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THE TOTAL SUMMATION OF PERTURBATION SERIES AT HIGH ENERGIES AND DIFRACTION SCATTERING

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THE TOTAL SUMMATION OF PERTURBATION SERIES AT HIGH ENERGIES AND DIFRACTION SCATTERING

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Summary

It is shown, that the introduction of the spin leads to an essential change in the structure of the j-plane and in the behaviour of the amplitudes off the mass shell. The reason for this is a singular character of the interaction at small distance. High-energy summation of the <u>all</u>, logarithmic terms of <u>all</u> the diagrams in pseudoscalar mesodynamics gives a new representation for the scattering amplitudes. In addition to the moving Regge poles, there appears a standing singularity. In the case of finite renormalization of the coupling constant and the wave function it is a square root branch point the position of which is determined by the bare coupling constant. The amplitudes do not vanish with the growth of external mass ($M^2 \gtrsim S \to \infty$).

This representation can serve as a basis for phenomenological description and the study of the properties of the Regge trajectories. For example, here we have the conspiration of π and π_c - trajectories and the proportionality of the residue of the Regge trajectories in meson-baryon and baryon-baryon scattering to their position $j = \alpha(t)$. This leads to the dips in the angular distributions when $\alpha(t) = 0$.

I. Introduction

For many years a reasonable description of the strong interaction on the basis of perturbation theory was considered to be impossible. Up to the present time the following three problems were solved by different authors:

1) Summation of senior logarithmic terms[1-5] (usually in the ladder type graphs).

2) Total summation of all the logarithmic terms of ladder graphs in φ^3-theory[2,7].

3) Summation of the senior logarithms in the so-called "double-logarithmic" approach[8,9] which can be reasonable in quantum electrodynamics but can not be applied to hadrons interactions.

In the first two cases there was a hope that the ladder approximation would reflect at least some of the essential features of the real situation because of its similarity to the potential description of scattering. However, except this hope the matter had no serious progress. This more than modest success in solvig the above particular problems reduced theoretists to such pessimism that till now some of the listeners come to us after the seminars and ask with confidence: "What diagrams did you sum up?"

Our success in investigating the Feynman graph asymptotics (see[11] and references therein) gives us the possibility to sum up <u>all</u> the logarithmic terms of <u>all</u> scattering diagrams without any exclusion for the φ^3- theory and mesodynamics

* Recently the same ladder approximation has been used for investigating the asymptotics of the deep inelastic ep - scattering[9,10].

$$\mathcal{L}_{int} = g\bar{\psi}\gamma^5\psi\varphi + h\varphi^4 \qquad (1)$$

in the limit $m^2/s,\ t/s \to 0$ i.e. in the diffraction region at high energy. It was found that the summation of junior logarithms does not change the character of the j - plane singularity of the amplitude only in φ^3- theory. For mesodynamics, in addition to the standing branch point due to senior terms there appears moving Regge poles. In this paper we discuss only the principal results of such summation in the following problems:

1) Elastic and quasielastic scattering of hadrons in the diffractional region (Sec.4).

2) Asymptotics of the hadron amplitudes in the limit of the large mass of the external particles (Sec.5).

3) γp- and $\gamma\gamma$- forward scattering at high energy and large "mass" of the photons (Sec. 6) which are connected with the deep inelastic ep- and ee- scattering.

The solution of these problems strongly depends on the type of interaction. The physical meaning of this dependence and two hypotheses which permit us to perform the summation are discussed in Section 7.

2. The Leading Singularities of Diagrams

The asymptotics of the Feynman graphs was studed by us in the papers[11]. For the description of the elastic scattering (Fig.1): $p_1^2 = p_1'^2 = m_1^2,\quad p_2^2 = p_2'^2 = m_2^2$

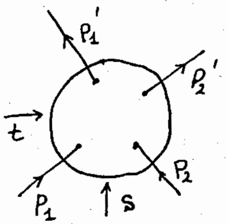

Fig.1

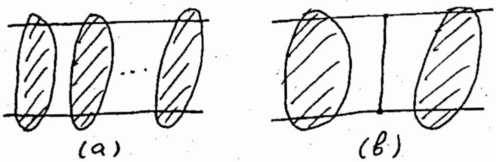

Fig.2

we use the symmetric energy variable $\nu = \frac{s-u}{2}$, three independent momenta

$$p = \frac{p_1' + p_1}{2}\,;\quad q = \frac{p_2' + p_2}{2}\,;\quad r = \frac{p_1 - p_1'}{2} = \frac{p_2' - p_2}{2}\,;\ 2pq = \quad;\ pr = qr = 0 \qquad (2)$$

and the relativistic classification of the two-particle states in t-channel and the transition amplitudes between them. There are two conserved quantum numbers: the signature $\sigma = (-)^J$ and the intrinsic parity or normality σP ($\sigma P = +1$ for normal states, where the parity $P = (-)^J$ and $\sigma P = -1$ for axial states, where $P = (-)^{J+1}$). In particular, $\sigma P = +1$ if there is at least one two-pion intermediate state in t-channel.

The states of a baryon - antibaryon system (B) are described by wave functions $\bar{u}\,O\,u$, where O are the usual combinations of the γ- matrices denoted by S, V, T, A and P. The two-meson state is denoted by M. The transition amplitudes between these states ("the invariant amplitudes") are labelled by two indices corresponding to the states of the pair 1, 1' and 2, 2'. Within the accuracy ν^{-1} and taking into account the Dirac equation, the projection operators on these states are

$$R_S=\tfrac{1}{4}1\times 1;\quad R_V=\frac{\hat{q}\times\hat{p}}{2\nu};\quad R_A=-\frac{\gamma^5\hat{q}\times\hat{p}\gamma_5}{2\nu}$$

$$R_P=-\tfrac{1}{4}\gamma^5\times\gamma^5;\quad R_T=R_{T+}+R_{T-};\quad R_{T-}=\frac{2\nu}{t}R_P \tag{3}$$

$$R_{T+}=\frac{2\nu}{t}\left(R_S+\frac{4M^2}{\nu}R_V-\frac{2M^2}{\nu}R_{SV}\right);\quad R_{SV}=\frac{\hat{q}\times 1+1\times\hat{p}}{4M}\;.$$

It is shown[12] that $GP=-1$ for the states P,A T^- and $GP=+1$ for the states M,S,V,T^+. The invariant amplitudes are normalized so that the situation would be similar to the case of scalar graphs. This corresponds to the normalization to the first Born term. In view of (3),

$$f_{MB}=\frac{1}{2M}f_{MS'}+\frac{\hat{p}}{\nu}f_{MV} \tag{4}$$

$$f_{BB}=\frac{R_S}{M^2}f_{SS}+\frac{2R_V}{\nu}f_{VV}+\frac{2R_{SV}}{\nu}f_{SV}+\frac{2R_A}{\nu}f_{AA}+\frac{R_P}{M^2}f_{PP}$$

and, for instance,

$$\frac{d\sigma_{MM}}{d\Omega}= \tag{4a}$$

$$\frac{d\sigma_{MB}}{d\Omega}=\frac{4\pi^2}{\nu}\left[\left(1-\frac{t}{4M^2}\right)|f_{MS'}|^2+|f_{MV}|^2+2\,\mathrm{Re}\left(f^*_{MS'}f_{MV}\right)\right].$$

For the investigation of the asymptotical behaviour of the graphs the Mellin transformation is very suitable. It must be done for the positive and negative signature separately

$$f^{\pm}(\nu,t)=-\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}dj\,\frac{(-\nu)^j\pm(\nu)^j}{2\sin\pi j}\,\pi\cdot\frac{\Phi(j,t)}{\Gamma(j+1)}\;. \tag{5}$$

Using the usual α - parametrization of propagators in the exponential form (to each line there corresponds a parameter α) we can write down the oontribution of any graph

$$\Phi^{\pm} = H\int_0^\infty \frac{\prod d\alpha}{\mathcal{D}^2(\alpha)}\, g(j,t,\alpha)\,|A|^j(\theta(A)\pm\theta(-A))\,e^{J(\alpha,t,m)},$$

where H is the product of the factors $(g/_{4\pi})$ and $(h/_{16\pi^2})$ and A,D,J are functions of α of the same form as for the scalar graphs. The numerators of spinor propagators turn into a pre-exponential factor $g(\alpha,j,t)$ with polinomial dependence on j.

The asymptotic behaviour of the graph is determined mainly by the two-particle separations of the graph in t-channel, which subdivide the graph into kernels. <u>The kernel is a subgraph with 4 external lines which has no two-particle separation in t-channel. Any scattering graph is eithera kernel or a set of kernels (Fig.2) linked by pairs of meson or baryon lines.</u>

The asymptotics of $f^{\pm}(\nu,t)$, is evidently determined by the most right singularity of $\Phi(j,t)$ in j-plane. We have shown[11] that the leading singularity of the positive signature part of any graph in mesodynamics is a pole at j = 0 which originates from integration over α in the region where $A(\alpha)\rightsquigarrow 0$ together with the set of α-parameters corresponding to a kernel or a connected union of kernels, that is the leading pole arising from "the asymptotical regime" of kernels or unions of kernels. For each graph we systematically take into account all the poles at j = 0 and drop out the part which is regular when

Re $j > -1$. This means that each graph is considered with the accuracy $O(1/\nu)$, i.e. all the powers of the logarithm are taken into account. Each kernel or union of kernels V in asymptotical regime generates by itself a simple pole j^{-1}, but the divergent parts* inside V increase the power of the pole at $j = 0$. Moreover, there is some sort at factorization of the asymptotics of the divergent parts. The simple pole $1/j$ for the asymptotical object without divergences turns to

$$\oint \prod_{1}^{\kappa} \frac{d\xi_i}{\xi_i} I(\xi_i) \frac{1}{j+\xi_1+\cdots\xi_\kappa}$$

for the same object but with the nonoverlaping partially divergent parts $\Gamma_1 \cdots \Gamma_\kappa$ where $I_i(\xi_i)$ is the Mellin transform of the asymptotics of Γ_i when all components of the momenta tend to infinity. These are the only sources of singularity in Re $j > -1$ for the positive signature part of the amplitude.

In a theory of the type φ^3 (e.g. $\varphi^2\chi$) the most right singularities of the graphs are at the point $j = -1$. For the negative signature part they are generated by the asymptotical regime of the simplest of the kernels, "the steps", if any (Fig.2b).

The "asymptotical regime" (i.e. $\alpha_\kappa \to 0$) means topologically the contraction of the corresponding object into a point.

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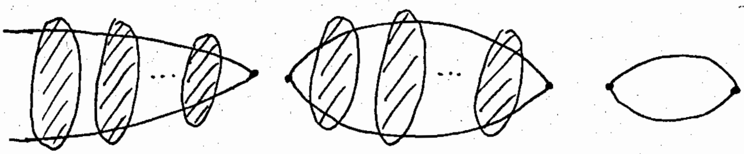
* Notice, that when Re $j > 0$ all kernels and unions of kernels are convergent.

The larger the number of asymptotical objects the higher the order of the pole. But simultaneously can work only the objects which either have no common lines or are entirely contained inside one another.

To take into account all the orders of the poles at $j = 0$ it is necessary to sum up over all the possibilities of the asymptotical and nonasymptotical regime of all the kernels and the unions. The contraction of the objects in the asymptotical regime transforms the initial graph into a weakly connected graph consisting of the components of the type of Fig.3.



(a) (b) (c)

Fig.3

The contribution of these noncontracted objects, which are regular in Re $j > -1$ (in the φ^3 -theory- Re $j > -2$) will be denoted by $C^n(t)$ (Fig.3a) and B^n(Fig.3b). The simplest of them $B^0(t)$ (Fig.3c) and $C^0(t)$ are written down in the expression (15)

In studing the asymptotical behaviour in the mass of the external particle we restrict ourselves to the case of elastic scattering with asymptotically large mass of the first particle, i.e. $p_1 = p_1' = M^2 \to \infty$ It is useful to introduce a new variable $\omega = -\nu/M^2$. The following two cases are of interest:

$$(a)\quad M^2 \to \infty, \quad s\nu \to \infty$$
$$(b)\quad M^2 \to \infty, \quad |\omega| \lesssim 1 . \qquad (7)$$

For the investigation of these limits the Mellin transformation with respect to M^2 - variable of the type (5),(6) is found to be useful again. This transformation reduces the problem of the asymptotical behaviour to the one of leading singularities in the plane of Mellin parameter ℓ. For any graph in mesodynamics in the limit (7a), these singularities are poles at $\ell = 0$ generated by the asymptotical regime (by the contraction) of those kernels and unions of kernels which contain both vertices 1 and $1'$. The order of the pole can be increased by the divergent parts inside the contracted objects. In the φ^3-theory the most right singularity is a pole at $\ell = -1$ which is generated only by the asymptotical regime of the line connecting the vertices 1 and $1'$, if any. In the limit (7b) the above parameter ℓ changes by $\ell + j$ that is poles shift to the left by j.

3. General Scheme of Summation

The method of summation we have used[12-14] is, in fact, a method of solution of the exact Bethe-Salpeter equation. Really, the results of investigation of the Feynman integral asymptotics summarized above reduce the general problem of summation of the logarithmic terms to the problem of a ladder type diagrams. It is natural to assume that the kernel of the t - channel Bethe-Salpeter equation (i.e. the sum of all graphs without two-particle separation in t - channel) has the same asymptotical property as the above mentioned kernels of the graphs. (For the discussion of this assumption see Sec.7)

The main difference between φ^3 - theory and mesodynamics is in the behaviour of the kernel with increasingly large mass of external particles. In φ^3- theory the contribution of the kernel vanishes with the growth of the mass therefore in the equation we can use the properties of the kernel on (or near) the mass shell. In this case the kernel is of the fredholmean type. So, the solution has the moving Regge poles only. Namely this hypothesis is usually made in all variants of the multiperipherical, multireggion, fire ball and parton models[15-18].

In mesodynamics the kernel turns out to be a nonfredholmean one and the contribution of the region of large values of the external mass is important. Such a kernel can be broken into a relatively simple nonfredholmean part V_{NF} which corresponds to the asymptotical regime of the kernel and a fredholmean remainder V_F. The nonfredholmean part V_{NF} is not quadratically integrable but the divergence is only logarithmic one.Since V_{NF} corresponds to the asymptotical regime of the kernel the picture of the j-plane singularities in this case is complicated by the internal divergences. That is why the unavoidable question arises what is the sum of these divergences, i.e. what is the character of the renormalizations in field theory? Because of the asymptotical character of its series, which is displayd just there the perturbation theory is <u>unable to answer in principle this question.</u> Thus we need a hypothesis. It seems natural and meaningful from the physical view point to assume that the wave function and charge renormalization constants Z are finite. (For the argument of this hypothesis see, e.g. refs.[13,19]). This, makes it possible to reduce the problem of the asymptotical regime of the general kernel to the one of the irreducible kernel, i.e. the kernel without internal divergent parts, but with bare coupling constants g' and h' in the vertices and with bare wave functions of the external to the kernel lines. Thus, the total contribution of the contracted objects after having been multiplied by Z^{-2} (i.e. by Z_2^{-2}, Z_3^{-2} or $Z_2^{-1} Z_3^{-1}$ depending on the kind of the external lines) is a series in the bare coupling constants. The fredholmean part is kept, as before to be a series in the renormalized constants.

4. <u>Scattering of Hadrons in the Diffractional Region</u>

The summation of all the orders of the leading poles of all the graphs (under the above mentioned assumption) gives for the amplitudes of positive signature in mesodynamics and of negative one in φ^3 - theory the following result

$$\Phi(j,t) = C(t)\left[u(j) - B(t)\right]^{-1}\tilde{C}(t) + R(t). \quad (8)$$

In φ^3-theory $u(j)=(j+1)/g^2$ and B(t), C(t) (and B(t)) are the series in the coupling constant ($g/4\pi$)^2, each term of which being regular in $Re\, j > -2$ and corresponding to the graphs of fig.3b or fig.3a. Thus, the amplitude has the simple Regge poles and the ambiguity in breaking the kernel contribution into the asymptotical and nonasymptotical parts makes it possible to supply the correct threshold behaviour

$$B(t) \sim \left(1-\frac{t}{4}\right)^{j+\frac{1}{2}} \quad \text{near} \quad t \sim 4 \,. \tag{9}$$

In mesodynamics all the quantities entering (8) are matrices, $u(j)$ and B(t) being symmetrical the first index of C(t) corresponds to the external line the second one to the internal line of the graph and $\tilde{C}$ is the transposed matrix of C . For $G\,P_{=+1}$ (the index T implies T^+)

$$B(t)=\begin{pmatrix} B_{\mu\mu} & \sqrt{2j}\, B_{\mu V} & \frac{M}{2}\sqrt{-2jt}\, B_{\mu T} \\ \sqrt{2j}\, B_{V\mu} & B_{VV} & \frac{M}{2}\sqrt{-t}\, B_{VT} \\ \frac{M}{2}\sqrt{-2jt}\, B_{T\mu} & \frac{M}{2}\sqrt{-t}\, B_{TV} & B_{TT} \end{pmatrix}$$

$$u(j)=\begin{pmatrix} u_{\mu\mu} & \sqrt{2j}\, u_{\mu V} & 0 \\ \sqrt{2j}\, u_{V\mu} & u_{VV} & 0 \\ 0 & 0 & u_{TT} \end{pmatrix} \tag{10}$$

$$C(t)=\begin{pmatrix} C_{\mu\mu} & \sqrt{2j}\, C_{\mu V} & \frac{M}{2}\sqrt{-jt}\, C_{\mu T} \\ C_{V\mu} & \sqrt{j}\, C_{VV} & -4M\sqrt{-j/t}\, C_{VT} \\ C_{S\mu} & \sqrt{j}\, C_{SV} & 2M\sqrt{-j/t}\, C_{ST} \end{pmatrix} .$$

For $\sigma P = -1$ they are simple functions

$$u_{AA}(j);\ B_{AA}(t);\ \sqrt{j}\,C_{AA}(t)\,, \tag{11a}$$

or (T implies T)

$$u_{TT}(j);\ B_{TT}(t);\ 2M\sqrt{-j/t}\,C_{PT}(t)\,. \tag{11b}$$

The matrices B, C and R are the series of B^n, C^n and R^n (see Fig.3) in all sorts of the kernels and in their number. The example of the series for C(t) is shown on Fig.4.

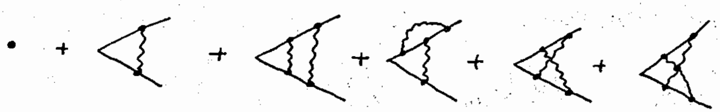


Fig.4

where the wavy lines correspond to mesons, the solid one – to baryons and all the lines are in nonasymptotical regime. It is naturally assumed that the functions B, C and R do not contain the leading singularity of the amplitude as each term of the series.

It is clear from (8) that the amplitude has moving poles which are determined by zeros of det $(U(j) - B(t))$. Thus, in the channel with $\sigma P = +1$ there are three such poles*.

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* A similar situation must take place in quantum electrodynamics, too. Therefore, the conclusion of Cheng and Wu[20] about the nonregge character of electrodynamics based on the analysis of the first several terms of the perturbation series seems to be rather hurry.

The main feature of the mesodynamics is the nontrivial function $\mathcal{U}(j)$ which is a consequence of nonfredholmean character of the kernel in the theory. The function $\mathcal{U}^{-1}(j)$ is the series in j^{-1}. We have succeeded in summing it under the assumption about finiteness of the renormalization constant. It turns out that $\mathcal{U}(j)$ possesses the square-root branch points, the positions of which are determined by the bare coupling constants (i.e. by the constants to be measured at small distances) certain of them being situated at Re $j>0$.

The representation (8), has all the main features of the usual Regge picture. It is clear, in particular, that the position and character of the leading singularity for all scattering processes (including quasielastic[21]) are universal and determined by the t-channel quantum numbers. The factorization theorem for the asymptotics

$$f_{xx}\, f_{yy} = f_{xy}^{2} \qquad (x, y = M, S, V) \tag{12}$$

is the direct consequence of the condition det $(\mathcal{U} - \mathcal{B}) = 0$ determining the poles of the amplitudes and a similar condition for the branch points.

In addition

$$\mathcal{U}_{T^+T^+} = \mathcal{U}_{T^-T^-} \tag{13a}$$

and when t = 0

$$\mathcal{B}_{T^+T^+} = \mathcal{B}_{T^-T^-}\, ;\; C_{PT^-} = C_{ST^+} = C_{VT^+}\,. \tag{13b}$$

The isotopic structure of the theory does not introduce any essential difficulties[12].

The representation (8) can help in solving the following problems:

A) It can serve as a basis for a correct phenomenological description of high energy scattering which requires to include the standing branch points in addition to the Regge poles.

B) A more detailed investigation of the functions B, C and $U(j)$ permits, in principle, to answer the questions about certain properties of the Regge trajectories and residues, for instance the question about their behaviour at large momentum transfers and large mass of the external particles.

C) In the case of a small coupling constant (for instance for μ^+e^- - system) one can use the first terms of the series for B, C and U for studing the properties of the Regge trajectories and the bound states.

For the time being, nothing can be said which of the singularities, the standing branch point or the moving pole - is the leading one. We believe that the standing branchpoint is the most suitable candidate for the Pomeranchuk singularity and moving poles for the P' and other trajectories. This hypothesis better reflects the peculiar role of the pomeranchon and in addition, is more self-consistent, because the rescattering of such pomeranchons gives again the same pomeranchon and there are no complication due to the Gribov many--reggion diagrams. But this does not mean of course that the standing cut is the leading one in the nonvacuum channels as well. In the approximation of the small g' and h' the effective coupling constants in nonvacuum channels are smaller than in the vacuum one and even negative .

The presence of the additional parameters g', h' means, that the problem of the nature of the pomeranchon can be solved either by experiment or, probably, by a nonperturbative approach . Our hypothesis predicts the total cross-section decreasing as $(\ln \nu/\nu_0)^{-3/2}$ and the ratio $\mathrm{Im} f / \mathrm{Re} f = \ln \nu/\nu_0$. The choice of the normalization constant ν_0 is essential in this aspect. If it is not too large (e.g. $\nu_0 \sim m_\pi^2$) the factor $(\ln \nu/\nu_0)^{-3/2}$ for nowaday energies changes rather weakly. The camparison of the hypothesis with the experiment for scattering made by Freedman[22] gives the accuracy of 0.2 mb for the total cross-section, 0.5 mb for the sum rules and 15% for the real part of the amplitude. Now a more complete fit including different processes and angular distributions is necessary. The Serpukhov measurements,[23] seems, at first sight, to contradict the picture of the logarithmically decreasing cross section. However the contributions of the electromagnetic processes[24] at these energies can reach the order of several millibarns and does not permit to make a decisive conclusion on the basis of these data.

It is rather curious that the hypothesis about the leading branchpoint in vacuum channel is consistent with the weak coupling approximation in g' and h'. Its position at $j=1$ is provided by[12] ($h' \ll g'$)

$$3\left(\frac{g'}{4\pi}\right)^2 \approx 0.14 \quad \text{for } SU_2 \qquad 3\left(\frac{g_f'}{4\pi}\right)^2 + \frac{5}{3}\left(\frac{g_d'}{4\pi}\right)^2 \approx 0.12 \quad (14)$$

This corresponds to $g/g' \approx 25 \div 30$. In the other channels the effective coupling oonstants are mach smaller than in the vacuum one and for the axial amplitude it is even negative. This corresponds to the cut along the imaginary axis, i.e. in this channel we deal with the repulsion. Thus we shall often use this approximation in what follows.

The expressions for B(t) and C(t) allow to find the connec-tions between different amplitudes and to ohoose among the dif-ferent solutions of the kinematical oonstrains. In particular, the factor $j = \alpha(t)$ appearing in different amplitudes of and NN-scattering (see C(t) in (IO)) suppresses the corresponding amplitudes at $\alpha(t) = 0$ and leads to the dips in the angular distributions. The same mechanism eliminates the pole at $\alpha(t) = 0$ in the conspiring T^+ and T^- amplitudes (see (IO)) whioh corresponds to the π and π_c trajectories. At t = 0 these amplitudes are equal and give a nonvanishing contribution to the cross seotion.

The processes with isospin 2 in t-ohannel have no NN inter-mediate states. The suppression of this channel can be explai-ned by the smallness of the coupling constant h. The splitting of A - meson could find its explanation in the nondiagonality of the octet representation in t - ohannel in the scheme of SU_3-symmetry.

In conclusion of this section we write down the nonvanishing elements of C(t) and B(t)

$$C^{\circ}_{OO'} = 1$$

$$B^{\circ}_{\mu\mu} = b(j,t,1) - \Gamma(j);\ B^{\circ}_{\mu V} = B^{\circ}_{\mu T} = 0;\ B_{VT} = -b(j+1,t,M) \tag{15a}$$

$$B^{\circ}_{VV} = (1-j)\,b(j,t,M) - \Gamma(j) + 2M^2 b(j+1,t,M);\ B^{\circ}_{TT} = jB(j,t,M)$$

$$B^{\circ}_{T^+T^+} = 2M^2 b(j+1,t,M) - j\,b(j,t,M);\ B^{\circ}_{AA} = (1-j)\,b(j,t,M) - \Gamma(j) \tag{15b}$$

$$\left[B^{\circ}_{VV} = B^{\circ}_{AA} + B^{\circ}_{T^+T^+} + B^{\circ}_{T^-T^-}\right],$$

where

$$b(j,t,M) = \Gamma(j)\int_0^1 dx\,\left[M^2 - x(1-x)t\right]^{-j}. \tag{15c}$$

5. Asymptotics of the Scattering Amplitude in the External Mass.

This scholastic, at first sight, problem becames recently of great actuality in connection with the investigation of deep inelastic ep-scattering and in connection with the moving Regge cuts due to the rescattering of reggions [25,26]. The modern method of finding such cuts assumes that the contribution of the amplitude vanishes fast enough with growth of the masses of the external particles. It is interesting to understand to what extent such a behaviour is justified in field theory.

Let us first consider the limit $(7a)\ M^2 \to \infty, \nu \to \infty$ (Fig.1) in the φ^3 -theory. The diagrams in which the vertices

1 and 1$'$ are linked by one line are here of asymptotical importance and the leading singularity in ℓ-plane is generated by asymptotical regime of this line. In the limit (7a) this leads to the behaviour

$$-\frac{1}{M^2}\,\omega^{\alpha(t)}C(t). \tag{16}$$

In the limit (7b) ($|\omega|\lesssim 1$) the asymptotics is found to be simple : $\sim M^{-2}$. Thus, in the φ^3-theory there is a necessary decrease with increasing mass[25,26].

In mesodynamics the leading singularity in ℓ-plane is generated by the asymptotical regime of the kernels and unions of kernels which contain both vertices 1 and 1$'$. Using, as before, the hypothesis of finite renormalization we obtain in the limit (7a)

$$f^{+}\sim \omega^{\alpha(t)}M^{2\ell_0(\alpha_0(t))}C(t), \tag{17a}$$

where α (t) is the position of a leading singularity determined by (8)*. In the case of the leading branch point α does not depend on t and eq. (17a) is to be multiplied by $(\ln\nu)^{-3/2}$. For small values of bare charges considered in (14)

$$\ell_0(\alpha)=\mathcal{U}^{-1}(\alpha)\simeq 0.1\div 0.2\,. \tag{17b}$$

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* The matrix C(t) is written down in (IO) and $\ell_0(\alpha)$ has the same form as $\mathcal{U}(j)$ but this matrix structure is nonessential for us at the moment.

In the limit (7b)

$$f(M^2;\omega,t)\sim (M^2)^{\ell_0(\omega)} \quad . \tag{18a}$$

6. Deep Inelastic Electro-Hadron Processes in Mesodynamics

Here we will describe the asymptotics of γp- and $\gamma\gamma$-forward scattering in the limit of high energy and large "mass" of photon which are measured (or can be measured) in the experiments on deep inelastic ep- and ee- scattering or on scattering of light by nuclei (see Fig.5).

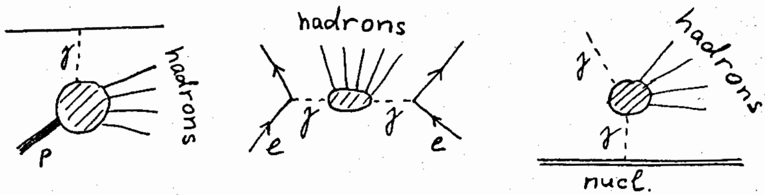

Fig.5

At the available energies these processes are to be considered in the first nonvanishing order with respect to the fine structure constant α . One of the consequence of inclusion of the photon into the play is the appearence of a new kind of divergences which are not reduced to the hadron constant renormalization. As before these divergences work only when they are inside the contracted objects i.e. inside the kernels and unions of kernels in asymptotical region. The summation of these divergences for each photon vertex results in a shift of the singularity in

ℓ-plane generated by the contracted objects to the left by a distance $\xi \approx \left(\frac{g'}{4\pi}\right)^2$ (for small g'). Notice for definitenes that in γp-process one should distinguish between the scattering amplitudes of the scalar (S) and the transverse (T) photons while in $\gamma\gamma$-scattering only the diagonal amplitudes with definite polarization of each of the photons are to be distinguished among them. In other words the first process is characterized by the cross sections σ_S and $\sigma_\perp$ and the second one by $\sigma_{\perp\perp}, \sigma_{S\perp}, \sigma_{\perp S}$ and σ_{SS}. For the sake of simplicity we give here only the result in the limit of weak coupling.

When the photon "mass" M^2 is not too large (or on the mass shell) there is only a slight modification of representation(8). Namely,

$$C_S(t) \rightarrow M^2 C_S'(t)$$

$$C_\perp(t) \rightarrow \left[C_\perp'(t) + F u(j)\left(j+2\xi-u^{-1}(j)\right)^{-1}\right], \tag{19}$$

where F is a number of order $\left(g'/4\pi\right)^2$. Thus, for the transverse photons an additional standing pole in the j-plane appears (the second term in (19)) which, probably, is not a leading one. When the photon mass becomes large the character of dependence (17a), (18a) is conserved but the number ℓ_0 for transversal photons decreases by 2ξ. Therefore, the electro-hadron amplitudes are "almost automodel" (the approximate scaling low holds). For the scalar photons the situation is more complicated but we can conclude, that when $M^2 \rightarrow \infty$

$$\frac{\sigma_S}{\sigma_\perp} \to 0 \qquad \text{for } \gamma p\text{-scattering},$$

$$\frac{\sigma_{S\perp}}{\sigma_{\perp\perp}} \to 0, \quad \frac{\sigma_{SS}}{\sigma_{\perp S}} \to 0 \qquad \text{for } \gamma\gamma\text{-scattering}. \tag{20}$$

In the φ^3-theory the photon formfactors do not work and the electrohadron amplitudes practically behave in just the same manner as the hadron ones.

7. Discussion

All the results enumerated above are based on two hypothesis:

I. The renormalization of the charges and the wave functions in quantum field theory is finite. This hypothesis does not contradict perturbation theory [13,19] and is natural from the physical point of view.

II. The character of the singularity of the Bethe-Salpeter kernel is the same as the singularity of each member of the series forming the kernel, i.e. each graph without two-particle separation in t-channel.

In the φ^4-theory the second hypothesis is connected with first one. It is probable that a similar connection exists in mesodynamics too. In the limit of weak coupling the leading singularity shifts from j = 0 (for separate graphs) at a distance of order g' (for amplitudes). We do not see any reason for the singularities at the points j = -1, -2... (for the separate graphs) to move much faster. So, we believe that the Exp.(8) gives a leading singularity for the amplitude at least, for not too large g and h. With increasing g and h the sum of the singularities at $j = -1$ can be comparable to the sum of the singularities at $j = 0$. The result of the works[25,26] means in our language that when the leading singularity of the graphs shifts due to summation from zero to a point $j = j_0$ then if the amplitude vanishes with mass there exists the class of graphs in which the singularity at the point $j = -1$ shifts to the point $j = 2j_0 - 1$, i.e. it reaches the first one at $j_0 = 1$. This fact siverely complicates the considerations. However, the results of the investigation of the asymptotics by the external mass rise the question about the validity of this statement.

Without recourse to the hypotheses mentioned our results are an exact asymptotical summation of the ladder type diagrams. In this case, of course, $g' = g$, $h' = h$ and the series for B(t) and C(t) turn out to be some other (in the series of Fig.4, for instance, only the terms a,b,c are kept there). Thus, many of the results of the ladder approximation [1,2,7,9,10] correctly describe the general situation in the φ^3-theory. The attempts[18] of summation in mesodynamics, we are aware of, use the assumption, which reduces it to the level of φ^3-theory).

The amplitudes with negative signature in mesodynamics are more complicated. They have, as follows from[11] an accumulation of poles at $j = 0$ due to pinch singularities similar to those found in [1,27]. Nevertheless, the experimental fact of the signature degeneracy of trajectories gives the hint that the accumulating poles come to $j = 0$ from the left. This is a consequence of the sign definiteness of the kernel of an integral equation[27,1]. A similar situation takes place for the negative signature amplitudes in the φ^3-theory, but the poles accumulated here at $j = -1$. All this demonstrates that the region $\mathrm{Re}\, j < 0$ is some sort of a "dump" where different unusual singularities could appear. For this reason one can not make any conclusion about the structure of the j - plane when $\mathrm{Re}\, j < 0$.

Thus, we see that in the φ^3-theory we have delt with a rather "natural", from the widely accepted point of view situation:

> there are only Regge poles, the amplitude vanishes with the growth of external mass, the scaling low holds for γp and $\gamma\gamma$ - scattering.

However, the simplest attempt of taking into account the spin leads to mesodynamics with a number of "unusual" properties

> There are standing branch points in addition to the Regge poles, the amplitude does not vanish with the growth of external mass, the scaling low holds only approximately.

This difference has a deep physical nature. The quantum mechanical analog of the φ^3-theory is a nonsingular at $r \approx 0$ potential of the Yukawa type. All the usual approximate arguments appeal to the images which are familiar from quantum mechanical problems with such potential. But the φ^3-theory can by no means be considered as a physical one because it has neither spin nor vacuum. The only renormalizable theory, which can pretend to the description of experiments is mesodynamics. The quantum mechanical analog of it is an exponentially vanishing at large r potential which, however, is singular at small r (of the type r^{-2}). It is just the singularity which is responsible for the above mentioned "unusual" features of mesodynamics. From this point of view the standing branch point is connected with "deeper" internal parts of the particle than the Regge poles. Really, the position of the branch point is determined by the bare coupling constants, which are measured at small distances, whereas the movement of the poles, i.e. the function B(t) is determined by the renormalized constants g and h which are measured from low energy experiments. It can happen that the scaling low in mesodynamics is exact one. But this would be due to the compensation of the singular interaction by "spreading" of the hadron with respect to the photon (i.e. in a vanishing "formfactor" vanishing in a definite way when all three momenta simultaneously tend to infinity).

A similar difference between φ^3-theory and mesodynamics was observed in the problem of the electromagnetic correction to the strong interaction. The enhancement of the role of the rigid photon due to the singular interaction results there in a rather noticeable difference in the cross sections of particle and antiparticle scattering[24].

In conclusion we would like to stress once more, that the introduction of spin is not a simple technical complication but carries with itself serious modification of principle. Attempts to consider the stated above problems in mesodynamics (e.g.[18]) are connected with the assumptions about the behaviour of the amplitude in the unphysical region which is valid only for a regular interaction. We have no need for such a hypothesis.

References

1. R.J.Eden, P.U.Landshoff, J.C.Polkinghorne, R.Oehme. "The analytic S-matrix". Cambridge, (1966).
2. И.Ф. Гинзбург, В.В. Серебряков. ЯФ, 3, 164 (1966).
   В.Г. Васев, И.Ф. Гинзбург. ЯФ, 5, 669 (1967).
3. B.A.Arbuzov, A.A.Logunov, A.N.Tavkhelidze, R.N.Faustov. Phys.Lett., 2, 150 (1962).
4. P.G.Federbush, M.T.Grisaru. Ann.Phys., 22, 263, 299 (1963).
5. R.F.Sawyer. Phys.Rev., 131, 1384 (1963).
6. J.D.Bjorken, T.T.Wu. Phys.Rev., 136, 2566 (1963).
7. J.C.Polkinghorne. Jorn Math.Phys., 5, 431 (1964).
8. А.И. Ахиезер, В.Б. Берестецкий, "Квантовая электродинамика" "Наука" (1969).
9. H.Abarbanel, M.L.Goldberger, S.B.Treiman. Phys.Rev.Lett., 22, 500 (1969).
10. G.Altarelli, H.Rubinstein. Phys.Rev., 187, 2111 (1969).
11. И.Ф. Гинзбург, А.В. Ефремов, В.Г. Сербо. Препринт ТФ-46 ИМ СО АН СССР Новосибирск (1968).
    И.Ф. Гинзбург, А.В. Ефремов, В.Г. Сербо. ЯФ, 2, 451 (1969).
    И.Ф. Гинзбург, В.Г. Сербо. ЯФ, 2, 868 (1969).

12. В.М. Буднев, И.Ф. Гинзбург, В.Г. Сербо. Препринт ТФ-52 ИМ СО АН СССР, Новосибирск, (1970), направлено в ТМФ.

13. A.V.Efremov, I.F.Ginsburg, V.G.Serbo. JINR Preprint , Е2-4572, Dubna (1969).

14. И.Ф. Гинзбург, А.В. Ефремов, В.Г. Сербо. Материалы совещания по аналитическим свойствам, Серпухов, СТФ 69-101, стр. 182.

15. D.Amati, S.Fubini, A.Stanghellini, M.Tonin. Nuovo Cim., 22, 569 (1961).

16. К.А. Тер-Мартиросян. ЖЭТФ, 44, 341 (1963).
T.W.Kibble. Phys.Rev., 131, 2282 (1963).
G.F.Chew, M.L.Goldberger, F.E.Low. Phys.Rev.Lett., 22, 208 (1969).
G.F.Chew, W.R.Frazer. Phys.Rev., 181, 1914 (1969).

17. И.М. Демин, И.И. Ройзен, Д.С. Чернавский. УФН, 101, №3 (1970).

18. S.D.Drell, D.Levy, T.M.Yan. Phys.Rev.Lett.,22, 744 (1969).

19. П.И. Фомин, В.И. Трутень. ТМФ, 3, (1970), ЖТЭФ, 59 (1970)

20. H.Cheng, T.T.Wu. Phys.Rev.Lett., 22, 666 (1969);
Phys.Rev., 182, 1852, 1868, 1873,1899(1969)

21. В.М. Буднев, И.Ф. Гинзбург ТМФ, 3, 171 (1970).

22. D.D.Freedman. Nuovo Cim., 63A, 483 K (1969).

23. J.V.Allaby, Yu.B.Bushin, S.P.Denisov, at el. Phys.Lett., 30B, 500 (1969).

24. L.D.Soloviev. Proc. of XV Int. Conf. on High Energy Physics, Kiev, 1970.

25. S.Mandelstam. Nuovo Cim., 30, 1113, 1127, 1143 (1963).

26. В.Н. Грибов. ЖЭТФ, 53, 654 (1967); ЯФ, 5, 197 (1967).

27. В.Н. Грибов, И.Я. Померанчук. ЖЭТФ, 43, 1556 (1963).
В.Г. Вакс, А.И. Ларкин. ЖЭТФ, 45, 800 (1963).

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