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FACTORIZATION OF DUAL AMPLITUDES AND LOOPS IN THE COHERENT STATE MODEL

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In a recent paper [1] an operator technique for building the dual N-point tree diagram with the aid of N-2 five-dimensional oscillators has been proposed. The purpose of the present paper is to build loops in this scheme. The creation (a^+_μ, b^+) and annihilation (a_μ, b) operators, corresponding to such an oscillator, satisfy the following commutation relations:

$$
\begin{aligned}
&[a^+_\mu, a_\nu] = g_{\mu\nu} \qquad g_{\mu\nu} = 0 \quad \mu \neq \nu \\
&[b^+, b] = 1 \qquad g_{00} = -g_{11} = -g_{22} = -g_{33} = 1
\end{aligned}
\tag{1}
$$

(the all remaining commutators are equal to zero). In this oase the Hamiltonian has the form:

$$
H = -g^{\mu\nu} a^+_\mu a_\nu - b^+ b . \tag{2}
$$

Let us define the coherent states of the oscillator (2) as follows:

$$|\beta\rangle \equiv |\beta_0,\beta_1,\beta_2,\beta_3,\beta_4\rangle = e^{-\frac{1}{2}\sum_{i=0}^{4}|\beta_i|^2} e^{-g^{\mu\nu}\bar{\beta}_\mu a_\nu^+ - \bar{\beta}_4 b^+} |0\rangle$$

$$\langle\beta| \equiv \langle\beta_0,\beta_1,\beta_2,\beta_3,\beta_4| = e^{-\frac{1}{2}\sum_{i=0}^{4}|\beta_i|^2} \langle 0| e^{\sum_{\mu=0}^{3}\beta_\mu a_\mu + \beta_4 b} , \qquad (3)$$

where $|0\rangle$ is the vacuum state, β_i are arbitrary complex numbers and $\bar{\beta}_i$ their complex conjugates. These states are normalized and form a complete set:

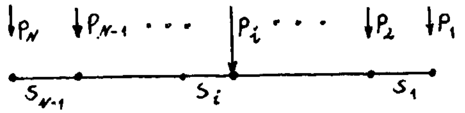
$$\langle\beta|\beta\rangle = 1$$

$$\frac{1}{\pi^5}\int |\beta\rangle\langle\beta| \prod_{i=0}^{4} d\,\mathrm{Re}\,\beta_i \, d\,\mathrm{Im}\,\beta_i = I , \qquad (4)$$

where I is the unite operator and for every β_i the integration is taken over the whole complex plane. In order to reveal the N+2-point amplitude factorization in oscillator operators we write it down in the form:

$$B_{N+2} = \int_0^1 \prod_{i=1}^{N} \langle 0|\Gamma_i|0\rangle_i \prod_{i=1}^{N-1} x_i^{-\alpha(s_i)-1} dx_i . \qquad (5)$$

The operator Γ_i depends only upon the i-th oscillator operators and corresponds to the diagram



with the following rules of correspondence:

To every knot $\overset{\downarrow p_k}{\overline{\quad\cdot\quad}}$ there corresponds the operator

$$V^{L,D}(p_k\cdot a_i, b_i) = (1+b_i)^{-\sqrt{2\alpha'}\frac{p_k\cdot a_i}{b_i} + \rho_{ik}^{L,D}} , \qquad (6)$$

where one should write the index L for $k>i$ and D for $k<i$. The constants ρ_{ki}^{L} are defined as follows:

$$\rho_{i\,i+1}^{L} = -1-\alpha(0)-2\alpha' p^2$$

$$\rho_{i\,i+2}^{L} = \alpha(p^2) \qquad \rho_{ij}^{L} = 0 \quad \text{for} \quad j > i+2 \qquad (7)$$

(α' being the slope of the linear Regge-trajectory). The constants ρ_{ik}^{D} are arbitrary.

To the vertex $\overset{\downarrow p_i}{\overline{\qquad}}$ there corresponds the operator:

$$e^{\sqrt{2\alpha'}p_i\cdot a_i^+ + b_i^+} :e^{-H_i}: (1+b_i)^{-2\alpha'\frac{p_i\cdot a_i}{b_i} + \rho_{ii}} , \qquad (8)$$

where $\rho_{ii} = 2\alpha' p_i^2 \equiv 2\alpha' p^2$

and
$$H_i = -g^{\mu\nu} a_{\mu i}^+ a_{\nu i} - b_i^+ b_i . \qquad (9)$$

To every element $\underset{s_k}{\bullet\!\!-\!\!\!-\!\!\!-\!\!\bullet}$ there corresponds the operator $x_k^{H_i}$.

Using these rules of correspondence we find out the operator Γ_i in the form

$$\Gamma_i = \Big(\prod_{k=i}^{N-1} x_k^{H_i}\Big) \prod_{n=i+1}^{N} \Big(1+\prod_{l=i}^{n-1} x_l b_i\Big)^{-2\alpha'\frac{p_n\cdot a_i}{b_i} + \rho_{in}^{L}} \cdot$$
$$e^{\sqrt{2\alpha'}p_i\cdot a_i^+ + b_i^+} :e^{-H_i}: \Big(\prod_{k=1}^{i-1} x_k^{H_i}\Big) \prod_{n=1}^{i} \Big(1+\prod_{l=1}^{n-1} x_l b_i\Big)^{-2\alpha'\frac{p_n\cdot a_i}{b_i} + \rho_{in}^{D}} . \qquad (10)$$

Substituting eq.(10) into eq.(5) we get the well known expression for B_{N+2} :

$$B_{N+2} = \int_0^1 \prod_{i=1}^{N-1} \prod_{n=i+1}^{N} \Big(1-\prod_{l=i}^{n-1} x_l\Big)^{-2\alpha' p_n\cdot p_i + \rho_{ni}^{L}} x_i^{-\alpha(s_i)-1} dx_i . \qquad (11)$$

Let us denote the N-particle coherent state by $|\chi_N\rangle$ and define it in the following way:

$$|\chi_N\rangle = \int_0^1 \prod_{i=1}^{N} \Gamma_i |0\rangle \prod_{i=1}^{N-1} x_i^{-\alpha(s_i)-1} dx_i \qquad (12)$$

Using the expression (10) for Γ_i we get:

$$|\chi_N\rangle = \int d\varphi_{N+2}(x)\, e^{\sum_{i=1}^{N}(\sqrt{2\alpha'}\, p_i \cdot a_i^+ + b_i^+)\prod_{k=i}^{N-1} x_k} \tag{13}$$

$$\left(\prod_{k=N}^{N-1} x_k \equiv 1\right),$$

where $d\varphi_{N+2}(x)$ denotes the integrand of the Bardaokoi-Ruegg expression for the N+2-point funotion. Then B_{N+M+2} can be written as follows:

$$B_{N+M+2} = \langle \tilde{\chi}_M | \frac{D_{MN}}{H-\alpha(s_N)} | \chi_N \rangle \quad . \tag{14}$$

where

$$D_{MN} = \prod_{i=1}^{N}\prod_{n=N+1}^{N+M} (1 - b_i b_n^+)^{-\frac{a_i \cdot a_n^+}{b_i b_n^+} + \rho_{in}^{L}} \tag{15}$$

$$\langle \tilde{\chi}_M | = \int d\tilde{\varphi}_{M+2}(x) \langle 0 | e^{\sum_{i=N+1}^{N+M}(\sqrt{2\alpha'}\, p_i \cdot a_i + b_i)\prod_{k=N+1}^{i-1} x_k} \tag{16}$$

and the sign tilde denotes, that we have changed the numeration of the variables x_i, p_i, a_i according to the oonvention $M \to N+1$, $M-1 \to N+2, \dots 1 \to N+M$.

The formula (14) is remarkable by its full factorization in both external momenta and harmonic oscillators. Just as in the case of four-point function, when the initial and final states have been characterized by coherent states, in the more general case, when N particles turn into M particles, there is also a possibility of presenting the initial and final states wave functions as superpositions of coherent states.

Recently the possibility of building loops in dual models has been intensively studied [3],[6],[7]. In particular, in ref. [6],[7] using the method of factorization with the aid of an infinite number of oscillators exponentially divergent expressions for the loops have been obtained. Below we shall show, that our method of factorization with final number of oscillators [1] allows us to build a convergent expression for the planar one-loop diagrams.

It is obvious, that the operator Γ_i takes into account the i-th particle contribution to the N-point dual amplitude. The fact, that Γ_i depends on the momenta of other particles should most probably be interpreted as a result of the averaging over the fields of the external particles, that has been performed before the operator Γ_i appears. It is clear from this point of view that the contribution of a given particle to the amplitude is determined rather by the appropriate oscillator, than the momentum. In principle, the oscillators corresponding to each particle can be built up, proceeding from the field operators of this particle. Such a possibility is disoussed in ref. [2].

Now it is easy to see, that the loops should be built up also with the aid of the operators Γ_i . From the above considerations it follows, that before integrating over the internal loop momentum, the loop L_N with N external partioles should be a product of N closed chains, each having only one external particle. If we cut any of these chains at any point, we should get a linear chain of the type Γ_i. This defines completely the way of oonstruoting closed chains ϕ_i :

$$\phi_i = Sp\, x_N^{H_i}\, \Gamma_i(x,p). \tag{17}$$

Then L_N is to be defined by the formula:

$$L_N = \int \prod_{i=1}^{N} \phi_i(x,p) \prod_{i=1}^{N} x_i^{\alpha(s_i(k))}\, dx_i\, d^4k \quad . \tag{18}$$

This method of constructing the loops well corresponds to the factorization (13). Indeed, from the symmetrized N'-point dual amplitude ($N' > N$) we can always single out some operator part with N external particles. Because of the commutativity of different oscillators, the trace of this operator chain is represented as the product of the traces of the operators $x_N^{H_i}\Gamma_i(x,p)$. The function (17) can be calculated with the aid of a complete set of coherent states(3).

$$Sp\, x_N^{H_i}\Gamma_i(x,p) = \prod_{n=1}^{i}\Big(1-\prod_{k=1}^{n-1}x_k\prod_{l=i}^{N}x_l\Big)^{-2\alpha' p_n\cdot p_i + \rho_{in}^{D}} \times$$
$$\prod_{n=i+1}^{N}\Big(1-\prod_{l=i}^{n-1}x_l\Big)^{-2\alpha' p_n\cdot p_i + \rho_{in}^{L}} \quad . \tag{19}$$

Substituting eq.(19) into eq.(18) we get

$$L_N = \int \mathcal{N}(x) \prod_{i=1}^{N} x_i^{\alpha(s_i(k))}\, dx_i\, d^4k \;, \tag{20}$$

where

$$\mathcal{N}(x) = \prod_{i=1}^{N}(1-x_i)^{-1-\alpha[(p_i+p_{i+1})^2]}\,(1-x_i x_{i+1})^{-2\alpha' p_i\cdot p_{i+2}+\alpha(p^2)} \times$$
$$\prod_{i=2}^{N-2}\Big(1-\prod_{k=i}^{N}x_k\Big)^{-2\alpha' p_i\cdot p_i}\prod_{i=3}^{N-1}\Big(1-\prod_{k=i}^{N+1}x_k\Big)^{-2\alpha' p_2\cdot p_1} \times \tag{21}$$
$$\prod_{i=4}^{N}\prod_{n=3}^{i-1}\Big(1-\prod_{k=1}^{n-1}x_k\prod_{l=i}^{N}x_l\Big)^{-2\alpha'(p_n\cdot p_i)}\prod_{i=1}^{N-3}\prod_{n=i+3}^{N}\Big(1-\prod_{l=i}^{n-1}x_l\Big)^{-2\alpha' p_n\cdot p_i} .$$

This form of $\mathcal{N}(x)$ shows, that the integral representation of the loop converges in some region of variation of the parameters ($p_i \cdot p_j$).

After performing in eq.(20) the integration over d^4k the expression for L_4 takes the form

$$L_4 = -\frac{\pi^2}{\alpha'^2}\int_0^1 \prod_{i=0}^{4} dx_i\, \frac{\omega^{-\alpha(0)-1}}{\ln^2\omega}\, e^{-\alpha' s\frac{\ln x_2 \ln x_4}{\ln\omega} - \alpha' t\frac{\ln x_1 \ln x_3}{\ln\omega} - \alpha' p^2\frac{\ln x_1x_3 \ln x_2x_4}{\ln\omega}} \times$$
$$\left[\frac{(1-x_1)(1-x_3)(1-x_2x_3x_4)(1-x_4x_2x_4)}{(1-x_1x_2)(1-x_2x_3)(1-x_3x_4)(1-x_4x_1)}\right]^{-1-\alpha(s)}\left[\frac{(1-x_2)(1-x_4)(1-x_1x_2x_3)(1-x_1x_3x_4)}{(1-x_1x_2)(1-x_2x_3)(1-x_3x_4)(1-x_4x_1)}\right]^{-1-\alpha(t)} \tag{22}$$
$$\times\Big[(1-x_1x_2)(1-x_2x_3)(1-x_3x_4)(1-x_4x_1)\Big]^{-2-\alpha(p^2)}\Big[(1-x_1x_2x_3)(1-x_2x_3x_4)(1-x_3x_4x_1)(1-x_4x_1x_2)\Big]^{\alpha(2p^2)+1} ,$$

where $\omega = x_1x_2x_3x_4$.

For $s \to -\infty$ the leading term in the asymptotic of L_4 has the form:

$$\Gamma(-\alpha(t))\,\Delta(t)\,(-\alpha' s)^{\alpha(t)}\,\ln(-\alpha' s) \quad , \tag{23}$$

where $\Delta(t)$ is a known function /8/.

It is clear, that this expression well corresponds to the general expression for L_4 , that has been derived in /3/.

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