{5}$$

where

$$p = (p^\mu) = (p^0, p^1, p^2, p^3) = (p^0, \vec{p}) \,, \quad \mathfrak{p} = |\vec{p}|$$
$$p_\perp^2 = (p^1)^2 + (p^2)^2 \,, \quad p^2 = (p^0)^2 - \mathfrak{p}^2 \quad . \tag{6}$$

Spacelike vectors with simple transformation properties are $(0, -p^2, p^1, 0) \to (0, -p^2, p^1, 0)$ and $(0, p^1, p^2, 0) \to \mathfrak{p}^{-1}(0, p^3p^1, p^3p^2, -p_\perp^2)$.

b) Canonical states

Applying to the states at rest a Lorentz transformation without rotations

$$L^{c}{}_{p}{}^{\mu}{}_{\nu} = \delta^\mu_\nu - \frac{1}{m^2 + m p^0}\left(\delta^0_\nu \delta^\mu_0 m^2 + \delta^\mu_0 m p_\nu + p^\mu p_\nu - \left(1 + \frac{2p_0}{m}\right) p^\mu \delta^0_\nu m \right) \tag{7}$$

we obtain the canonical states (Joos[9]). Explicitly we write down:

$$L^{c}{}_{p}{}^{\mu}{}_{\nu} = \begin{pmatrix} \frac{p^0}{m} & \frac{p^1}{m} & \frac{p^2}{m} & \frac{p^3}{m} \\ \frac{p^1}{m} & \left(\frac{p^0}{m}-1\right)\left(\frac{p^1}{\mathfrak{p}}\right)^2 + 1 & \left(\frac{p^0}{m}-1\right)\frac{p^1p^2}{\mathfrak{p}^2} & \left(\frac{p^0}{m}-1\right)\frac{p^1p^3}{\mathfrak{p}^2} \\ \frac{p^2}{m} & \left(\frac{p^0}{m}-1\right)\frac{p^1p^2}{\mathfrak{p}^2} & \left(\frac{p^0}{m}-1\right)\left(\frac{p^2}{\mathfrak{p}}\right)^2 + 1 & \left(\frac{p^0}{m}-1\right)\frac{p^2p^3}{\mathfrak{p}^2} \\ \frac{p^3}{m} & \left(\frac{p^0}{m}-1\right)\frac{p^1p^3}{\mathfrak{p}^2} & \left(\frac{p^0}{m}-1\right)\frac{p^2p^3}{\mathfrak{p}^2} & \left(\frac{p^0}{m}-1\right)\left(\frac{p^3}{\mathfrak{p}}\right)^2 + 1 \end{pmatrix} \tag{8}$$

Space-like vectors with simple transformation properties are the vectors perpendicular to t^0 and to the spatial momentum (they are invariant) and $(0, p^1, p^2, p^3) \to \frac{1}{m}(\mathfrak{p}^2, p^0p^1, p^0p^2, p^0p^3)$. In preparation of the limiting procedure $\mathfrak{p} \to \infty$ we introduce the notations

$$p^\mu = \alpha^\mu \mathfrak{p} \,, \quad \frac{\mathfrak{p}}{m} = \beta \,, \quad (\alpha^1)^2 + (\alpha^2)^2 + (\alpha^3)^2 = 1 \,, \quad \alpha^0 = \sqrt{1 + \beta^{-2}} \quad . \tag{9}$$

In this notation free parameters are α^1, α^2, β, m. For $p \to \infty$, α^μ, m remain finite and β goes to infinity.

3. The Wigner Rotation R_W in the Limit $p \to \infty$

Considering $\lim_{p\to\infty} L^{-1}_{\Lambda p} \Lambda L_p$ we use an indirect method. Helicity states and canonical states are treated separately.

a) Helicity states

It is sufficient to show

$$R^H_W t^c = t^c \quad (a) \qquad , \qquad \lim_{p\to\infty} R^H_W n^c_3 = n^c_3 \quad (b), \tag{10}$$

then it follows that R^H_W reduces to a rotation around the n^c_3-axis/3/. Eq. (10a) is satisfied by every boost matrix

$$L^{-1}_{\Lambda p} \Lambda L_p t^c = L^{-1}_{\Lambda p} \Lambda L_p \frac{\overset{\circ}{p}}{m} = L^{-1}_{\Lambda p} \Lambda \frac{p}{m} = t^c. \tag{11}$$

It remains to consider $R^H_W n^c_3$. For this reason we look at $\Lambda L^H_p n^c_3$. Using eq. (5) and eq. (9) we get

$$\Lambda L^H_p n^c_3 = \Lambda n^H_{3(p)} = \Lambda \begin{pmatrix} \frac{1}{\alpha^0} \\ \alpha^i \end{pmatrix} \alpha^0 \beta = (\lambda^\mu{}_0 \frac{1}{\alpha^0} + \lambda^\mu{}_i \alpha^i) \alpha^0 \beta . \tag{12}$$

If we use

$$(\Lambda p)^\mu = p'^\mu = \gamma^\mu p', \; \delta = \frac{p'}{m}, \; \gamma^\mu = \lambda^\mu{}_\nu \alpha^\nu \beta \delta^{-1}, \; \gamma^0 = \sqrt{1+\delta^{-2}}, \tag{13}$$

then for $L^{H}_{\Lambda p} n_3^0$ we may write down

$$L^{H}_{\Lambda p} n_3^0 = n^{H}_{3\,(\Lambda p)} = \begin{pmatrix} \frac{1}{\gamma^0} \\ \gamma^i \end{pmatrix} \gamma^0 \delta \tag{14}$$

Now we can derive

$$n^{H}_{3\,(\Lambda p)} = \Lambda\, n^{H}_{3\,(p)} + \varepsilon^{H}$$

$$\varepsilon^{H} = \lambda^{\nu}{}_{0} \frac{1}{\beta} + (\gamma^{0} - \alpha^{0})\, \delta \gamma^{\nu} - \frac{1}{\delta}\, \delta^{\nu}_{0} \,. \tag{15}$$

With the help of this relation eq. (10b) can be written

$$R^{H}_{w} n_3^0 = L^{H-1}_{\Lambda p} \Lambda L^{H}_{p} n_3^0 = L^{H-1}_{\Lambda p} \Lambda\, n^{H}_{3\,(p)} = L^{H-1}_{\Lambda p} \left(n^{H}_{3\,(\Lambda p)} - \varepsilon^{H} \right) = n_3^0 - L^{H-1}_{\Lambda p} \varepsilon^{H} \,. \tag{16}$$

A direct calculation leads to $\lim\limits_{p \to \infty} L^{H-1}_{\Lambda p} \varepsilon^{H} = 0$ which is the desired result.

b) Canonical states

In this case the Wigner rotation does not contract to a rotation around an always fixed axis. We show at first:

$$\lim_{p \to \infty} R^{c}_{w}(\Lambda, p)\, e_{\alpha} = e_{\gamma} \,, \tag{17}$$

where $e_{\alpha} = (0, \alpha^1, \alpha^2, \alpha^3) = (0, p^{-1} \vec{p})$, $e_{\gamma} = \lim\limits_{p \to \infty} e_{\Lambda p}$, $e_{\Lambda p} = \left(0, \frac{\vec{\Lambda p}}{|\vec{\Lambda p}|}\right)$.

To prove eq. (17) it is convenient to use a α-basis consisting of the vectors

$$t^0 = (1,0,0,0) \quad \text{time-like vector}$$

$$e_\alpha = (0,\alpha^1,\alpha^2,\alpha^3) \quad \text{long. vector}$$

and two transversal vectors e_{t_1} and e_{t_2}. In this system the action of L^c_p on e_α is

$$L^c_p e_\alpha = \beta t^0 + \beta\alpha^0 e_\alpha = \begin{pmatrix} \beta \\ \beta\alpha^0 \\ 0 \\ 0 \end{pmatrix}_\alpha . \tag{18}$$

For simplicity we assume that the Lorentz transformation $\lambda^\delta{}_\varepsilon$ is also given in the α -basis, then we have

$$\Lambda L^c_p e_\alpha = (\lambda^\delta{}_0 \beta + \lambda^\delta{}_1 \beta\alpha^0) . \tag{19}$$

On the other hand we must determine $L^c_{\Lambda_p} e_{\Lambda_p}$. In the α -basis p is given by $p = m(\beta\alpha^0 t^0 + \beta e_\alpha)$ and $\Lambda_p = p'$ by $p' = (\delta\gamma^\delta m) = m(\lambda^\delta{}_0\beta\alpha^0 + \lambda^\delta{}_1\beta)$. The action of the boost on the longitudinal vector e_{Λ_p} is

$$L^c_{\Lambda_p} \begin{pmatrix} 0 \\ \frac{1}{\delta}(\lambda^i{}_0 \beta\alpha^0 + \lambda^i{}_1\beta) \end{pmatrix}_\alpha = \begin{pmatrix} \delta \\ (\lambda^i{}_0\beta\alpha^0 + \lambda^i{}_1\beta)\,\gamma^0 \end{pmatrix}_\alpha , \tag{20}$$

$$\gamma^0 = \delta^{-1}(\lambda^0{}_0\beta\alpha^0 + \lambda^0{}_1\beta) = \sqrt{1 + \frac{m^2}{p'^2}} ,$$

where $\quad \delta^2 = \left|\frac{p'^2}{m^2}\right| = (\lambda^i{}_0\beta\alpha^0 + \lambda^i{}_1\beta)(\lambda^i{}_0\beta\alpha^0 + \lambda^i{}_1\beta)$.

It is now possible to write

$$L^c_{\Lambda_p} e_{\Lambda_p} = \Lambda L^c_p e_\alpha + \varepsilon^c \tag{21}$$

$$\varepsilon^c = \beta(\lambda^\delta{}_0(\alpha^0\gamma^0 - 1) + \lambda^\delta{}_1(\gamma^0 - \alpha^0))_\alpha + \beta \begin{pmatrix} \lambda^0{}_0\left(\frac{\alpha^0}{\gamma^0} - 1\right) + \lambda^0{}_1\left(\frac{1}{\gamma^0} - \alpha^0\right) \\ 0 \end{pmatrix}_\alpha .$$

It follows easily

$$\left(\Lambda L^c_p\right)^{-1} L^c_{\Lambda p}\, e_{\Lambda p} = e_\alpha + \left(\Lambda L^c_p\right)^{-1} \varepsilon^c \, . \tag{22}$$

Direct calculations show $\left(\Lambda L^c_p\right)^{-1} \varepsilon^c \underset{p\to\infty}{=} 0$. This agrees with eq.(17).

4. Determination of the rotation angles

Every Lorentz transformation may be reduced in the form $\Lambda = R L^c_k$ where R is a pure rotation and L^c_k is a pure Lorentz transformation. First we consider the Wigner rotation corresponding to L^c_k and second we complete our result by an examination of the rotation R .

a) Canonical states

It is easy to see that the special Wigner rotation $R^c_w(L^c_k, p) = L^{c-1}_{L^c_k p} L^c_k L^c_p$ leaves the vector $\vec{m}$ invariant

$$R^c_w(L^c_k, p)\, \vec{m} = \vec{m} \, , \tag{23}$$

where

$$\vec{m} = \frac{\vec{k} \times \vec{p}}{|\vec{k} \times \vec{p}|} \, , \tag{24}$$

because $\vec{m}$ is a transversal vector for all Lorentz transformations in $R^c_w(L^c_k, p)$. Therefore $\vec{m}$ is the rotation axis. The rotation angle is then given by

$$\cos\alpha = -e_\alpha e_\gamma = \frac{\vec{p}}{p} \lim_{p\to\infty} \frac{\vec{\Lambda}_p}{|\vec{\Lambda}_p|} , \tag{25}$$

where

$$\Lambda = L^c_K , \quad \vec{L^c_K p} = \vec{p} + \left(\frac{1}{M}\sqrt{p^2+m^2} + \left(\frac{k^0}{M} - 1\right) \frac{\vec{k}\vec{p}}{k^2} \right) \vec{k} \tag{26}$$

L^c_K is characterized by $k = (k^0, \vec{k})$, $k^2 = M^2$.

An explicit calculation gives

$$\cos\alpha = \frac{(\vec{k}\vec{n}_\alpha)^2 + (k^0+M)(\vec{k}\vec{n}_\alpha + M)}{(k^0+M)(k^0+\vec{k}\vec{n}_\alpha)} , \quad \vec{n}_\alpha = \frac{\vec{p}}{p} . \tag{27}$$

The same result is given by /11/ for the transformation of light-like vectors. Another possibility of obtaining this result is the use of horospheric coordinates introduced in /12/. Now it is very easy to prove

$$\cos\alpha = \cos(\theta - \theta') \tag{28}$$

where θ and θ are two angles which are well known from the abberation of light. Using the formula

$$\cos\theta = \frac{\cos\theta' - \beta}{1 - \beta\cos\theta'} \tag{29}$$

and taking into account $\beta = \frac{k}{k^0}$ we obtain the result (28). Now we have to consider the Wigner rotation for a pure rotation R. An inspection of the explicit expression for the Wigner rotation given in/9/ shows

$$R^{c}_{w(R,p)} = R \, . \tag{30}$$

b) Helicity states

The two Wigner rotations (for helicity states and canonical states) are connected by

$$R^{H}_{w(\Lambda,p)} = L^{H-1}_{\Lambda p} \Lambda L^{H}_{p} = L^{H-1}_{\Lambda p} L^{c}_{\Lambda p} R^{c}_{w(\Lambda,p)} L^{c-1}_{p} L^{H}_{p} \, , \tag{31}$$

where

$$T_p = L^{H-1}_{p} L^{c}_{p} = \begin{pmatrix} \left(\frac{p^3}{p}-1\right)\left(\frac{p^1}{p_\perp}\right)^2+1 & \left(\frac{p^3}{p}-1\right)\frac{p^1 p^2}{p_\perp^2} & -\frac{p^1}{p} \\ \left(\frac{p^3}{p}-1\right)\frac{p^1 p^2}{p_\perp^2} & \left(\frac{p^3}{p}-1\right)\left(\frac{p^2}{p_\perp}\right)^2+1 & -\frac{p^2}{p} \\ \frac{p^1}{p} & \frac{p^2}{p} & \frac{p^3}{p} \end{pmatrix} . \tag{32}$$

$L^{H-1}_{p} L^{c}_{p}$ is a rotation around the axis $\vec{p} \times \vec{n}^{\,0}_3$ with the angle $\cos\psi = \frac{p^3}{p} = \alpha^3$, especially it transforms the vector $\frac{\vec{p}}{p} = \vec{n}_\alpha$ into the vector $\vec{n}^{\,0}_3$. At first we consider again the special Wigner rotation. With the help of eqs. (32), (17) the vectors n^0_3, n^0_1, n^0_2 transform in the limit $p \to \infty$ in the following way:

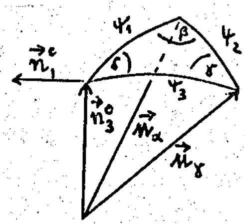
$$
\begin{array}{ccccccc}
\vec{n}_3^{\,0} & \xleftarrow{T_{L_K P_\infty}} & \vec{w}_\gamma & \xleftarrow{R_W^c(L_K^c,\, P_\infty)} & \vec{w}_\alpha & \xleftarrow{L_{P_\infty}^{c-1} L_{P_\infty}^{H}} & \vec{n}_3^{\,0} \\
 & \vec{w}_\gamma \times \vec{n}_3^{\,0} & & \vec{w}_\alpha \times k & & \vec{n}_3^{\,0} \times \vec{w}_\alpha & \text{axis} \\
 & \cos\psi_3 = \gamma^3 & & \psi_2 = \alpha & & \cos\psi_1 = \alpha^3 & \text{angle}
\end{array}
\tag{33}
$$

$$
\vec{n}_1^{\,0}\cos\varphi + \vec{n}_2^{\,0}\sin\varphi \longleftarrow \vec{n}_1^{\,0}
$$

$$
-\vec{n}_1^{\,0}\sin\varphi + \vec{n}_2^{\,0}\cos\varphi \longleftarrow \vec{n}_2^{\,0}
$$

$$
\vec{w}_\alpha = \frac{\vec{p}}{p}\,, \quad \vec{w}_\gamma = \lim_{p\to\infty} \frac{\vec{\Lambda}_p}{|\vec{\Lambda}_p|}\,, \quad P_\infty = \lim_{p\to\infty} P \ (\text{formally}),
$$

Geometrically this looks on the sphere as follows



The transformation of $\vec{n}_1^{\,0}$ is now a parallel shift along the three circles. For this reason we may write down the value of the angle

$$
\varphi = \pi - (\beta + \gamma + \delta) \quad . \tag{34}
$$

An explicit calculation gives

$$
\begin{aligned}
\cos\varphi &= g(\vec{w}_\alpha, \vec{w}_\gamma)\,, \\
g(\vec{q}, \vec{q}\,') &= \frac{(\vec{q}\,\vec{q}\,')_\perp^2}{q_\perp^2\, q_\perp'^2} + \frac{(\vec{q}\times\vec{q}\,')_3^2}{q_\perp^2\, q_\perp'^2\,(\vec{q}\times\vec{q}\,')^2} \cdot \\
&\quad \cdot \left\{ (\vec{q}\,\vec{q}\,')_\perp (q q' - \vec{q}\,\vec{q}\,') + (\vec{q}\,' q^3 - \vec{q}\, q'^3)(\vec{q}\,' q - \vec{q}\, q') \right\} .
\end{aligned}
\tag{35}
$$

For a general Lorentz transformation $\Lambda = R_2 R_1 L^c_k$ the Wigner rotation reduces

$$
\begin{aligned}
R^H_w(\Lambda,p) &= R^H_w(R_2,p'')\, R^H_w(R_1,p')\, R^H_w(L^c_k,p) \\
&= T_{R_2 p''}\, R^c_{w(R_2,p'')}\, T^{-1}_{p''}\, T_{p''}\, R^c_{w(R_1,p')}\, T^{-1}_{p'}\, R^H_{w(L^c_k,p)} \qquad (36)\\
&= R_{n^o_3}(\phi_2)\, R_{n^o_3}(\phi_1)\, R^H_{w(L^c_k,p)}
\end{aligned}
$$

where: $T_{p'} = L^{H-1}_{p'} L^c_{p'}$, $p' = L^c_k p$, $p'' = R_1 p'$. $R_{n^o_3}(\phi)$ is a rotation around n^o_3 -axis with the angle ϕ. Taking into account $R^H_w(L^c_k, p_\infty) = R_{n^o_3}(\varphi)$ then we have

$$
\lim_{p\to\infty} R^H_w(\Lambda,p) = R_{n^o_3}(\phi_2)\, R_{n^o_3}(\phi_1)\, R_{n^o_3}(\varphi) = R_{n^o_3}(\phi_2+\phi_1+\varphi). \qquad (37)
$$

It is possible to calculate the angles ϕ_2 and ϕ_1 in the same manner if we choose

- R_1 as a rotation which transforms $\vec{p}' \to \vec{p}''$ or $\vec{n}_\gamma \to \vec{n}_\delta$ around the axis $\vec{n}_\gamma \times \vec{n}_\delta$,
- R_2 as a rotation around the axis $\vec{p}''$ or $\vec{n}_\delta$ with the angle ε.

Finally we have

$$
\begin{aligned}
\varphi &= \arccos g(\vec{n}_\alpha, \vec{n}_\gamma) \\
\phi_1 &= \arccos g(\vec{n}_\gamma, \vec{n}_\delta) \qquad (38)\\
\phi_2 &= \varepsilon .
\end{aligned}
$$

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