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A MODEL-INDEPENDENT ANALYSIS OF THE FORWARD $\pi^{\pm}p$ SCATTERING DATA

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In this note we present a new analysis of the experimental data on forward elastic $\pi^{\pm} p$ scattering, using a method previously applied to $K^{\pm} p$ scattering and described in greater detail elsewhere /1/.

The dispersion relation for the $\pi^{\pm} p$ forward scattering amplitude $F_{\pm}(\omega) = D_{\pm}(\omega) + i A_{\pm}(\omega)$ with one subtraction at $\omega = 0$ /2/, in its finite-contour form /1/, can be written

$$H(\omega) \equiv D_{-}(\omega) + I(\omega) = C + \frac{2f^2}{\omega - \omega_n} - \frac{\omega}{\pi} J(\omega), \tag{1}$$

where $C = D_{\pm}(0) + 2f^2/\omega_n$, f is the πNN coupling constant, $\omega_n = (m_n^2 - m_p^2 - m_\pi^2)/2m_p$,

$$I(\omega) = -\frac{\omega}{4\pi^2} P \int_{m_\pi}^{W} \frac{k'}{\omega'} \left[\frac{\sigma_-(\omega')}{\omega' - \omega} - \frac{\sigma_+(\omega')}{\omega' + \omega} \right] d\omega', \tag{2}$$

$$J(\omega) = -\mathrm{Im} \int_{S(W)} \frac{F_-(\omega')\,d\omega'}{\omega'(\omega'-\omega)} \tag{3}$$

and $S(W)$ is the semicircle $\omega = W\exp(i\phi)$, $0 < \phi < \pi$. We choose $W = 65$ GeV, the highest energy at which $\sigma_\pm(\omega)$ are known experimentally.

We parametrize $J(\omega)$ as follows [1]. The conformal transformation

$$\xi = \frac{\sqrt{W+\omega} - \sqrt{W-\omega}}{\sqrt{W+\omega} + \sqrt{W-\omega}} \tag{4}$$

maps the ω plane, cut along the real axis for $|\omega| > W$, into the interior of the unit circle $|\xi| < 1$. Thus the representation

$$J(\omega) = \sum_{n=0}^{\infty} a_n \xi^n \tag{5}$$

is valid in the entire ω plane and can be shown [3] to be the most rapidly convergent power series expansion of $J(\omega)$. Truncating this series, we represent $H(\omega)$ in terms of the parameters C, f^2, $a_0, \dots, a_N$.

A knowledge of $D_\pm$ at an energy ω determines $H(\mp\omega)$ from eq. (1), since the existing $\sigma_\pm$ data [4,5] accurately define $I(\omega)$ for all ω. Using the most reliable of the many measurements of the forward $\pi^\pm p$ scattering amplitudes [4,6], we determined $|D_\pm|$ and hence two possible values of $H(\omega)$ at each of 59 values of ω. To ensure rapid convergence of the series (5), we used only data for $|\omega| < 10$ GeV, corresponding to $|\xi| < 0.08$.

The parametrization defined above was fitted by the least-

-squares method to these values of $H(\omega)$. For a given number of parameters, the unknown signs of $\pm|D|$ were fixed by finding that set of signs which leads to the best fit. The optimum fit obtained in this way, with $\chi^2 = 45.3$, corresponds to the parameter values

$$C = (-0.220 \pm 0.017)\ \text{fm}, \qquad f^2 = 0.110 \pm 0.034,$$
$$a_0 = (-0.348 \pm 0.164)\ \text{GeV}^{-2}, \qquad a_1 = (-0.164 \pm 0.020)\ \text{GeV}^{-3}. \quad (6)$$

The values found for C and f^2 are compatible with those obtained from conventional dispersion relation calculations /2/.

The values of a_0 and a_1 give information on the asymptotic behaviour of the amplitudes $F_{\pm}(\omega)$. Predictions of these parameters from specific models for the asymptotic behaviour may be obtained as follows. Consider the expansion

$$J(\omega) = \sum_{n=0}^{\infty} b_n \omega^n . \quad (7)$$

Comparing (5) and (7), we see that $a_0 = b_0$ and $a_1 = b_1 (d\omega / d\xi)_{\xi=0}$. From eq. (3) it follows that

$$b_n = -\text{Im} \int_{S(W)} \frac{F_-(\omega')\, d\omega'}{\omega'^{(n+2)}} . \quad (8)$$

For each specific high-energy model, the contour integral (8) may be evaluated explicitly (see ref. /1/ for details). The resulting values of a_0 and a_1 for various models /7-11/ are shown in Table 1, where the corresponding values of $\Delta\sigma \equiv \sigma_-(\infty) - \sigma_+(\infty)$ are also given for comparison.

It is clear that our set of parameter values (6) is inconsistent with all the existing models. In particular, our value of a_0

favours those models which satisfy the Pomeranchuk theorem, while a_1 favours those which do not. However, this conclusion must be regarded with caution. An ambiguity in the value of a_0 exists for some models which violate the Pomeranchuk theorem, due to the non-uniqueness of the analytic continuation of $F_{\pm}(\omega)$ to complex ω /1/. The value of a_1, on the other hand, is found to be accurately and unambiguously determined by each model. However, the value of a_1 obtained from our fit is determined in practice almost entirely by a small number of experimental $D_{\pm}$ points between about 5 and 10 GeV. In this respect more accurate data on $D_{\pm}$ in this energy range would be especially useful in constraining the asymptotic behaviour. This is especially so because a_1 is extremely sensitive to the model. In any case, our predicted parameter values (6), including a_1 appear to be quite stable; all of them are virtually unchanged in the 5-parameter fit.

From eq. (1) we also obtain an accurate and stable model-independent prediction for $D_{\pm}(\omega)$ at moderate energies, which is found to agree well with conventional dispersion relation calculations /6,12-14/. One of us (O.V.D.) is indebted to Dr. V.S. Stavinsky for critical remarks.

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Table 1

Parameter values predicted by various models for the asymptotic behaviour of the amplitudes.

Reference	$\Delta\sigma$ (mb)	a_0 (GeV^{-2})	a_1 (GeV^{-3})
Phillips and Rarita [7]	0	-0.357	-0.00173
Barger and Phillips [8]	0	-0.223	-0.00129
Barger and Phillips [9]	2.0	1.57	-4.61
Arnowitt and Rotelli [10]	0.80 ± 0.36	0.45 ± 0.32	-0.141 ± 0.001
Horn [11]	1.3 ± 0.3	1.02 ± 0.25	-0.152 ± 0.001