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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

PROBLEM OF QUASIPOTENTIAL SMOOTHNESS IN QUANTUM FIELD THEORY MODELS

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I. Recently, in the theoretical analysis of experimental data on high energy particle scattering one has extensively used the eikonal or Glauber representation [1] for the small angle scattering amplitude taken from nonrelativistic quantum mechanics.

In this connection there arose the problem of proving the validity of the eikonal approximation in quantum field theory in describing the particle scattering in relativistic energy region.

Note that in the framework of the quasipotential approach in quantum field theory the finding of the scattering amplitude is reduced to the solving of the quasipotential equation [2]. It is well known that in deriving the eikonal representation in this approach [3] of importance is the assumption on the smoothness of the local quasipotential [4] first siggested in paper [5].

Thus, two questions of principle arise:

1. Is it possible to obtain the eikonal representation by summing directly a certain class of the Feynmann graphs?

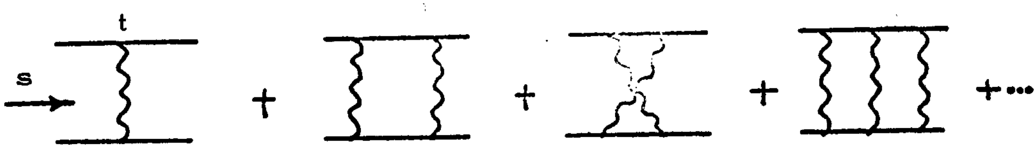
2. What is the status of the quasipotential smoothness in quantum field theory?

In this note we make an attempt to show what answer is obtained in studying some models of quantum field theory[x]. In our consideration we shall base on the results of our previous papers[6,7] in which the functional methods have been used in the framework of the straight-line paths approximation (SLPA). The approximation mentioned is close to the so-called $k_i k_j = 0$ approximation proposed in papers[8,9]. Employing the Feynmann interpretation of the functional integral as a sum over the paths one may say that in the framework of SLPA one takes into account the contribution to the high energy scattering amplitude from the trajectories which are the closest to the classical one (to approximately straight-line ones for small angles scattering).

For definitness we consider the model of interaction of scalar "nucleons" with a neutral vector field A_μ, where the interaction Lagrangian is of the form

$$L_{int} = ig : A_\mu (\psi^* \partial_\mu \psi - \partial_\mu \psi^* \psi) : + \frac{g^2}{2} : A_\mu^2 \psi^* \psi : \qquad (1)$$

II. As a first step we consider the asymptotic behaviour of the scattering amplitude for two nucleons taking into account the multi-meson exchange, neglecting the radiative corrections and the contributions of the closed nucleon loops.



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x/The basic results of this paper have been reported by one of the authors (A.T.) in the rapporteur's talk at the XV International Conference on High-Energy Physics, Kiev-1970.

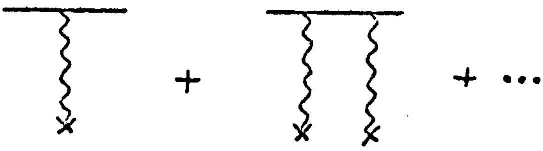
At high energies and fixed momentum transfers, i.e. as $s \to \infty$, and t is fixed, the main logarithmic terms in the asymptotic form of the scattering amplitude are cancelled and the scattering amplitude takes on the form of the Glauber representation /6,7/.

$$f^{(0)}(s,t) = is \int d^2\vec{\rho}\, e^{i\vec{\rho}\vec{q}} \left(1 - e^{2i\chi^{(0)}(\vec{\rho})}\right), \tag{2}$$

where the phase is real and is of the form

$$\chi^{(0)}(\vec{\rho}) = \frac{1}{s}\int_{-\infty}^{\infty} V(s, \sqrt{\rho^2 + z^2})\,dz = \frac{g^2}{8\pi^2}\int \frac{d^2\vec{k}\, e^{-i\vec{k}\vec{\rho}}}{\vec{k}^2 + \mu^2}. \tag{3}$$

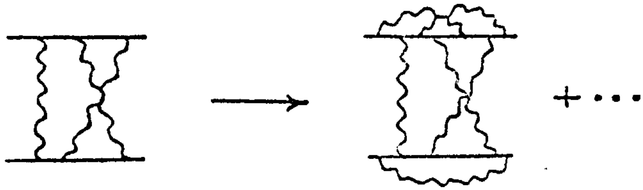
The obtained result means that the retardation effects vanish in some respect and to the scattering amplitude there corresponds the sum of the quasipotential graphs



As we see, in this case the corresponding quasipotential is purely real and is of the form of the Yukawa potential which has a singularity for zero impact parameter $\vec{\rho}$.

The relativistic eikonal representation has been obtained by a number of authors /6,7,10/ by various methods in the framework of the usual field theory models.

III. The next step is the account of the radiative corrections to the nucleon lines



In the framework of SLPA the contribution from the radiative corrections to the scattering amplitude is factorized /11,7/ .

$$f(s,t) = H(t)\, f^{(0)}(s,t) . \tag{4}$$

In the region $|t/m^2| \ll 1$ the factor $H(t)$ depends exponentially upon t

$$H(t) = e^{\alpha t} , \tag{5}$$

$$\alpha = \frac{g^2}{m^2} \left(\ln \frac{m^2}{\mu^2} + \frac{1}{2} \right) . \tag{6}$$

This results in a diffraction peak for small angle scattering.

It is interesting to note that it is just the account of the meson "cloud" of nucleons that leads to the appearance of diffraction type factors in the scattering amplitude. This fact has been noticed in other approaches , as well /12,13/.

We stress that the account of the radiative corrections leads naturally to a smooth complex quasipotential. In fact, writting the scattering amplitude (4) in the eikonal form (2) we get for the eikonal function the following relation

$$e^{-2i\chi(\vec{\rho})} = \int \frac{d^2\vec{b}}{4\pi\alpha} e^{-\frac{\vec{b}^2}{4\alpha}} e^{2i\chi^{(0)}(\vec{\rho}+\vec{b})} . \tag{7}$$

Hence, it is not difficult to show that $\chi(\vec{\rho})$ is a complex quantity with positive-definite imaginary part $|e^{2i\chi(\vec{\rho})}| < 1$ according to the unitarity condition.

Expanding now the exponential under the integral sign in eq. (7) in a power series in $\chi^{(0)}$ we obtain for $\chi(\vec{\rho})$ the following expression

$$\chi(\vec{\rho}) = \frac{g^2}{8\pi^2} \int \frac{d^2 k\, e^{-i\vec{k}\vec{\rho}}}{\vec{k}^2+\mu^2} e^{-\alpha\vec{k}^2} +$$

$$+ i\left(\frac{g^2}{8\pi^2}\right)^2 \int \frac{d^2\vec{k}_1 d^2\vec{k}_2}{(\vec{k}_1^2+\mu^2)(\vec{k}_2^2+\mu^2)} e^{i\vec{\rho}(\vec{k}_1+\vec{k}_2)} \left(e^{-\alpha(\vec{k}_1+\vec{k}_2)^2} - e^{-\alpha\vec{k}_1^2} e^{-\alpha\vec{k}_2^2}\right) + \dots \tag{8}$$

The first term of this expression is purely real and corresponds to the scattering on the Yukawa potential, the centre of forces of which is randomly distributed according to the Gaussian law. The second term contributes to the imaginary part of the quasipotential.

There arises the important problem of improving the range of applicability of SLPA in considering the radiative corrections and the contributions of closed nucleon loops. In particular, it is interesting to study in the framework of the field theory models a possible energy dependence of the slope parameter of the diffraction peak which is due to the account of the radiative corrections.

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