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ON THE BEHAVIOUR OF THE ELASTIC SCATTERING AMPLITUDE AT MOMENTUM TRANSFERS DECREASING AS $(\ln s)^{-2}$

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О поведении амплитуды упругого рассеяния при значениях передачи импульса, убывающих как $(\ln s)^{-2}$

В работе показано, что при $t \approx \frac{\text{const}}{\ln^2 s}$ амплитуда имеет такое же поведение, что и ее поведение при $t = 0$.

Препринт Объединенного института ядерных исследований.
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Nguyen van Hieu E2-5337

On the Behaviour of the Elastic Scattering Amplitude at Momentum Transfers Decreasing as $(\ln s)^{-2}$

We prove that in any interval

$$-4k^2 + t_1(s) \leq t \leq - t_1(s),$$

where

$$t_1(s) = \frac{\text{const}}{\ln^2 s},$$

there must exist at least one value t_0 such that the magnitude of the elastic scattering amplitude at this momentum transfer has the same behaviour as its behaviour at $t = 0$: $\left|\frac{F(s, t_0)}{F(s, 0)}\right| \geq \text{const}, \; s \to \infty$.

This conclusion is true also for the imaginary part of the amplitude. If the real part of the amplitude at $t = 0$ is bounded below by some power s^{-n}, then for it we have the same result.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1970

In a recent paper [1] Kinoshita showed that at momentum transfers decreasing as

$$- t(s) = |t(s)| \lesssim \frac{\text{const}}{\ln^2 s} \qquad (1)$$

the real part of the elastic scattering amplitude has the same behaviout as that at $t = 0$

$$\left| \frac{\text{Re}\, F(s, t(s))}{\text{Re}\, F(s, 0)} \right| \gtrsim \text{const} . \qquad (2)$$

In proving this statement Kinoshita used the following representation of Ref $F(s,t)$:

$$\text{Re}\, F(s,t) = D^{+}(s,t) - D^{-}(s,t) ,$$

the functions $D^{\pm}(s,t)$ being non-negative at

$$D^{\pm}(s,0) \geq 0 .$$

Further he proved that for values t decreasing as in Eq. (1) the functions $D^{\pm}(s,t(s))$ have the same behaviours as that at $t=0$:

$$\left|\frac{D^{\pm}(s,t(s))}{D^{\pm}(s,0)}\right| \geq \epsilon^{\pm} .$$

Since $D^{+}(s,0)$ and $D^{-}(s,0)$ are different, then due to the last inequality we can believe that for rather small s-independent constants in Eq. (1) the function $D^{-}(s,t(s))$ cannot cancel $D^{+}(s,t(s))$, and for these values of the constant in Eq. (1) we must have the inequality (2).

In this note we prove rigorously that in any interval

$$-4k^{2} + t_{1}(s) \leq t \leq -t_{1}(s) , \tag{3}$$

where k is the 3-momentum of particles in the c.m.s., and

$$t_{1}(s) = \frac{a^{2}}{\ln^{2} s} , \tag{4}$$

there must exist at least one value t_0 such that at this momentum transfer the magnitude of the scattering amplitude decreases not faster or increases not slower than that at $t = 0$:

$$\left|\frac{F(s,t_{0})}{F(s,0)}\right| \gtrsim \beta(a), \quad s \to \infty . \tag{5}$$

Remember that the function

$$f_{s}(t) = F(s,t)$$

is analytic in the Martin ellipse [2] with the foci at $t=0$, $t=-4k^2$ and the major semiaxis $2k^2+\delta^2$, $\delta^2>0$. For convenience we put

$$w = t + 2k^2$$

$$f_s(t) = g_s(w) . \tag{6}$$

New function $g_s(w)$ is analytic in the ellipse E with the foci at

$$w_c = \pm c , \quad c = 2k^2 , \tag{7}$$

and the major semiaxis

$$a = 2k^2 + \delta^2 . \tag{8}$$

Its minor semiaxis is

$$b = \sqrt{a^2 - c^2} . \tag{9}$$

Let t_1 be some positive number, $t_1 < c$ and we consider the ellipse E' with foci at

$$w_c' \pm c' , \quad c' = c - t_1 = 2k^2 - t_1 , \tag{10}$$

which has the same minor semiaxis b. Its major semiaxis is

$$a' = \sqrt{c'^2 + b^2} = \sqrt{a^2 + c'^2 - c^2} \ . \tag{11}$$

It is easy to check that for rather large s the ellipse E' will contain the point $w=c$ (i.e. $t=0$) if

$$t_1 < \delta^2 \ . \tag{12}$$

We assume that this condition is satisfied.

By means of the conformal mapping

$$\xi = \frac{w + \sqrt{w^2 - c'^2}}{c'} \tag{13}$$

we transform the ellipse E' with the cut $[-c', c']$ into a ring with the internal radius 1 and the external radius R

$$R = \frac{a' + \sqrt{a'^2 - c'^2}}{c'} \ . \tag{14}$$

The point $w = c$ (i.e. $t=0$) goes to the point $\xi = r$,

$$r = \frac{c + \sqrt{c^2 - c'^2}}{c'} \ . \tag{15}$$

We put

$$h_s(\xi) = g_s(w),$$

and introduce some notations:

$$m = \max_{|\xi|=1} |h_s(\xi)| = \max_{-c' \le w \le c'} |g_s(w)| = \max_{-4k^2+t_1 \le t \le -t_1} |f_s(t)|, \tag{16}$$

$$M = \max_{|\xi|=R} |h_s(\xi)| = \max_{w \in \partial E'} |g_s(w)| = \max_{t \in \partial E} |f_s(t)|, \tag{17}$$

where ∂E and $\partial E'$ are the boundaries of the ellipses E and E', resp. Applying the Hadamard three circles theorem/3,4/ we have

$$\left(1-\frac{\ln r}{\ln R}\right) \ln \frac{m}{|h_s(r)|} + \frac{\ln r}{\ln R} \ln \frac{M}{|h_s(r)|} \ge 0 . \tag{18}$$

From eqs. (7), (8), (10), (11), (14), and (15) it is easy to show that at

$$\frac{\ln r}{\ln R} \approx \sqrt{\frac{t_1}{\delta^2}} . \tag{19}$$

If we choose

$$t_1 = t_1(s) = \frac{\delta^2 \gamma^2}{\ln^2 s}, \tag{20}$$

where γ is some constant, then from Eq. (18) we get

$$\ln \frac{m}{|f_s(0)|} \gtrsim - \frac{\gamma}{\ln s} \ln \frac{M}{|f_s(0)|}. \tag{21}$$

Since $F(s,t)$ for every t in the ellipse E satisfies the dispersion relation in s with two subtractions /5/ and $|F(s,0)|$ is bounded below by some power s^{-n} /6/, then we have

$$\ln \left| \frac{M}{f_s(0)} \right| \lesssim \kappa \ln s . \tag{22}$$

From Eq. (21) it follows that

$$\ln \frac{m}{|f\ (0)|} \gtrsim - \gamma \kappa , \tag{23}$$

$$\max_{-4k^2+t_1\leq t\leq -t_1} \ln \left| \frac{f_s(t)}{f_s(0)} \right| \geq -\kappa \frac{a}{\delta} . \tag{24}$$

Therefore

$$\max_{-4k^2+t_1\leq t\leq -t_1} \left| \frac{F(s,t)}{F(s,0)} \right| \geq e^{-\kappa \frac{a}{\delta}} . \tag{25}$$

Thus the inequality (5) has been proved. Moreover, we found the explicit expression for the constant $\beta(a)$ in Eq. (5):

$$\beta(a) = e^{-\kappa \frac{a}{\delta}} . \tag{26}$$

Suppose that in any interval of the type (3) the magnitude of $F(s,t)$ reaches its maximum at the end point

$$t = -t_1(s) .$$

Then we have

$$\left| \frac{F(s,-t_1(s))}{F(s,0)} \right| \geq \beta(a) . \tag{5'}$$

It is obvious that our conclusions are true also for the imaginary part $\mathrm{Im}\,F(s,t)$. If we suppose that at $t=0$ the real part of the amplitude is bounded below by some power s^{-n}, then for it we have the same results.

In conclusion we note that the constant α in Eq. (4) can be chosen to be arbitrarily small.

References

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