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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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REPRESENTATION FOR THE RELATIVISTIC SCATTERING AMPLITUDE AT HIGH ENERGIES

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I. Introduction

Recently processes of hadron interactions at high energies have been a subject of intensive theoretical and experimental investigation.

The derivation of closed expressions for scattering amplitudes in the framework of some reasonable physical assumptions in the high energy region is of special interest. In order to obtain expressions of such a kind one can use the unitarity condition for the scattering amplitude with a definite parametrization of inelastic channel contributions[1-5], the Logunov-Tavkhelidze quasipotential equation[6-10], the functional integration method in quantum field theory[11-14], summation of field theoretic perturbation theory graphs[15,16], the reggeon diagram technique[17-19].

In the present paper we derive a new representation for the scattering amplitude at high energies, starting from the equations of the quasipotential type[x], which were considered in refs.[20-23].

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[x] For simplicity we confine our considerations to the case of scattering of spinless particles of equal masses.

$$\psi_{\vec{q}}(\vec{p}) = (2\pi)^3 \delta^{(3)}(\vec{p}\,(-)\,\vec{q}) + \frac{1}{(2\pi)^3} \frac{1}{2E_q - 2E_p + i\epsilon} \int V[(\vec{p}\,(-)\,\vec{k})^2; E_q]\,\psi_{\vec{q}}(\vec{k})\,d\Omega_{\vec{k}} \quad (1.1)$$

$$A(\vec{p},\vec{q}) = -\frac{m}{4\pi} V[(\vec{p}\,(-)\,\vec{q})^2; E_q] + \frac{1}{(2\pi)^3} \int \frac{V[(\vec{p}\,(-)\,\vec{k}); E_q]\, d\Omega_{\vec{k}}\, A(\vec{k},\vec{q})}{2E_q - 2E_k + i\epsilon} . \quad (1.2)$$

Here

$$d\Omega_{\vec{k}} = \frac{d\vec{k}}{\sqrt{1 + \vec{k}^2/m^2}} ; \qquad E_k = \sqrt{m^2 + \vec{k}^2} , \quad (1.3)$$

$$\vec{p}\,(-)\,\vec{k} = \vec{p} - \vec{k}\,[\sqrt{1 + \vec{p}^{\,2}/m^2} - \frac{\vec{p}\,\vec{k}}{m^2(1 + \sqrt{1 + \vec{k}^2/m^2})}] . \quad (1.4)$$

The amplitude $A(\vec{p},\vec{q})$ is normalized to the differential cross section of elastic scattering in the following manner:

$$\frac{d\sigma}{d\Omega} = |A(\vec{p},\vec{q})|^2 ; \quad E_p = E_q . \quad (1.5)$$

Comparing eqs. (1.1)-(1.2) with the nonrelativistic Schoedinger

$$\psi_{\vec{q}}(\vec{p}) = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{q}) + \frac{1}{(2\pi)^3} \frac{1}{2E_q - 2E_p + i\epsilon} \int V[(\vec{p}-\vec{k})^2] \psi_{\vec{q}}(\vec{k}) d\vec{k} \quad (1.6)$$

and Lippmann-Schwinger equations

$$A(\vec{p},\vec{q}) = -\frac{m}{4\pi} V[(\vec{p}-\vec{q})^2] + \frac{1}{(2\pi)^3} \int \frac{V[(\vec{p}-\vec{k})^2] d\vec{k} A(\vec{k},\vec{q})}{2E_q - 2E_k + i\epsilon} \quad (1.7)$$

it is easy to see that the relativization (1.3)-(1.4) reduces essentially to the replacement[x/]

$$d\vec{k} \to d\Omega_{\vec{k}}$$

$$\vec{p} - \vec{k} \to \vec{p}(-)\vec{k} \quad (1.8)$$

$$\frac{\vec{k}^2}{2m} \to \sqrt{m^2 + \vec{k}^2}.$$

The replacement (1.8) has a clear geometric character. Indeed, on the left hand sides of eqs. (1.8) we see the images of three-dimensional Euclidean momentum space, while on the right hand sides we deal with quantities and operations which are defined on the upper sheet of the hyperboloid

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x/ A nonrelativistic potential is replaced, of course, by the quasipotential.

$$k_0^2 - \vec{k}^2 = m^2 \qquad (1.9)$$

i.e. in the Lobachevsky space.

Such a "geometric" point of view suggests a new approach to the notion of the relativistic coordinate and relativistic configuration space[24-26]. All the considerations , we undertake below in the framework of the quasipotential formalism, generalize in the spirit of Lobachevsky geometry the treatment of the Schroedinger equation (1.3) which leads to the eikonal or Glauber representation for the nonrelativistic scattering amplitude. For the reader's convenience a nonrelativistic case is considered in detail (Section 2). A separate section (Section 4) is devoted to the development of the special kind of "operator Fourier-transform" which is the main tool in deriving the relativistic eikonal representation.

2. Eikonal Representation for the Nonrelativistic Scattering Amplitude

Let us put in eq. (1.6)

$$\begin{aligned} \vec{p} - \vec{q} &= \vec{\Delta} \\ \vec{k} - \vec{q} &= \vec{\lambda}, \end{aligned} \qquad (2.1)$$

$$\psi_{\vec{q}}(\vec{\Delta} + \vec{q}) \equiv \phi_{\vec{q}}(\vec{\Delta}). \qquad (2.2)$$

Taking into account that

$$d\vec{k} = d\vec{\lambda} \tag{2.3}$$

we obtain

$$\phi_{\vec{q}}(\vec{\Delta}) = (2\pi)^3 \delta^{(3)}(\vec{\Delta}) - \frac{1}{2(E_{\vec{\Delta}+\vec{q}} - E_q - i\epsilon)} \frac{1}{(2\pi)^3} \int V(\vec{\Delta}-\vec{\lambda})\phi_{\vec{q}}(\vec{\lambda})\, d\vec{\lambda} . \tag{2.4}$$

Consider in detail the energy denominator (Green function):

$$G = \frac{-1}{2(E_{\vec{\Delta}+\vec{q}} - E_q + i\epsilon)} = \frac{m}{-\vec{\Delta}^2 - 2\vec{\Delta}\vec{q} + i\epsilon} . \tag{2.5}$$

Let the incident plane wave move along the z-axis

$$\vec{q} = (0,0,q) . \tag{2.6}$$

In this case, evidently

$$G = \frac{m}{2q\left(-\Delta_3 - \frac{\vec{\Delta}^2}{2q} + i\epsilon\right)} . \tag{2.7}$$

Now, assuming that at large q the term $\vec{\Delta}^2/2q$ in the denominator of (2.7) can be omitted, we get the equation:

$$\phi_{\vec{q}}(\vec{\Delta}) = (2\pi)^3 \delta^{(3)}(\vec{\Delta}) - \frac{m}{2q(\Delta_3 - i\epsilon)} \cdot \frac{1}{(2\pi)^3} \int V(\vec{\Delta} - \vec{\lambda}) \phi_{\vec{q}}(\vec{\lambda}) \, d\vec{\lambda} . \qquad (2.8)$$

As will be seen below, this is equivalent to neglect of the second order derivatives of the slowly varying part of the wave function in the traditional derivation of the eikonal representation in configuration space (see, for instance, /27/).

The expression obtained for the scattering amplitude is a good approximation in the kinematical region restricted by the condition

$$\frac{\vec{\Delta}^2}{2q} \approx 0. \qquad (2.9)$$

We find this expression in an explicit form. After neglecting the term $\vec{\Delta}^2/2q$, the Green function becomes one- dimensional, because the action of the potential is transmitted through it only along the z -axis. Indeed, performing the Fourier transformation of the quantity

$$G = - \frac{m}{2q(\Delta_3 - i\epsilon)} \qquad (2.10)$$

we get the following expression:

$$-\frac{1}{(2\pi)^3}\int e^{i\tilde{\tilde{\Delta}}\tilde{\tilde{\rho}}+i\Delta_3 z}\frac{m}{2q(\Delta_3-i\epsilon)}\,d\tilde{\tilde{\Delta}}\,d\Delta_3 =$$

$$=\frac{-im}{2q}\,\delta^{(2)}(\tilde{\tilde{\rho}})\,\theta(z). \tag{2.11}$$

A similar transformation of the whole eq. (2.8) gives:

$$\phi_{\vec{q}}(\tilde{\tilde{\rho}},z)=1+\frac{m}{2iq}\int_{-\infty}^{\infty}\theta(z-z')\,V\left(\sqrt{\tilde{\tilde{\rho}}^{2}+z'^{2}}\right)\phi_{\vec{q}}(\tilde{\tilde{\rho}},z')\,dz', \tag{2.12}$$

where

$$V(r)=V\left(\sqrt{\tilde{\tilde{\rho}}^{2}+z^{2}}\right)=\frac{1}{(2\pi)^3}\int e^{i\vec{r}\vec{\Delta}}\;V(\vec{\Delta})\,d\vec{\Delta}, \tag{2.13}$$

$$\phi_{\vec{q}}(\tilde{\tilde{\rho}},z)=\frac{1}{(2\pi)^3}\int e^{i\tilde{\tilde{\rho}}\tilde{\tilde{\Delta}}+iz\Delta_3}\;\phi_{\vec{q}}(\vec{\Delta})\,d\vec{\Delta}. \tag{2.14}$$

The solution of eq. (2.12) is of the common eikonal form

$$\phi_{\vec{q}}(\vec{r})=e^{\frac{m}{2iq}\int_{-\infty}^{\infty}\theta(z-z')\,V\left(\sqrt{\tilde{\tilde{\rho}}^{2}+z'^{2}}\right)dz'}. \tag{2.15}$$

One can evaluate now the scattering amplitude in this approximation. According to the general quantum-mechanical formulae:

$$A(\vec{p},\vec{q}) = -\frac{m}{4\pi}\int e^{-i\vec{p}\vec{r}} V(\vec{r})\psi_{\vec{q}}(\vec{r})d\vec{r} =$$

$$= -\frac{m}{4\pi}\frac{1}{(2\pi)^3}\int V\left[(\vec{p}-\vec{k})^2\right]\psi_{\vec{q}}(\vec{k})d\vec{k}. \tag{2.16}$$

Taking into account eqs. (2.2), (2.13)-(2.15) we have:

$$A(\vec{\Delta}+\vec{q},\vec{q}) = -\frac{m}{4\pi}\frac{1}{(2\pi)^3}\int V(\vec{\Delta}-\vec{\lambda})\,\phi_{\vec{q}}(\vec{\lambda})d\vec{\lambda} =$$

$$= -\frac{m}{4\pi}\int e^{-i\tilde{\Delta}\tilde{\rho}-i\Delta_3 z}\, d\tilde{\rho}\, dz\, V(\sqrt{\tilde{\rho}^2+z^2})\phi_{\vec{q}}(\tilde{\rho},z) = \tag{2.17}$$

$$= -\frac{m}{4\pi}\int e^{-i\tilde{\Delta}\tilde{\rho}}\, d\tilde{\rho}\int e^{-i\Delta_3 z}\, dz\, V(\sqrt{\tilde{\rho}^2+z^2})\, e^{\frac{m}{2iq}\int_{-\infty}^{\infty}\theta(z-z')V(\sqrt{\tilde{\rho}^2+z'^2})dz'}.$$

In the kinematical region restricted by the inequality (2.9), the momentum transfer vector $\vec{\Delta}$ is approximately transverse

$$\Delta_3 \approx \frac{\tilde{\Delta}^2}{2q} \approx 0; \quad \tilde{\Delta} = (\Delta_1, \Delta_2). \tag{2.18}$$

Under the condition (2.18), the z -integration in (2.17) can be carried out and the final expression for the scattering amplitude takes the form:

$$A(\vec{p},\vec{q}) = -q\,i\int_0^\infty \rho\, d\rho\, J_0(\rho\Delta)\left\{ e^{\frac{m}{2iq}\int_{-\infty}^{\infty} V(\sqrt{\rho^2+z^2})\,dz} - 1 \right\}. \tag{2.19}$$

Formula (2.19) is known as the eikonal or Glauber representation for the scattering amplitude. We note that the crucial point in deriving representation (2.19) is the transition to the "one-dimensional" Green function (2.10)-(2.11), since in such a way the equation for the function $\phi_{\vec{q}}(\tilde{\tilde{\rho}},z)$ can be solved exactly. $\phi_{\vec{q}}(\tilde{\tilde{\rho}},z)$ is obviously just the "slowly varying part" of the wave function we are dealing with when deriving the representation (2.19) in the usual way, starting from the Schroedinger differential equation.

3. Quasipotential Equation in the High Energy Approximation

We now consider the relativistic equation (1.1). In complete analogy with (2.1)-(2.2) we put

$$\vec{p}\,(-)\,\vec{q} = \vec{\Delta}$$
$$\vec{k}\,(-)\,\vec{q} = \vec{\lambda}, \tag{3.1}$$

$$\psi_{\vec{q}}(\vec{\Delta}(+)\vec{q}) \equiv \phi_{\vec{q}}(\vec{\Delta}). \qquad (3.2)$$

Since[x]

$$\sqrt{1+(\vec{p}(-)\vec{k})^2} = E_p E_k - \vec{p}\vec{k} \qquad (3.3)$$

is a relativistic invariant, then

$$(\vec{p}(-)\vec{k})^2 = (\vec{\Delta}(-)\vec{\lambda})^2. \qquad (3.4)$$

Making use of (3.4) and the relation

$$d\Omega_{\vec{k}} = \frac{d\vec{k}}{\sqrt{1+\vec{k}^2}} = d\Omega_{\vec{\lambda}} = \frac{d\vec{\lambda}}{\sqrt{1+\vec{\lambda}^2}} \qquad (3.5)$$

we get the following equation for $\phi_{\vec{q}}(\vec{\Delta})$

$$\phi_{\vec{q}}(\vec{\Delta}) = (2\pi)^3 \delta^{(3)}(\vec{\Delta}) -$$

$$- \frac{1}{2(E_{\vec{\Delta}(+)\vec{q}} - E_q - i\epsilon)} \cdot \frac{1}{(2\pi)^3} \int V[(\vec{\Delta}(-)\vec{\lambda})^2]\phi_{\vec{q}}(\vec{\lambda})\, d\Omega_{\vec{\lambda}}\,. \qquad (3.6)$$

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[x] In what follows we use the system of units where $\hbar=c=m=1$.

Taking into account (3.3), the Green function in eq. (3.6) can be rewritten in the following manner

$$G = \frac{1}{2(E_{\vec{q}} - E_{\vec{\Delta}(+)\vec{q}} + i\epsilon)} = \frac{1}{2(E_{\vec{q}} - E_{\vec{\Delta}} E_{\vec{q}} - \vec{\Delta}\vec{q} + i\epsilon)} . \tag{3.7}$$

In the high energy limit $E_q \gg 1$ we have approximately:

$$E_q \approx q + \frac{1}{2q} ; \qquad q = |\vec{q}| . \tag{3.8}$$

Assuming, as before, the vector $\vec{q}$ to be along the z -axis (relation (2.6)) and using (3.8) one obtains:

$$G \approx \frac{1}{2q\left(1 - \Delta_0 - \Delta_3 - \frac{\Delta_0 - 1}{2q^2} + i\epsilon\right)} ; \; \Delta_0 \equiv E_{\vec{\Delta}} \tag{3.9}$$

instead of (3.7). Further, we assume that at high energies the term $\frac{\Delta_0 - 1}{2q^2}$ in the denominator of (3.9) can be neglected (cf. Section 2). Finally the Green function takes the form

$$G \approx \frac{1}{2q(1 - \Delta_0 - \Delta_3 + i\epsilon)} . \tag{3.10}$$

Because of this, we deal now with the following equation for

$\phi_{\vec{q}}(\vec{\Delta})$

$$\phi_{\vec{q}}(\vec{\Delta}) = (2\pi)^3 \delta^{(3)}(\vec{\Delta}) + \tag{3.11}$$

$$+ \frac{1}{2q(1-\Delta_0-\Delta_3+i\epsilon)} \frac{1}{(2\pi)^3} \int V[(\vec{\Delta}(-)\vec{\lambda})^2]\phi_{\vec{q}}(\vec{\lambda})\, d\Omega_{\vec{\lambda}}.$$

This equation is an analogue of the nonrelativistic equation (2.8). We shall see that it can be solved explicitly by applying to it a special kind of Fourier transform.

The development of the corresponding formalism is the subject of the following section.

When one deals with the approximate expression (3.10) for the Green function, a question arises about the conditions under which our neglect of the $\frac{\Delta_0 - 1}{2q^2}$ - term is valid. In order to answer this equation, we express our earlier variables in terms of Mandelstam invariant variables s and t. Obviously

$$s = 4(\vec{q}^2 + m^2) \approx 4\vec{q}^2$$

$$t = (E_p - E_q)^2 - (\vec{p} - \vec{q})^2 = \tag{3.12}$$

$$= 2(1 - E_p E_q + \vec{p}\vec{q}) = 2(1-\Delta_0).$$

Therefore

$$\frac{\Delta_0 - 1}{2\vec{q}^{\,2}} = \left| \frac{t}{s} \right| \qquad (3.13)$$

and consequently the neglect of the terms (3.13) means that the obtained expression for the scattering amplitude can be valid in the region where the kinematic invariants s and t are restricted by the condition

$$\left| \frac{t}{s} \right| \ll 1, \qquad \text{(cf. (2.9))}. \qquad (3.14)$$

4. Lorentz Group, Horospherical Coordinates and Operator Type Fourier Transform

Consider the two-dimensional spinor representation of the Lorentz group, realized by the complex 2×2 unimodular matrices

$$a = \begin{pmatrix} \alpha, & \beta \\ \gamma, & \delta \end{pmatrix}, \quad \det a = \alpha\delta - \gamma\beta = 1. \qquad (4.1)$$

By means of the Pauli matrices σ_i ($i = 1,2,3$), the unit matrix $\sigma_0 = \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$ and the 4-vector Δ_μ, one can construct the spin-tensor

$$\hat{\Delta} = \Delta_\mu \sigma_\mu = \begin{pmatrix} \Delta_0 + \Delta_3 , & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 , & \Delta_0 - \Delta_3 \end{pmatrix}$$

which transforms under the Lorentz rotations $\Delta'_\mu = L_\mu^{\nu} \Delta_\nu$ in the following way

$$\hat{\Delta}' = a \hat{\Delta} a^+ = \Delta'_\mu \sigma_\mu . \tag{4.2}$$

We choose now in the group (4.1) a three-parameter subgroup of the triangle matrices of the type

$$K = \begin{pmatrix} e^{a/2} , & 0 \\ e^{a/2} \gamma , & e^{-a/2} \end{pmatrix}, \quad \begin{matrix} a \text{ is real} \\ \\ \gamma = \gamma_1 + i\gamma_2 . \end{matrix} \tag{4.3}$$

If we assume $\Delta^2 = 1$ and put $\Delta_\mu = (1, \vec{0})$ into the eq. (4.2), we obtain a relation which maps the space of group parameters (4.3) onto the upper sheet of the hyperboloid

$$\Delta_0^2 - \vec{\Delta}^2 = 1$$

$$\begin{pmatrix} \Delta_0 + \Delta_3 , & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 , & \Delta_0 - \Delta_3 \end{pmatrix} = K K^+ = \begin{pmatrix} e^a , & e^a(\gamma_1 - i\gamma_2) \\ e^a(\gamma_1 + i\gamma_2) , & e^{-a} + e^a \tilde{\tilde{\gamma}}^2 \end{pmatrix} \qquad (4.4)$$

$\tilde{\tilde{\gamma}}$ denotes the two-vector (γ_1, γ_2). Equating the corresponding elements of the matrices in eq. (4.4) we get:

$$\Delta_0 + \Delta_3 = e^a$$

$$\Delta_0 - \Delta_3 = e^{-a} + e^a \, \tilde{\tilde{\gamma}}^2 \qquad (4.5)$$

$$\tilde{\tilde{\Delta}} = (\Delta_1, \Delta_2) = e^a \tilde{\tilde{\gamma}} .$$

The relations (4.5) determine the horospherical[x/] coordinate system on the surface $\Delta^2 = 1$, $\Delta_0 > 0$ /28/.

[x/] By definition, the horosphere in the Lobachevsky space, realized on the upper sheet of the hyperboloid $\Delta^2 = 1$, is the two-dimensional surface determined by the following equation

$\Delta\xi = \text{const}, \; \xi^2 = \xi_0^2 - \vec{\xi}^2 = 0$.

in particular, the equation

$$a = 0 \qquad (4.6)$$

gives the horosphere $\Delta_0 + \Delta_3 = 1$.

The remarkable property of the horosphere is the Euclidean character of its intrinsic geometry.

Since the matrices (4.3) form a group, then in the space of parameters $(a, \tilde{\tilde{\gamma}})$ and, according to (4.5), on the upper sheet of the hyperboloid $\Delta^2 = 1$ some group operation is also induced.

When applied to the four-vectors of the type $\Delta_\mu = (\sqrt{1+\vec{\Delta}^2}, \vec{\Delta})$ we shall denote this operation by the symbol $\oplus$ and write

$$\vec{\Delta}' = \vec{\Delta} \oplus \vec{q} \tag{4.7}$$

(4.7) is equivalent to the relation

$$\begin{aligned} a' &= a + c \\ \tilde{\tilde{\gamma}}' &= e^{-c} \tilde{\tilde{\gamma}} + \tilde{\tilde{\mu}}, \end{aligned} \tag{4.8}$$

where $(a', \tilde{\tilde{\gamma}}')$, $(a, \tilde{\tilde{\gamma}})$ and $(c, \tilde{\tilde{\mu}})$ are horospherical coordinates of the four-vectors Δ'_μ, Δ_μ and q_μ respectively. The validity of all the group properties for the $\oplus$-operation can be verified by straightforward calculations[x/] using eq. (4.8). The inverse transformation is defined as

$$\vec{\Delta}' = \vec{\Delta} \oplus \vec{q}^{\,-1}, \tag{4.9}$$

[x/] We emphasize that the shift transformations (3.1) with which we dealt earlier do not form a group.

where

$$\vec{q}^{\,-1} \equiv (-c\,,\, -e^{c}\,\tilde{\tilde{\mu}}\,). \tag{4.10}$$

In what follows, we shall call the set of $\oplus$ -operations the group of horospherical shifts and denote it by $T(3)$.

It is easy to see from (4.5) and (4.8) that in the nonrelativistic limit

$$a\,,\ |\gamma| \ll 1$$

the horospherical coordinates transform into Cartesian coordinates of the three-dimensional Euclidean momentum space and the group $T(3)$ into the Abelian group of translations of this space.

The volume element of the hyperboloid $d\Omega_{\vec{\Delta}} = \dfrac{d\vec{\Delta}}{\sqrt{1+\vec{\Delta}^{2}}}$ in the horospherical coordinates (4.5) reads

$$d\,\Omega_{\vec{\Delta}} = e^{2a}\, da\, d\tilde{\tilde{\gamma}}\,. \tag{4.11}$$

Since $T(3)$ is a subgroup of the Lorentz group, then evidently

$$d\,\Omega_{\vec{\Delta}\oplus\vec{q}} = d\,\Omega_{\vec{\Delta}}\,, \tag{4.12}$$

(cf. (3.5)). The property (4.12) allows one to interpret (4.11) as the right invariant volume element on the group $T(3)$ itself.

By means of (4.5), (4.8)-(4.10) it is easy also to show that

$$\Delta_0 \lambda_0 - \vec{\Delta}\vec{\lambda} = \sqrt{1+(\vec{\Delta} \oplus \vec{\lambda}^{-1})^2} . \qquad (4.13)$$

Comparing (4.13) with (3.3), we are led to the important relation:

$$(\vec{\Delta}(-)\vec{\lambda})^2 = (\vec{\Delta} \oplus \vec{\lambda}^{-1})^2 . \qquad (4.14)$$

Now, taking into account (4.14), we can understand the integral term in the equation (3.11) as a convolution on the group $T(3)$

$$\frac{1}{(2\pi)^3} \int V[(\vec{\Delta} \oplus \vec{\lambda}^{-1})^2]\, \phi_{\vec{q}}(\vec{\lambda})\, d\Omega_{\vec{\lambda}} \equiv V(\vec{\Delta}^2) * \phi_{\vec{q}}(\vec{\Delta}^2). \qquad (4.15)$$

Finally, after transforming to the horospherical coordinates (4.5), equation (3.11) takes the form:

$$\phi_{\vec{q}}(a, \tilde{\tilde{\gamma}}) = \delta(a)\, \delta^{(2)}(\tilde{\tilde{\gamma}}) - \frac{1}{2q(e^a - 1 - i\epsilon)} V(a, \tilde{\tilde{\gamma}}) * \phi_{\vec{q}}(a, \tilde{\tilde{\gamma}}). \qquad (4.16)$$

An important property of (4.16) is the one-dimensional nature of the Green function

$$G = -\frac{1}{2q(e^a - 1 - i\epsilon)} \qquad (4.17)$$

(cf. (2.10)).

The presence of the convolution operation in (4.16) suggests that the Fourier transform defined on the group T(3) should be applied to this equation. Let us explain the meaning of this notion.

A usual three-dimansional Fourier transform is an expansion in terms of plane waves, which are one-dimensional unitary representations of the Abelian group of translations of the Euclidean 3-space, or in other words, the unitary solutions of the functional equation

$$U(\vec{\Delta}_1 + \vec{\Delta}_2) = U(\vec{\Delta}_1)U(\vec{\Delta}_2).$$

Since the group T(3) is non-Abelian, it has no non-trivial one-dimensional representations, i.e. the functional equation

$$U(\vec{\Delta}_1 \oplus \vec{\Delta}_2) = U(\vec{\Delta}_1)\,U(\vec{\Delta}_2) \qquad (4.18)$$

permits only operator solutions. The matrix elements of these operators form the basis in terms of which the functions on the group T(3) can be expanded. The corresponding expansions then play the role of the Fourier transform on the given group. The Fourier transforms obtained are the operators acting in the same space where the operators U from (4.18) act.

Let us assume now that we have at our disposal some complete and orthogonal set of "state vectors" $|\tilde{\tilde{\rho}}\rangle$:

$$\int d\tilde{\tilde{\rho}}\; |\tilde{\tilde{\rho}}\rangle\langle\tilde{\tilde{\rho}}| = \hat{I}$$
$$\langle\tilde{\tilde{\rho}}|\tilde{\tilde{\rho}}'\rangle = \delta^{(2)}(\tilde{\tilde{\rho}} - \tilde{\tilde{\rho}}'); \quad \tilde{\tilde{\rho}} = (\rho_1, \rho_2). \qquad (4.19)$$

Define in the space of these vectors an operator $\hat{U}_z(\vec{\Delta}) = U_z(a, \tilde{\tilde{\gamma}})$ putting

$$<\tilde{\tilde{\rho}}_1 \mid \hat{U}_z(a, \tilde{\tilde{\gamma}}) \mid \tilde{\tilde{\rho}}_2 >= e^{iaz + i\tilde{\tilde{\gamma}}\tilde{\tilde{\rho}}_2 - a} \delta^{(2)}(\tilde{\tilde{\rho}}_1 - e^{-a}\tilde{\tilde{\rho}}_2). \qquad (4.20)$$

With the help of (4.20) it is easy to obtain

$$\hat{U}_z(a_1+a_2, e^{-a_2}\tilde{\tilde{\gamma}}_1 + \tilde{\tilde{\gamma}}_2) = \hat{U}_z(a_1, \tilde{\tilde{\gamma}}_1)\hat{U}_z(a_2, \tilde{\tilde{\gamma}}_2) \qquad (4.21)$$

$$\hat{U}^+_z(a, \gamma) = \hat{U}_z(-a, -e^a\tilde{\tilde{\gamma}}) = \hat{U}^{-1}_z(a, \tilde{\tilde{\gamma}}). \qquad (4.22)$$

Thus the operators $\hat{U}_z(a, \tilde{\tilde{\gamma}})$ realize the unitary representation of the group T(3). Further, let $f(\vec{\Delta}) = f(a, \tilde{\tilde{\gamma}})$ be some function on the group T(3). According to the above considerations its Fourier transform on this group is determined by the following expression

$$\frac{1}{(2\pi)^3} \int f(\vec{\Delta})\, d\Omega_{\vec{\Delta}} <\tilde{\tilde{\rho}}_1 \mid \hat{U}_z(\vec{\Delta}) \mid \tilde{\tilde{\rho}}_2 > \equiv <\tilde{\tilde{\rho}}_1 \mid \hat{f}(z) \mid \tilde{\tilde{\rho}}_2 >. \qquad (4.23)$$

It turns out that there exists also the inversion formula making it possible to calculate $f(\vec{\Delta})$ from the known matrix elements $<\tilde{\tilde{\rho}}\;|\hat{f}(z)|\tilde{\tilde{\rho}}\;>$

$$f(\vec{\Delta}) = \int dz\, d\tilde{\tilde{\rho}}_1\, d\tilde{\tilde{\rho}}_2 < \tilde{\tilde{\rho}}_1 \,|\, \hat{f}(z)\hat{U}_z^+(\vec{\Delta})\,|\tilde{\tilde{\rho}}_2 > . \qquad (4.24)$$

As is clear from (4.24), it is necessary in fact to know only the integral of $<\tilde{\tilde{\rho}}_1 \;|\hat{f}(z)|\tilde{\tilde{\rho}}_2 >$ over the parameter $\tilde{\tilde{\rho}}_1$ in order to find $f(\tilde{\tilde{\Delta}})$

$$\int d\tilde{\tilde{\rho}}_1 < \tilde{\tilde{\rho}}_1 \;|\hat{f}(z)|\tilde{\tilde{\rho}}_2 > \equiv f(z, \tilde{\tilde{\rho}}_2) \qquad (4.25)$$

and there is no need to know the matrix element $<\tilde{\tilde{\rho}}_1|\hat{f}(z)|\tilde{\tilde{\rho}}_2>$ itself. This point reflects the circumstance that the formula (4.25) also can be inverted, i.e. it is possible to reproduce the whole matrix $<\tilde{\tilde{\rho}}_1\;|\hat{f}(z)|\tilde{\tilde{\rho}}_2>$:

$$<\tilde{\tilde{\rho}}_1|\hat{f}(z)|\tilde{\tilde{\rho}}_2 > = \frac{1}{2\pi}\int e^{ia(z-z')}\, dz' da\, \delta^{(2)}(\tilde{\tilde{\rho}}_1 - e^{-a}\tilde{\tilde{\rho}}_2) f(z', \tilde{\tilde{\rho}}_2). \qquad (4.26)$$

Using (4.18), it is easy to obtain the "convolution theorem" for the transformation (4.23)-(4.24).

Namely, if

$$\hat{f}_1(z) = \frac{1}{(2\pi)^3} \int f_1(\vec{\Delta})\hat{U}_z(\vec{\Delta})\, d\Omega_{\vec{\Delta}}$$

and

$$\hat{f}_2(z) = \frac{1}{(2\pi)^3} \int f_2(\vec{\Delta})\hat{U}_z(\vec{\Delta})\, d\Omega_{\vec{\Delta}}$$

then

$$\hat{f}_1(z)\hat{f}_2(z) = \frac{1}{(2\pi)^3} \int d\Omega_{\vec{\Delta}} \{ f_1(\vec{\Delta}) * f_2(\vec{\Delta}) \} \hat{U}_z(\vec{\Delta}), \tag{4.27}$$

where the "convolution" of the functions f_1 and f_2 is defined by the integral

$$f_1(\vec{\Delta}) * f_2(\vec{\Delta}) = \frac{1}{(2\pi)^3} \int f_1(\vec{\Delta} \oplus \vec{\lambda}^{-1}) f_2(\vec{\lambda})\, d\Omega_{\vec{\lambda}} \quad . \tag{4.28}$$

In the non-relativistic limit, as is easily seen, all the relations connected with the Fourier expansion on the group T(3) transform into corresponding formulae of the usual Fourier analysis.

5. High Energy Representation for the Quasipotential Scattering Amplitude

Now we apply the Fourier transform on the group T(3) to the equation (4.16).

We begin putting by definition (cf. (4.23)).

$$<\tilde{\tilde{\rho}}_1|\hat{\phi}_{\vec{q}}(z)|\tilde{\tilde{\rho}}_2> = \frac{1}{(2\pi)^3}\int \phi_{\vec{q}}(a,\tilde{\tilde{\gamma}})<\tilde{\tilde{\rho}}_1|\hat{U}_z(a,\tilde{\tilde{\gamma}})|\tilde{\tilde{\rho}}_2> e^{2a}\, da\, d\tilde{\tilde{\gamma}} \tag{5.1}$$

$$<\tilde{\tilde{\rho}}_1|\hat{V}_q(z)|\tilde{\tilde{\rho}}_2> = \frac{1}{(2\pi)^3}\int V_q(a,\tilde{\tilde{\gamma}})<\tilde{\tilde{\rho}}_1|\hat{U}_z(a,\tilde{\tilde{\gamma}})|\tilde{\tilde{\rho}}_2> e^{2a}\, da\, d\tilde{\tilde{\gamma}}. \tag{5.2}$$

Using the formulae (4.20), (4.24) and (4.27), after some simple calculations we get from (4.16)

$$<\tilde{\tilde{\rho}}_1|\hat{\phi}_{\vec{q}}(z)|\tilde{\tilde{\rho}}_2> = <\tilde{\tilde{\rho}}_1|\tilde{\tilde{\rho}}_2> +$$

$$+\frac{1}{2\pi i}\int_{-\infty}^{\infty} da \frac{e^{ia(z-z')}\, dz'}{e^{a}-1-i\epsilon}\, \delta(\tilde{\tilde{\rho}}_1 - e^{-a}\tilde{\tilde{\rho}}_2)\frac{1}{2qi}\int d\tilde{\tilde{\rho}}\, <\tilde{\tilde{\rho}}|\hat{V}_q(z')\hat{\phi}_{\vec{q}}(z')|\tilde{\tilde{\rho}}_2>. \tag{5.3}$$

By means of the relations (4.25)-(4.26) it is easy to show that equation (5.3) can be written in the form

$$<\tilde{\tilde{\rho}}_1|\hat{\phi}_{\vec{q}}(z)|\tilde{\tilde{\rho}}_2> = <\tilde{\tilde{\rho}}_1|\tilde{\tilde{\rho}}_2> +$$

$$+\frac{1}{2qi}\int_{-\infty}^{\infty} dz'\,\hat{\theta}(z-z')<\tilde{\tilde{\rho}}_1|\hat{V}_q(z')\hat{\phi}_{\vec{q}}(z')|\tilde{\tilde{\rho}}_2>, \tag{5.4}$$

where

$$\hat{\theta}(z-z') = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{ia(z-z')}}{e^{a}-1-i\epsilon} da \tag{5.5}$$

is an analogue of the step function in the finite difference calculus, sutisfying a non-homogeneous finite-difference equation of the first order

$$\Delta_z \hat{\theta}(z-z') = \delta(z-z')$$

$$\Delta_z = \frac{e^{-i\frac{d}{dz}}-1}{-i}. \tag{5.6}$$

Taking into account (5.6) we have from (5.4)

$$<\tilde{\rho}_1|\Delta_z \hat{\phi}_{\vec{q}}(z)|\tilde{\rho}_2> = \frac{1}{2qi} <\tilde{\rho}_1|\hat{V}_q(z)\hat{\phi}_{\vec{q}}(z)|\tilde{\rho}_2>$$

or

$$\Delta_z \hat{\phi}_{\vec{q}}(z) = \frac{1}{2qi} \hat{V}_q(z)\hat{\phi}_{\vec{q}}(z). \tag{5.7}$$

The boundary condition for the operator $\hat{\phi}_{\vec{q}}(z)$ follows from (5.4):

$$\hat{\phi}_{\vec{q}}(z)|_{z\to-\infty} = 1. \tag{5.8}$$

It is clear that quantity $\hat{\phi}_{\vec{q}}(z)$ is a relativistic analogue of the "slowly varying part" of the wave function (cf. Sect. 2). The formal solution of the equation (5.4) and (5.7) is an "ordered" exponential

$$\hat{\phi}_{\vec{q}}(z) = \sum_{n=0}^{\infty} \frac{1}{(2qi)^n} \int \hat{\theta}(z-z_1)\hat{\theta}(z_1-z_2)\ldots\hat{\theta}(z_{n-1}-z_n)\times$$

$$\times \hat{V}_q(z_1)\ldots\hat{V}_q(z_n)dz_1\ \ldots\ dz_n \equiv \tag{5.9}$$

$$= P_z \exp\left[\frac{1}{2qi}\int_{-\infty}^{\infty}\hat{\theta}(z-z')\hat{V}_q(z')dz'\right].$$

For simple quasipotentials the series (5.9) can be calculated explicitly.

Now we proceed to consider the quasipotential scattering amplitide $A(\vec{p},\vec{q})$. According to /24/

$$A(\vec{p},\vec{q}) = -\frac{1}{4\pi}\frac{1}{(2\pi)^3}\int V[(\vec{p}(-)\vec{k})^2]\ \psi_{\vec{q}}(\vec{k})\, d\,\Omega_{\vec{k}}\,.$$

By means of (3.2), (3.4), (4.14) and (4.15) we find from this expression that

$$A(\vec{p},\vec{q}) = A(\vec{\Delta},\vec{q}) = A(\vec{\Delta}^2; q) =$$

$$= -\frac{1}{4\pi} V_q(\vec{\Delta}^2) * \phi_{\vec{q}}(\vec{\Delta}). \tag{5.10}$$

Further, due to the "convolution theorem" and formula (4.24), we conclude that

$$A(\vec{\Delta}^2, \vec{q}\ ) = -\frac{1}{4\pi}\int dz\, d\tilde{\tilde{\rho}}_1\, d\tilde{\tilde{\rho}}_2 \langle \tilde{\tilde{\rho}}_1 \mid \hat{V}_q(z)\phi_{\vec{q}}(z)U_z^+(\vec{\Delta}\ )\mid \tilde{\tilde{\rho}}_2 \rangle. \qquad (5.11)$$

This expression allows some simplifications. Namely, considering (4.20), let us rewrite (5.11) in the form:

$$A(\vec{\Delta}^2, q) = -\frac{1}{4\pi}\, e^{-a}\int d\tilde{\tilde{\rho}}\, e^{-i\tilde{\tilde{\gamma}}\tilde{\tilde{\rho}}}\int dz\, d\tilde{\tilde{\rho}}_1\, e^{-iaz}\langle \tilde{\tilde{\rho}}_1 \mid \hat{V}_q(z)\,\hat{\phi}_{\vec{q}}(z)\mid \tilde{\tilde{\rho}}\rangle. \qquad (5.11')$$

Now, let us consider the condition (3.14). Since (see (2.6), (3.12) and (4.6))

$$e^a = (p_0 + p_3)(q_0 - q_3) \approx 1 + \frac{1}{s} + \frac{t}{s}$$

then at $s \gg 1$ and $\left|\frac{t}{s}\right| \ll 1$, we have

$$\begin{aligned} &e^a \approx 1, && a \approx 0 \\ &\tilde{\tilde{\gamma}} \approx \tilde{\tilde{\Delta}}, && |t| \approx \tilde{\tilde{\gamma}}^2. \end{aligned} \qquad (5.12)$$

In other words, in this approximation the momentum transfer vector $\vec{\Delta}$ belongs to the horosphere (4.6). This fact can be thought of as a particular "transversality condition" in the relativistic case.

After inserting (5.12) into (5.11') we obtain:

$$A(s,t) = -\frac{1}{4\pi} \int d\tilde{\tilde{\rho}}\, e^{-i\tilde{\tilde{\Delta}}\tilde{\tilde{\rho}}} \int d\tilde{\tilde{\rho}}_1\, dz \, \langle \tilde{\tilde{\rho}}_1 | \hat{V}_q(z)\, \hat{\phi}_{\vec{q}}(z) | \tilde{\tilde{\rho}} \rangle =$$

(5.13)

$$= -\frac{1}{4\pi} 2\pi \int_0^\infty \rho\, d\rho\, J_0(\sqrt{-t}\,\rho) \int dz\, d\tilde{\tilde{\rho}}_1 \langle \tilde{\tilde{\rho}}_1 | \hat{V}_q(z)\, \hat{\phi}_{\vec{q}}(z) | \tilde{\tilde{\rho}} \rangle .$$

(In the last equation the azimuthal symmetry has been taken into account). It is possible to carry out the z -integration in eq. (5.13) explicitly. Namely, on the basis of (5.7) and (5.9) we have:

$$\int_{-\infty}^{\infty} dz \langle \tilde{\tilde{\rho}}_1 | \hat{V}_q(z)\, \hat{\phi}_{\vec{q}}(z) | \tilde{\tilde{\rho}} \rangle =$$

(5.14)

$$= 2qi \int_{-\infty}^{\infty} dz \langle \tilde{\tilde{\rho}}_1 | \Delta_z P_z \exp\left[\frac{1}{2qi} \int_{-\infty}^{\infty} \hat{\theta}(z-z')\hat{V}_q(z')dz'\right] | \tilde{\tilde{\rho}} \rangle \equiv I.$$

Now, due to (5.6), we can easily obtain that

$$I = 2qi\,\{\langle \tilde{\tilde{\rho}}_1 | P_z \exp\left[\frac{1}{2qi} \int_{-\infty}^{\infty} \hat{V}_q(z)dz\right] | \rho \rangle - \delta^{(2)}(\tilde{\tilde{\rho}}_1 - \tilde{\tilde{\rho}})\}, \qquad (5.15)$$

where, by definition,

$$P_z \exp\left[\frac{1}{2qi}\int_{-\infty}^{\infty}\hat{V}_q(z)\,dz\right] = 1 +$$

$$+\frac{1}{2qi}\int_{-\infty}^{\infty}\hat{V}_q(z_1)\,dz_1 + \sum_{n=2}^{\infty}\int\hat{\theta}(z_1-z_2)\ldots\hat{\theta}(z_{n-1}-z_n)\,\hat{V}_q(z_1)\ldots\hat{V}_q(z_n)\cdot dz_1\ldots dz_n\,. \tag{5.16}$$

After inserting (5.15) into (5.13) we obtain the sought - for high energy representation for the quasipotential scattering amplitude:

$$A(s,t) =$$

$$= -qi\int_0^{\infty}\rho\,d\rho\,J_0(\sqrt{-t}\,\rho)\{\int d\tilde{\rho}_1 < \tilde{\rho}_1 \,|\, P_z \exp\left[\frac{1}{2qi}\int_{-\infty}^{\infty}V_q(z)\,dz\right]|\,\tilde{\rho} > -1\}. \tag{5.17}$$

Obviously this formula can be considered as the direct generalization of the eikonal representation (2.19) to the relativistic case.

6. Concrete Examples

Now consider the case when the matrix $<\tilde{\rho}_1|\,\hat{V}_q(z)\,|\tilde{\rho}_2>$ is diagonal

$$<\tilde{\tilde{\rho}}_1 | \hat{V}_q(z) | \tilde{\tilde{\rho}}_2> = \delta^{(2)}(\tilde{\tilde{\rho}}_1 - \tilde{\tilde{\rho}}_2) V_q(z, \tilde{\tilde{\rho}}_1). \qquad (6.1)$$

It tirns out that under the condition (6.1) the following relation holds

$$<\tilde{\tilde{\rho}}_1 | P_z \left[\frac{1}{2qi} \int_{-\infty}^{\infty} \hat{V}_q(z) dz \right] | \tilde{\tilde{\rho}} > =$$

$$= \delta^{(2)}(\tilde{\tilde{\rho}}_1 - \tilde{\tilde{\rho}}) \exp\left\{ i \int_{-\infty}^{\infty} \ln\left(1 - \frac{\hat{V}_q(z, \tilde{\tilde{\rho}})}{2q}\right) dz \right\}. \qquad (6.2)$$

Equality (6.2) can be proved starting from the expansion of the P_z -exponential in powers of the potential by means of successive applications of the identity

$$\hat{\theta}(z-z')\hat{\theta}(z-z'') = \hat{\theta}(z-z')\hat{\theta}(z'-z'') +$$

$$+ \hat{\theta}(z-z'')\hat{\theta}(z''-z') + \frac{1}{i}\hat{\theta}(z-z')\delta(z'-z''). \qquad (6.3)$$

Another method, connected with the use of (5.7), is also possible. Due to the diagonality of $\hat{V}_q(z)$ the operator $\hat{\phi}$ is evidently also diagonal

$$<\tilde{\tilde{\rho}}_1 \,|\hat{\phi}_{\vec{q}}(z)|\,\tilde{\tilde{\rho}}_2> = \delta^{(2)}(\tilde{\tilde{\rho}}_1 - \tilde{\tilde{\rho}}_2)\,\phi_{\vec{q}}(z, \tilde{\tilde{\rho}}_1) \tag{6.4}$$

and

$$\Delta_z \phi_{\vec{q}}(z, \tilde{\tilde{\rho}}) = \frac{1}{2qi} V_q(z, \tilde{\tilde{\rho}})\,\phi_{\vec{q}}(z, \tilde{\tilde{\rho}}). \tag{6.5}$$

The solution of the " c -number" equation (6.5) is given by the expression

$$\phi_{\vec{q}}(z, \tilde{\tilde{\rho}}) = \exp\{ i \int_{-\infty}^{\infty} \hat{\theta}(z-z')\, \ell n \left(1 - \frac{V_q(z', \tilde{\tilde{\rho}})}{2q}\right) dz' \} \tag{6.6}$$

as is easy to verify with the help of (5.6). Now, taking into consideration the fact that the left-hand side of (6.2) can be represented in the form

$$<\tilde{\tilde{\rho}}_1 \,|\hat{\phi}_{\vec{q}}(\infty)|\,\tilde{\tilde{\rho}}_2> = \delta^{(2)}(\tilde{\tilde{\rho}}_1 - \tilde{\tilde{\rho}}_2)\,\phi_{\vec{q}}(\infty, \tilde{\tilde{\rho}}_2) \tag{6.7}$$

and that $\hat{\theta}(\infty)=1$, we get the formula (6.2).

Representation (5.17) in the case of quasipotentials (6.1) is of the following form

$$A(s,t) = -q\, i \int_0^\infty \rho\, d\rho\, J_0(\sqrt{-t}\,\rho)\left\{ e^{\, i \int_{-\infty}^{\infty} \ln\left(1 - \frac{V_q(z,\tilde{\tilde{\rho}})}{2q}\right) dz} - 1 \right\}. \qquad (6.8)$$

In the approximation $\frac{V(z,\tilde{\tilde{\rho}})}{2q} \ll 1$ this expression coincides with the non-relativistic formula (2.19).

Now we choose $V_q(z, \tilde{\tilde{\rho}})$ in the form of a "potential well":

$$V_q(z, \tilde{\tilde{\rho}}) = -V_0\, \theta\left(r_0 - \sqrt{z^2 + \tilde{\tilde{\rho}}^{\,2}}\right), \qquad (6.9)$$

where r_0 is the radius of the well and V_0 is its "depth"x/. Simple calculations led to:

$$A(s,t) = -q\, i \int_0^{r_0} \rho\, d\rho\, J_0(\rho\sqrt{-t}\,)\left\{ e^{2i\omega\sqrt{r_0^2 - \rho^2}} - 1 \right\}, \qquad (6.10)$$

where

$$\omega = \ln\left(1 + \frac{V_0}{2q}\right). \qquad (6.11)$$

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x/ The quantity V_0, in general, can be complex and energy-dependent.

Formula (6.10) differs from the corresponding non-relativistic expression (see, for instance,[29]) only by the type of dependence on the parameter $V_0 / 2q$.

We note in conclusion that the technique of decomposition in terms of representations of $\Gamma(3)$ group which is developed in this paper can be applied, for instance, for the study of asymptotic behaviour of perturbation theory graphs.

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Представление для релятивистской амплитуды рассеяния при высоких энергиях

Получено представление для релятивистской амплитуды рассеяния при высоких энергиях в рамках квазипотенциального подхода. Ключевым моментом во всем рассмотрении является фурье-анализ на трехпараметрической неабелевой группе трансляций, вложенной в качестве подгруппы в группу Лоренца. Полученное представление обобщает эйкональное приближение к квантовой механике.

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Representation for the Relativistic Scattering Amplitide at High Energies

In the framework of the quasipotential approach high energy representation for the relativistic scattering amplitude is obtained. A basic point is an application of the Fourier analysis in terms of the non-abelian group of translations embedded as a subgroup in the Lorentz group. The representation obtained corresponds to the eikonal one in quantum mechanics.

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