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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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PHOTON-PHOTON SCATTERING WITH COLLIDING $e^{\pm}$ e^{-} BEAMS. I

1970

Фотон-фотонное рассеяние в экспериментах на $e^{\pm}e^{-}$ встречных пучках. I

Обсуждается фотон-фотонное рассеяние вперед на основе процесса аннигиляции $e^{+}+e^{-} \to \gamma$ + адроны. На основе автомодельности, векторной доминантности и партонной модели дается приближенная оценка сечения и показывается, что оно может быть сравнимо с сечением глубоко-неупругого процесса $e^{+}+e^{-} \to H+$ адроны, где H - выделенный адрон. При помощи кварковой модели вычислены сечения некоторых одномезонных процессов.

Сообщения Объединенного института ядерных исследований

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Photon-Photon Scattering with Colliding $e^{\pm}e^{-}$ Beams. I

We discuss photon-photon forward scattering by studying the annihilation process $e^{+}+e^{-} \to \gamma$ + hadrons. Using automodelity, vector meson dominance and the parton model we give a rough estimate of the deep inelastic cross-section showing that it might be comparable to the cross-section of the deep inelastic reaction $e^{+}+e^{-} \to H$ + anything, H being a singled out hadron. Some of the one meson contributions are calculated in the quark model.

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PHOTON-PHOTON SCATTERING WITH COLLIDING $e^{\pm}$ e^{-} BEAMS. I

I. Introduction

The importance of studying inleastic lepton-hadron processes was stressed in last years/1/. As electron-positron and electron-electron storage rings will be considerably developed in the next few years, further new interesting experiments will be feasible if the energies and luminosities anticipated are achieved /2/. First of all the reactions

$$e^- + e^+ \rightarrow \gamma + \text{hadrons}, \quad (1)$$

$$e^\pm + e^- \rightarrow e^\pm + e^- + \text{hadrons} \quad (2)$$

should be studied. Process (1) is of third order in the electromagnetic coupling constant, and its cross section most likely does not decrease faster than the cross-section for

$$e^- + e^+ \rightarrow \text{hadrons}. \quad (3)$$

The significance of processes (2), which can be considered as generalized Bhabha and Möller scattering, is based on the increase of the cross section near forward direction with increasing energy. As a consequence the additional factor α compared to the cross-sections for process (3) will be compensated at higher energies as first shown by F. Low ten years ago/3/.

Both processes (1) and (2) have been discussed in the literature. Creutz and Einhorn /4/ have pointed out that process (1) is very convenient to study hadron systems with even charge conjugation parity. Particularly, they thoroughly investigated the $\gamma \pi^- \pi^+$ final state (see, also/5/) which should give information on even -C di-pion resonances such as the ϵ meson.

Processes (2) were proposed by Low as a method for investigating hadron systems in connection with the measurement of the lifetime of the π^0 /2/ (see also/6,7/).

Their present practical importance has been realized by the Orsay and the Novosibirsk groups/8,9/. However, a more detailed theoretical study of these processes is needed. They play an important role in the investigation of C -even hadronic corrections to pure quantum electrodynamics since their cross-sections are connected with the fundamental process of photon-photon scattering[x].

The investigation of the reaction (2) in the deep inelastic region will be postponed to a second paper. In this paper we discuss only the reaction (1).

In Section II the kinematical analysis is presented. We discuss how to minimize the background as well. In Section III using vector meson dominance (VMD) and the parton model we give a rough estimate of the deep inelastic cross section. Finally the section IV and Appendix are devoted to the discussion of one meson contributions.

II. Kinematics

In the one-photon exchange approximation there are two types of amplitudes as shown in fig. 1 (see Ref. 4).

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x/ An interesting possibility for experimental investigation of real photon-photon scattering process was proposed in/10/.

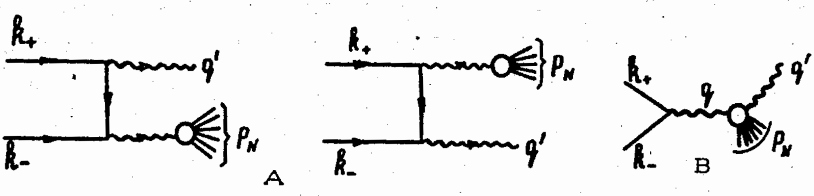

Fig. 1

For experiments which treat the charges symmetrically the interference term between these two types of diagrams vanished by charge conjugation invariance:

$$d\sigma = d\sigma^{(-)} + d\sigma^{(+)} \approx |A|^2 + |B|^2 \ . \tag{4}$$

Creutz and Einhorn /4/ pointed out that the magnitude of A up to order e^3 can be exactly calculated using Q.E.D. and the knowledge of the cross section for the reaction (3). Therefore we can measure the magnitude of B in a "charge simmetric" experiment.

The T-matrix element for the C-odd amplitude shown in Fig. 1 is given by

$$T_{fi}^{(-)} = \frac{(4\pi\alpha)^{3/2}}{q^2-2\nu}\, \bar{v}(k_+)\left(\gamma^\mu \frac{\hat{k}_+ - \hat{q}'}{2k_+q'}\gamma^\nu - \gamma^\nu \frac{\hat{k}_- - \hat{q}'}{2k_-q'}\gamma^\mu\right) u(k_-)\,\epsilon_\mu F_\nu \ , \tag{5}$$

$$q = k_+ + k_- \ , \quad \nu = qq' \ ,$$

where ϵ_μ is the photon polarization vector, $J_\nu(x)$ is the operator of the electromagnetic hadron current and

$$F_\nu = \langle N \,|\, J_\nu(0) \,|\, 0 \rangle . \tag{6}$$

Its contributions to the spin averaged cross sections with respect to the variables ν and Ω (solid angle of the final photon) reads

$$\frac{d\sigma^{(-)}}{d\nu\, d\Omega} = \frac{8\nu\alpha^3}{q^2} \frac{(1-\frac{q^2}{\nu})^2+\cos^2\theta}{\sin^2\theta} \rho\,(q^2-2\nu), \tag{7}$$

where ρ is defined as follows

$$\rho_{\mu\nu}(\tilde{\tilde{q}}) = \sum_N (2\pi)^4 \delta(k_+ + k_- - q' - p_N) < 0 | J_\mu(0) | p_N >< p_N | J_\nu(0) | 0 > =$$

$$= (-\tilde{\tilde{q}}^2 g_{\mu\nu} + \tilde{\tilde{q}}_\mu \tilde{\tilde{q}}_\nu)\, \rho\,(\tilde{\tilde{q}}^2), \tag{8}$$

where

$$\tilde{\tilde{q}} = q - q' \qquad \tilde{\tilde{q}}^2 = q^2 - 2\nu .$$

Using knowledge of $\rho\,(\tilde{\tilde{q}}^2)$ from reaction (3) this contribution can be removed.

The matrix element for the C -even amplitude (see Fig. 1) is given by

$$T_{fi}^{(+)} = \frac{(4\pi\alpha)^{3/2}}{q^2} \bar{v}(k_+)\gamma^\mu u(k_-)\epsilon^\lambda\, i \int dx < N | T\,(J_\mu(x) J_\lambda(0) | 0 > . \tag{9}$$

Its contribution to the differential cross section is, if we observe only the (final state) hadrons,

$$\frac{d\sigma^{(+)}}{d\nu\, d\Omega} = \frac{2\alpha^3\nu}{q^8} L^{\mu\nu} \rho_{\mu\nu;\lambda\sigma}\, \ell^{\lambda\sigma}, \tag{10}$$

where $\nu = q \cdot q'$, $L^{\mu\nu}$, $\ell^{\lambda\sigma}$ are the density matrices of the lepton pair and the photon respectively. The fourth rank-tensor represented by the diagram of Fig. 2 is defined as follows

$$\rho_{\mu\nu;\lambda\sigma} = \sum_N (2\pi)^4 \delta(q-q'-p_N) \int dx dy\, e^{-iq(x-y)} \Gamma_{\mu\nu;\lambda\sigma}(x,y), \tag{11a}$$

where

$$\Gamma_{\mu\nu;\lambda\sigma}(x,y) = \langle N | T(J_\mu(x) J_\lambda(0)) | 0 \rangle \langle N | T(J_\nu(y) J_\sigma(0) | 0 \rangle^* \tag{11b}$$

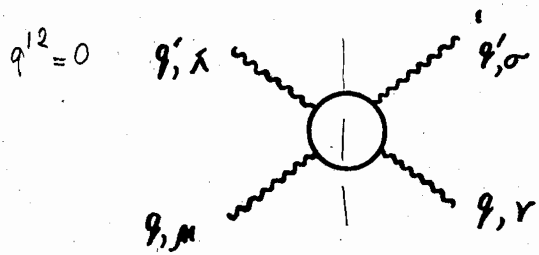

Fig. 2. Diagram for the $\rho_{\mu\nu;\lambda\sigma}(q,q')$ tensor showing its close connection with the absorptive part of the forward photon-photon scattering.

The $\rho_{\mu\nu;\lambda\sigma}$ tensor is closely related to the forward photon-photon scattering amplitude. From the hermicity of $\rho_{\mu\nu,\lambda\sigma}$

$$\rho_{\mu\nu;\lambda\sigma} = \rho^*_{\nu\mu;\sigma\lambda} \tag{12}$$

and from PT invariance

$$\rho_{\mu\nu;\lambda\sigma} = \rho_{\nu\mu;\sigma\lambda} \tag{13}$$

we find that the tensor (11a) is real and symmetric with respect to simultaneous change of $\mu \rightleftarrows \nu$ and $\lambda \rightleftarrows \sigma$. Current conservation, P, T and Lorentz-invariance imply that $\rho_{\mu\nu;\lambda\sigma}$ can be expressed in the form

$$\rho_{\mu\nu;\lambda\sigma} = \rho_1 G_{\mu\nu} G'_{\lambda\sigma} + \rho_2 Q_{\mu\nu} G'_{\lambda\sigma} + \\ + \rho_3 (G''_{\mu\lambda} G''_{\nu\sigma} + G''_{\mu\sigma} G''_{\nu\lambda}) + \rho_4 (G''_{\mu\lambda} G''_{\nu\sigma} - G''_{\mu\sigma} G''_{\nu\lambda}), \tag{14}$$

where ρ_i (i = 1,2,3,4) are real functions depending on the variables ν and q^2. The list of gauge-invariant tensors given in Eq. (14) reads as follows

$$G_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}, \tag{15a}$$

$$G'_{\lambda\sigma} = -g_{\lambda\sigma} + \frac{q_\lambda q'_\sigma + q'_\lambda q_\sigma}{q \cdot q'}, \tag{15b}$$

$$G''_{\mu\lambda} = -g_{\mu\lambda} + \frac{q'_\mu q_\lambda}{q \cdot q'}, \tag{15c}$$

$$Q_{\mu\nu} = Q_\mu Q_\nu \qquad Q_\mu = q_\mu - \frac{q^2}{q \cdot q'} q'_\mu \,. \tag{15d}$$

The measurement of the photon angular distribution gives only information on two particular combinations of our structure functions. To obtain further information on the structure functions one has to perform polarization experiments.

For collisions of polarized electrons and unpolarized positrons the density matrix of the lepton pair has the form

$$L^{\mu\nu} = k_+^{\mu} k_-^{\nu} + k_+^{\nu} k_-^{\mu} - \frac{q^2}{2} g^{\mu\nu} - im\, \epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta^{(-)}, \tag{16a}$$

where $s_\beta^{(-)}$ is the polarization vector of the electron. The density matrix of the photon can be expressed by the Stokes parameters

$$\begin{aligned} \ell^{\lambda\sigma} &= \frac{1}{2}(\epsilon^{\lambda}_{(1)} \epsilon^{\sigma}_{(1)} + \epsilon^{\lambda}_{(2)} \epsilon^{\sigma}_{(2)}) + \frac{\xi_1}{2}(\epsilon^{\lambda}_{(1)} \epsilon^{\sigma}_{(2)} + \epsilon^{\lambda}_{(2)} \epsilon^{\sigma}_{(1)}) - \\ &- i\frac{\xi_2}{2}(\epsilon^{\lambda}_{(1)} \epsilon^{\sigma}_{(2)} - \epsilon^{\lambda}_{(2)} \epsilon^{\sigma}_{(1)}) + \frac{\xi_3}{2}(\epsilon^{\lambda}_{(1)} \epsilon^{\sigma}_{(1)} - \epsilon^{\lambda}_{(2)} \epsilon^{\lambda}_{(2)}). \end{aligned} \tag{16b}$$

Using Eqs. (10)-(16) we obtain

$$\frac{d\sigma(q^2, \nu, s^{(-)}, \xi_1)}{d\nu\, d\Omega} = \frac{a^3 \nu}{q^6} \{ 2(\rho_1 + \rho_3) + \sin^2\theta [q^2 \rho_2 - (1-\xi_3)\rho_3] + \frac{4m\, s_3^{(-)}}{\sqrt{q^2}} \rho_4 \} . \tag{17}$$

Averaging over initial lepton spins and the final photon polarizations we arrive at

$$\frac{d\sigma^{(+)}}{d\nu\, d\Omega} = \frac{2a^3 \nu}{q^6} [2(\rho_1 + \rho_3) + \sin^2\theta\, (q^2 \rho_2 - \rho_3)] . \tag{18}$$

Obviously this gives informations only on the combinations

$$\overline{W}_1^{\gamma} = 2(\rho_1 + \rho_3) \tag{19a}$$

$$\overline{W}_2^{\gamma} = 2(q^2\rho_2 - \rho_3). \tag{19b}$$

For unpolarized beams the measurement of the linear polarization of the photon with respect to the plane spanned by the positron momentum $\vec{k}_+$ and the photon momentum $\vec{q}\,'$ provides us with additional information. The cross section for polarization transverse to this plane is given by

$$\left(\frac{d\sigma^{(+)}}{d\nu\, d\Omega}\right)^{\perp} = \frac{\alpha^3\nu}{q^6}\left[2(\rho_1+\rho_3) + q^2 \sin^2\theta\rho_2\right] \tag{20a}$$

for polarization in the plane

$$\left(\frac{d\sigma^{(+)}}{d\nu d\Omega}\right)^{||} = \frac{\alpha^3\nu}{q^6}\left[2(\rho_1+\rho_3) + \sin^2\theta(q^2\rho_2 - 2\rho_3)\right]. \tag{20b}$$

The structure function ρ_3 is obtained by the difference

$$\left(\frac{d\sigma^{(+)}}{d\nu\, d\Omega}\right)^{\perp} - \left(\frac{d\sigma^{(+)}}{d\nu\, d\Omega}\right)^{||} = \frac{2\alpha^3\nu}{q^6}\sin^2\theta\,\rho_3 . \tag{21}$$

Finally, the structure function ρ_4 can be measured by observing circularly polarized final photons from annihilations of polarized electrons and unpolarized positrons

$$\left(\frac{d\sigma^{(+)}}{d\nu\, d\Omega}\right)^{\circlearrowleft} = \frac{\alpha^3\nu}{q^6}\left[2(\rho_1+\rho_2) + \sin^2\theta(q^2\rho_2 - \rho_3) + \frac{4m s_3^{(-)}}{\sqrt{q^2}}\rho_4\right], \tag{22}$$

Performing such experiments one encounters the question of distinguishing the internal bremstrahlung process represented by the diagram B of Fig. 1, from the large background of external bremstrahlung and photons coming from π^0 decays. Because of the known features of the external bremstrahlung photons have to be observed at large angles with respect to the direction of the beam and the momenta of the other charged particles involved. To reduce the background one has to know something about the production mechanism of hadrons. (As to the background see the analysis of the inelastic Compton scattering given in Ref./11/). Bjorken and Brodsky have pointed out/12/ that there are two possible extremes. On the one hand the jet picture, where the distribution of the transverse momenta of secondaries relative to a particular axis is given by an exponential low.

Therefore, energetic photons measured at large angles with respect to the jet axis should mainly be due to internal bremstrahlung. On the other hand the statistical model predicts a distribution of secondaties which falls off with energy exponentially.

Accroding to this model the mean value of the energy of pions should be $\langle E_\pi \rangle \approx 400$ MeV. If production of hadrons exhibits such a "statistical" behaviour very energetic photons transversal to the beam direction should be observed.

Hopefully, measurements of reaction (3) will provide us with the necessary information to set up an experiment where photons produced by internal bremstrahlung can be distinguished from the large background.

III. Estimate for the Deep Inelastic Cross Section Using Automodelity, VMD and the Parton Model

In work/1/ the approximate automodelity or scale invariance principle was formulated for lepton-hadron processes at high energies and large momentum transfers.

It follows from this principle that asymptotics for the form factors ρ_i ($i = 1,2,3,4$) of our process transform under scale transformations

$$q \to \lambda q$$

$$q' \to \lambda q'$$

as homogeneous functions of corresponding dimensions.

It is easy to see that the tensor $\rho_{\mu\nu;\lambda\sigma}$ is dimensionless

$$[\rho_{\mu\nu;\lambda\sigma}] = 1$$

from where it follows that

$$[\rho_i] = 1 \quad , \quad i = 1,3,4$$

$$[\rho_2] = [m^{-2}] .$$

From the automodelity principle it follows that

$$\rho_i(\lambda^2 q^2, \lambda^2 \nu) = \rho_i(q^2, \nu) \qquad (i = 1,3,4)$$

$$\lambda^2 \rho_2(\lambda^2 q^2, \lambda^2 \nu) = \rho_2(q^2, \nu).$$

These requirements can be satisfied by putting

$$\rho_i = F_i\left(\frac{\nu}{q^2}\right) \qquad i = 1,3,4$$

$$\rho_2 = \frac{1}{q^2} F_2\left(\frac{\nu}{q^2}\right).$$

The experimental verification of these consequences of automodelity is interesting.

The vector meson dominance model can be used for the processes with real photons. It might, however, not be used for virtual photons in the far-off time like region. Therefore we use VMD only for the real photon as shown in Fig. 3.

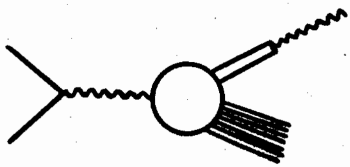

Fig. 3

Diagram of vector meson dominance for the real photon.

By this procedure the C -even part of the cross section for process (I) is connected with the cross section for the reaction

$$e^- + e^+ \to V + \text{hadrons}, \tag{23}$$

V being a singled out vector meson (see, e.g. Refs. 13). More precisely, the spin averaged part of the fourth rank tensor Eq. (14) can be written

$$\overline{W}^{\gamma}_{\mu\nu} \equiv \rho^{\lambda}_{\mu\nu;\lambda} = \sum_N (2\pi)^4 \delta(q-q'-p_N)\left(\frac{1}{2\gamma_V}\right)^2 < 0 | J_\mu(0) | p_N, V(q') > \times$$

$$\times < p_N, V(q') | J_\nu(0) | 0 >^* + \text{ interference terms} \tag{24}$$

Neglecting the interference term we obtain

$$\overline{W}_1^{\gamma} = \sum_V \frac{3 m_V \pi}{\gamma_V^2} \overline{W}_1^{V} \tag{25a}$$

and

$$\overline{W}_2^{\gamma} = \sum_V \frac{3\pi}{\gamma_V^2} \frac{1}{m_V} \overline{W}_2^V , \tag{25b}$$

where $\overline{W}_1^V$ $\overline{W}_2^V$ are the structure functions for the process of Eq. (23), defined as follows

$$\overline{W}_{\mu\nu} = \frac{1}{4\pi m_V} \sum_N (2\pi)^4 \delta(q - q' - p_N) < 0 \mid J_\mu(0) \mid p_N , V(q') > \times \tag{26a}$$

$$\times < p_N , V(q') \mid J_\nu(0) \mid 0 > =$$

$$= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \overline{W}_1^V + \frac{1}{m_V^2} (q'_\mu - \frac{qq'}{q^2} q_\mu)(q'_\nu - \frac{qq'}{q^2} q_\nu) \overline{W}_2^V . \tag{26b}$$

The invariant functions for the corresponding electroproduction process are defined similarly. According to the substitution low/12/ the structure functions for electroproduction $W_{\mu\nu}^V (q, q')$ are related to the annihilation structure functions $\overline{W}_{\mu\nu}^V (q', q')$ by

$$W_{\mu\nu}^V (q, q') = \overline{W}_{\mu\nu}^V (q, -q'). \tag{27}$$

The cross section for electroproduction on vector meson has the usual from:

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^2 \frac{\theta}{2}} (W_2^V \cos^2 \frac{\theta}{2} + 2 W_1^V \sin^2 \frac{\theta}{2}). \tag{28}$$

Integrating over the photon solid angle and introducing the new variable

$$\omega = \frac{q^2}{2\nu} . \tag{29}$$

we can rewrite cross-section Eqs. (18) in the form

$$\frac{d\sigma}{d\omega} = \frac{2\pi\alpha^3}{\omega^3 q^2}\left[\bar{W}_1^{\gamma} + \frac{1}{3}\bar{W}_2^{\gamma}\right]. \tag{30}$$

Using the relations of Eqs. (25) we obtain

$$\frac{d\sigma}{d\omega} = \frac{2\pi\alpha^3}{\omega^3 q^2}\sum_{V}\left(\frac{3m_V\pi}{\gamma_V^2}\bar{W}_1^V + \frac{\pi}{\gamma_V^2}\frac{1}{m_V}\bar{W}_2^V\right). \tag{31}$$

In the limit $q^2 \to \infty$, $\nu \to \infty$ and ω is fixed an automodel/1/ behaviour for $\bar{W}_1$ is assumed

$$\lim_{\substack{q^2,\nu\to\infty \\ \omega\approx \text{fixed}}} \frac{\nu}{m_V}\bar{W}_2^V = \bar{F}_2^V(\omega) \tag{32a}$$

$$m_V\bar{W}_1^V = \bar{F}_1^V(\omega). \tag{32b}$$

Obviously, the structure functions $\bar{W}_1^{\gamma}$ exhibit the same automodel or scale invariant behaviour

$$\bar{W}_1^{\gamma} \approx \sum_{V}\frac{3\pi}{\gamma_V^2}\bar{F}_1^V(\omega) \tag{33a}$$

$$\lim_{\substack{q^2,\nu\to\infty \\ \omega-\text{fixed}}} \bar{W}_2^{\gamma} \approx \sum_V \frac{3\pi}{2\gamma_V^2} \bar{F}_2^V(\omega). \tag{33b}$$

Therefore the deep inelastic cross-section for process (1) reads as follows

$$\frac{d\sigma}{d\omega} = \frac{6\pi^2\alpha^3}{\omega^3 q^2} \sum_V \frac{1}{\gamma_V^2} (\bar{F}_1^V(\omega) + \frac{1}{6}\bar{F}_2^V(\omega)) \tag{34}$$

showing automodel character explicitly.

We assume that the asymptotic form factors $\bar{F}_1^V(\omega)$ and $\bar{F}_2^V(\omega)$ can be obtained by analytic continuation of the form factors of deep inelastic electroproduction $F_1^V(\omega)$ and $F_2^V(\omega)$. This assumption was proposed by several authors. It is worth noticing that such a property can be shown in a simple field theoretic model. In Veneziano-type models, however, the asymptotic form factor $\bar{F}_2(\omega)$ diverges/14/. Supposing such an analytic continuation is possible we find (see Eq. (37))

$$F_1^V(\omega) = \bar{F}_1^V(\omega), \tag{35a}$$

$$F_1^V(\omega) = -\bar{F}_2^V(\omega). \tag{35b}$$

For annihilation and electroproduction processes the physical regions have a common boundary at $\omega = 1$, therefore we can estimate the magnitude of the cross section (34) near $\omega \approx 1$ using the knowledge of the asymptotic form factors of electroproduction. The parton model of Bjorken and Paschos/11/ should be useful in estimating the order of magnitude of $F^V_{1,2}(\omega)$. For spin 1/2 partons we have

$$F^V_1(\omega) = \frac{1}{2\omega} F^V_2(\omega). \qquad (36)$$

Therefore, the cross section (34) can be written as

$$\frac{d\sigma}{d\omega} = \frac{2\pi^2 a^3}{\omega^4 q^2} \sum_V \frac{1}{\gamma_V^2} F^V_2(\omega). \qquad (37)$$

Since the parton-antiparton could give the main contributions the magnitude of $F^V_2(\omega)$ is of the same order as for electroproduction on protons. The behaviour of $F^V_2(\omega)$ near $\omega \approx 1$, however, might be different from $F_2^{Proton}(\omega)$. In such parton models (the parton spin is 1/2) we have at $\omega = 1$ the following threshold theorems [13]:

for fermions

$$F_2(\omega) = C(1-\omega)^{2n+1} + \dots \quad n = 0,1,2,\dots \qquad (38a)$$

and for bosons, respectively

$$F_2(\omega) = C'(1-\omega)^{2n} + \dots \quad n = 0,1,2,\dots \qquad (38b)$$

Therefore assuming smooth behaviour near $\omega \approx 1$ we find that $\bar{F}_2(\omega)$ larger for bosons than for fermions. An explicit calculation shows that the most important contribution comes from the ρ meson. Using the experimental values of the factors γ_V /2/, we obtain

$$\frac{d\sigma^{\gamma}}{d\omega} : \frac{d\sigma^{V}}{d\omega} \gtrapprox 0,1 \; . \tag{39}$$

Integrating the cross section (37) over the region $\omega = 1,1 - 1,3$ at virtual photon mass square $q^2 \approx 50$ GeV^2 we find

$$\Delta\sigma \approx 10^{-35} - 10^{-36} \text{ cm}^2 . \tag{40}$$

IV. One-Meson Contributions

We briefly discuss the reactions

$$e^+ + e^- \to P + \gamma \tag{41a}$$

and

$$e^+ + e^- \to S + \gamma , \tag{41b}$$

where P and S denote pseudoscalar and scalar mesons respectively.

This processes are connected with the photon-photon scattering through one hadronic intermediate state. Measuring the cross sections for these processes we can obtain important information for verteces $P\gamma^*\gamma$ and/or $S\gamma^*\gamma$ (The asterisk denotes virtual photon). These verteces can be given for pseudoscalar mesons as

$$\Gamma^{P}_{\mu\nu} = \int dx\, e^{-iqx} < P(k) \,|\, T(J_{\mu}(x)\, J_{\nu}(0)) \,|\, 0 > = \epsilon_{\mu\nu\lambda\sigma}\, q^{\lambda} q'^{\sigma} G_P(q^2) \tag{42a}$$

and for scalar mesons, respectively as

$$\Gamma^{S}_{\mu\nu} = \int dx\, e^{-ipx} < S(k) \,|\, T(J_{\mu}(x), J_{\nu}(0) \,|\, 0 > = (-g_{\mu\nu} + \frac{q'_{\mu} q_{\nu}}{qq'})\, G_S\,(q^2\,). \tag{42b}$$

Using the vertex functions $G_P(q^2)$ and $G_S(q^2)$ for the corresponding cross sections we obtain the forms

$$\frac{d\sigma^P}{d\Omega} = \frac{\pi\alpha^3 \nu^3}{q^6}(1 + \cos^2\theta\,) \,|\, G_P(q^2\,) \,|^{\,2} \tag{43a}$$

and

$$\frac{d\sigma^S}{d\Omega} = \frac{\pi\alpha^3 \nu}{q^6}\,(1 + \cos^2\theta) \,|\, G_S(q^2\,) \,|^2. \tag{43b}$$

It is worth mentioning that $G_{\pi^0}(0)$ gives the lifetime of π^0

$$\frac{1}{\tau_{\pi^0}} = \frac{m_\pi^3}{64\pi} \,|\, G_\pi(0) \,|^2\,. \tag{44}$$

As to the contributions of processes (41a and b) to the inelastic form factors we obtain for pseudoscalar mesons

$$\rho_1 = 2\rho_4 = -2\rho_3 = -\frac{1}{q^2}\rho_2 = 2\pi\delta(q^2 - 2\nu - m_p^2)G_P(q^2) \tag{45a}$$

and for scalar mesons

$$\rho_1 = \rho_2 = 0$$

$$\rho_3 = \rho_4 = \pi\delta(q^2 - 2\nu - m_S^2)G_S(q^2). \tag{45b}$$

If the c.m. energy is near the values of vector meson masses we must use vector meson dominance for the virtual photon. The vertex functions (41a and b) near vector meson resonances have the forms

$$G_P(q^2) = \frac{m_V^2}{q^2 - m_V^2}\,\frac{f_{VP\gamma}}{\gamma_V} \tag{46a}$$

and

$$G_S(q^2) = \frac{m_V^2\nu}{q^2 - m_V^2}\,\frac{d_{VS\gamma}}{2\gamma_V}, \tag{46b}$$

where $f_{VP\gamma}$ and $d_{VS\gamma}$ denote the corresponding coupling constants related to the widths of the decays $V \to P\gamma$ and $V \to S\gamma$ respectively

$$\Gamma(V \to P\gamma) = \frac{f^2_{VP\gamma}}{3\pi} k^3 \qquad (47a)$$

and

$$\Gamma(V \to S\gamma) = \frac{d^2_{VS\gamma}}{12\pi} k^3. \qquad (47b)$$

The known predictions for widths (47a) obtained in the quark model are listed in Table I.

Table I

Predictions of the quark model for the widths

Process	$f_{VP\gamma}$	Γ in MeV	Experiment
$\rho^0 \to \pi^0\gamma$	$-1/3\,\mu_P$	0.12	-
$\omega \to \pi^0\gamma$	$-\mu_P$	1.17	1.20
$\phi \to \pi^0\gamma$	0	$2.68\ \sin^2\theta'$	< 0.014
$\rho^0 \to \eta\gamma$	$-\frac{1}{\sqrt{3}}\mu_P$	$5.0\ 10^{-2}$	-
$\omega \to \eta\gamma$	$-\frac{1}{3\sqrt{3}}\mu_P$	$6.3\ 10^{-3}$	-
$\phi \to \eta\gamma$	$-\frac{2}{3}\sqrt{\frac{2}{3}}\,\mu_P$	0.34	<0.32

As to the coupling constant $d_{VS\gamma}$ it can be calculated in the quark model, too. Such type of calculation is given in the Appendix.

At energies near the mass values of the vector mesons we can use Breit-Wigner resonance formula to predict the magnitude of the cross-sections at the resonance peacs. The Breit-Wigner formula for the cross-sections has the form

$$\sigma(E) = \frac{12\pi}{m_V^2} \frac{\Gamma(V \to ee)\Gamma(V \to f)}{\Gamma_V^2} \frac{1}{1 + \frac{m_V(2E - m_V)}{2\Gamma_V^2}} . \tag{47}$$

The predictions obtained by using this formula are given in Table II

Table II

Predictions for the cross-sections of $ee \to P\gamma$ and $ee \to S\gamma$

Process	$\sigma(m_\rho)$ in μb	$\sigma(m_\omega)$ in μb	$\sigma(m_\phi)$ in μb
$ee \to \pi\gamma$	$(1.5 \pm 0.5)\ 10^{-3}$	$(1.74 \pm 0.6)\ 10^{-1}$	$3.41 \sin^2 \theta'$
$ee \to \eta\gamma$	$(6.3 \pm 2.0)\ 10^{-4}$	$(7.9 \pm 2.4)\ 10^{-4}$	(4.3 ± 0.5)
$ee \to \epsilon\gamma$	-	-	10^{-2}-10^{-3}

The V.M.D. can be used for the real photon, too. We argue, however, that at the vector meson resonances the quark model is more reliable.

Appendix

Decay $\phi \to \epsilon + \gamma$ and the quark model.

We consider the process of radiative decay of a vector meson to a scalar ϵ-meson presented in Fig. 5.

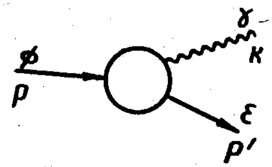

Fig. 5

The matrix element of the decay

The decay matrix element is written in the form

$$T_{fi} = e\, \epsilon^{\mu}_{(\gamma)} < \epsilon \,|\, J_{\mu}(0)\,|\, \phi > . \tag{A.1}$$

From the condition of Lorentz and gauge invariance it follows

$$T_{fi} = d\,[(\epsilon^{(\gamma)} p)(\epsilon^{(V)} \cdot k) - (\epsilon^{(\gamma)} \cdot \epsilon^{(V)})(p\, k)], \tag{A.2}$$

where d is an unknown constant connected with the width by the relation

$$\Gamma = \frac{d^2}{12\pi} \left(\frac{m^2_{\phi} - m^2_{\epsilon}}{2m_{\phi}} \right)^3 . \tag{A.3}$$

The aim of the present Appendix is the determination of this constant by means of the quark model. In the quark model the decay

$$1^- \to 0^+ + \gamma \tag{A.4}$$

corresponds to the electric dipole E_1 transition of the type

$$^3S_1 \overset{3}{\to} P_0 + \gamma \tag{A.5}$$

performed by the operator/15/

$$\vec{D} = \int \vec{x}\, \rho(\vec{x})\, d\vec{x} \;. \tag{A.6}$$

Since $\rho(\vec{x}) = J_0(\vec{x},0)$ we have

$$< \epsilon | \vec{D} | \phi > = \int \vec{x}\, d\vec{x} < \epsilon | J_0(\vec{x},0) | \phi > =$$

$$= (2\pi)^3\, \delta(\vec{k})\, d\, m_\phi\, \vec{\epsilon}^{(V)} . \tag{A.7}$$

On the other hand, assuming quarks to be point particles without proper electromagnetic structure/15/, we may put $\rho(\vec{x}) = e_1\, \delta(\vec{x}-\vec{r}_1) + e_2\, \delta(\vec{x}-\vec{r}_2)$, where $e_i, \vec{r}_i$ are the charges and coordinates of the quark ($i = 1$) and antiquark ($i = 2$). We find for the matrix element of the operator $\vec{D} = \vec{e}_1 \vec{r}_1 + \vec{e}_2 \vec{r}_2$ the following value

$$< \epsilon | \vec{D} | \phi > = < \epsilon | e_1 \vec{r}_1 + e_2 \vec{r}_2 | \phi > =$$

$$= (2\pi)^3\, \delta(\vec{k})\, 2 m_\phi \frac{1}{3\sqrt{3}} \int d\vec{r}\, \psi^*_\epsilon(\vec{r})\, \vec{r}\, \psi_\phi(\vec{r}), \tag{A.8}$$

where $\psi_\epsilon(\vec{r})$ and $\psi_\phi(\vec{r})$ are the wave functions of the relative motion of quark–antiquark pair in the meson:

$$\psi_{\epsilon}(\vec{r}) = \psi_{\epsilon}(r)\frac{\vec{\sigma}\cdot\vec{n}}{\sqrt{2}} \qquad \vec{n} = \frac{\vec{r}}{|\vec{r}|} \tag{A.9}$$

$$\psi_{\phi}(\vec{r}) = \psi_{\phi}(\vec{r})\frac{\vec{\sigma}\cdot\vec{\epsilon}^{(V)}}{\sqrt{2}}. \tag{A.10}$$

Comparing (7) and (8) we find

$$d = -\frac{8\pi}{9\sqrt{3}}\, e \int_0^{\infty} dr\, \psi_{\epsilon}(r) r^3 \psi_{\phi}(r).$$

Using the Dalitz oscillator wave functions we find

$$d = -1{,}23 \cdot e\, \frac{1}{\mathrm{GeV}}.$$

Hence, for the width Γ we have the value 40 keV, given in the paper.

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