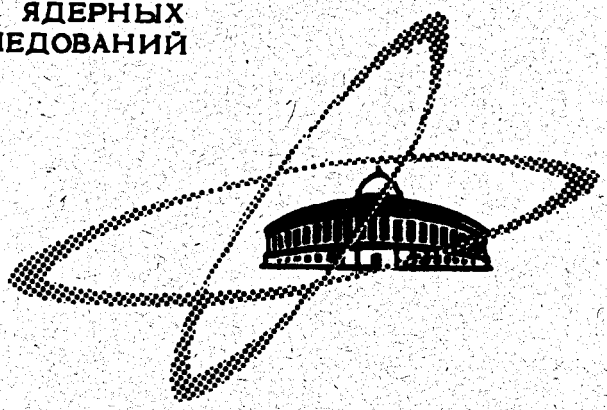


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ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна



E2 - 5337

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

Nguyen van Hieu

ON THE BEHAVIOUR
OF THE ELASTIC SCATTERING
AMPLITUDE AT MOMENTUM
TRANSFERS DECREASING AS $(\ln s)^{-2}$

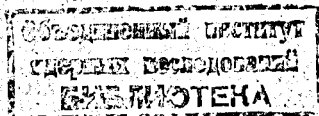
1970

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Submitted to "Nuclear Physics"



Нгуен Ван Хьеу

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О поведении амплитуды упругого рассеяния при значениях передачи импульса, убывающих как $(\ln s)^{-2}$

В работе показано, что при $t = \frac{\text{const}}{\ln^2 s}$ амплитуда имеет такое же поведение, что и ее поведение при $t = 0$.

**Препринт Объединенного института ядерных исследований.
Дубна, 1970**

Nguyen van Hieu

E2-5337

On the Behaviour of the Elastic Scattering Amplitude at Momentum Transfers Decreasing as $(\ln s)^{-2}$

We prove that in any interval

$$-4k^2 + t_1(s) \leq t \leq -t_1(s),$$

where

$$t_1(s) = \frac{\text{const}}{\ln^2 s},$$

there must exist at least one value t_0 such that the magnitude of the elastic scattering amplitude at this momentum transfer has the same behaviour as its behaviour at $t = 0$: $|\frac{F(s, t_0)}{F(s, 0)}| \geq \text{const}, s \rightarrow \infty$.

This conclusion is true also for the imaginary part of the amplitude. If the real part of the amplitude at $t = 0$ is bounded below by some power s^{-n} , then for it we have the same result.

Preprint. Joint Institute for Nuclear Research.

Dubna, 1970

In a recent paper^{/1/} Kinoshita showed that at momentum transfers decreasing as

$$- \operatorname{Im} t(s) = |t(s)| < \frac{\text{const}}{\ln^2 s} \quad (1)$$

the real part of the elastic scattering amplitude has the same behaviour as that at $t = 0$

$$\left| \frac{\operatorname{Re} F(s, t(s))}{\operatorname{Re} F(s, 0)} \right| > \text{const} . \quad (2)$$

In proving this statement Kinoshita used the following representation of $\operatorname{Re} F(s, t)$:

$$\operatorname{Re} F(s, t) = D^+(s, t) - D^-(s, t) ,$$

the functions $D^\pm(s, t)$ being non-negative at

$$D^\pm(s, 0) > 0 .$$

Further he proved that for values t decreasing as in Eq. (1) the functions $D^\pm(s, t(s))$ have the same behaviours as that at $t=0$:

$$\left| \frac{D^\pm(s, t(s))}{D^\pm(s, 0)} \right| \geq \epsilon^\pm$$

Since $D^+(s, 0)$ and $D^-(s, 0)$ are different, then due to the last inequality we can believe that for rather small s -independent constants in Eq. (1) the function $D^-(s, t(s))$ cannot cancel $D^+(s, t(s))$, and for these values of the constant in Eq. (1) we must have the inequality (2).

In this note we prove rigorously that in any interval

$$-4k^2 + t_1(s) \leq t \leq -t_1(s), \quad (3)$$

where k is the 3-momentum of particles in the c.m.s., and

$$t_1(s) = \frac{a^2}{\ln^2 s}, \quad (4)$$

there must exist at least one value t_0 such that at this momentum transfer the magnitude of the scattering amplitude decreases not faster or increases not slower than that at $t=0$:

$$\left| \frac{F(s, t_0)}{F(s, 0)} \right| \geq \beta(a), \quad s \rightarrow \infty. \quad (5)$$

Remember that the function

$$f_s(t) = F(s, t)$$

is analytic in the Martin ellipse^{/2/} with the foci at $t=0$, $t=-4k^2$ and the major semiaxis $2k^2 + \delta^2$, $\delta^2 > 0$. For convenience we put

$$w = t + 2k^2 \tag{6}$$

$$f_s(t) = g_s(w).$$

New function $g_s(w)$ is analytic in the ellipse E with the foci at

$$w_c = \pm c, \quad c = 2k^2, \tag{7}$$

and the major semiaxis

$$a = 2k^2 + \delta^2. \tag{8}$$

Its minor semiaxis is

$$b = \sqrt{a^2 - c^2}. \tag{9}$$

Let t_1 be some positive number, $t_1 < c$ and we consider the ellipse E' with foci at

$$w'_c = \pm c', \quad c' = c - t_1 = 2k^2 - t_1, \tag{10}$$

which has the same minor semiaxis b . Its major semiaxis is

$$a' = \sqrt{c'^2 + b^2} = \sqrt{a^2 + c'^2 - c^2}. \quad (11)$$

It is easy to check that for rather large s the ellipse E' will contain the point $w=c$ (i.e. $t=0$) if

$$t_1 < \delta^2. \quad (12)$$

We assume that this condition is satisfied.

By means of the conformal mapping

$$\xi = \frac{w + \sqrt{w^2 - c'^2}}{c'} \quad (13)$$

we transform the ellipse E' with the cut $[-c', c']$ into a ring with the internal radius 1 and the external radius R .

$$R = \frac{a' + \sqrt{a'^2 - c'^2}}{c'}. \quad (14)$$

The point $w=c$ (i.e. $t=0$) goes to the point $\xi=r$,

$$r = \frac{c + \sqrt{c^2 - c'^2}}{c'}. \quad (15)$$

We put

$$h_s(\xi) = g_s(w),$$

and introduce some notations:

$$\begin{aligned} m &= \max_{|\xi|=1} |h_s(\xi)| = \max_{-c' \leq w \leq c'} |g_s(w)| = \\ &= \max_{-4t^2 + t_1 \leq t \leq -t_1} |f_s(t)|, \end{aligned} \quad (16)$$

$$\begin{aligned} M &= \max_{|\xi|=R} |h_s(\xi)| = \max_{w \in \partial E'} |g_s(w)| = \\ &= \max_{t \in \partial E} |f_s(t)|, \end{aligned} \quad (17)$$

where ∂E and $\partial E'$ are the boundaries of the ellipses E and E' , resp. Applying the Hadamard three circles theorem^{3,4/} we have

$$\left(1 - \frac{\ln r}{\ln R}\right) \ln \frac{m}{|h_s(r)|} + \frac{\ln r}{\ln R} \ln \frac{M}{|h_s(r)|} > 0. \quad (18)$$

From eqs. (7), (8), (10), (11), (14), and (15) it is easy to show that at

$$\frac{\ln r}{\ln R} \approx \sqrt{\frac{t}{\delta^2}}. \quad (19)$$

If we choose

$$t_1 = t_1(s) = \frac{\delta^2 \gamma^2}{\ln^2 s}, \quad (20)$$

where γ is some constant, then from Eq. (18) we get

$$\ln \frac{m}{|f_s(0)|} \geq - \frac{\gamma}{\ln s} \ln \frac{M}{|f_s(0)|}. \quad (21)$$

Since $F(s,t)$ for every t in the ellipse E satisfies the dispersion relation in s with two subtractions^{/5/} and $|F(s,0)|$ is bounded below by some power s^{-n} ^{/6/}, then we have

$$\ln \left| \frac{M}{f_s(0)} \right| \leq \kappa \ln s. \quad (22)$$

From Eq. (21) it follows that

$$\ln \frac{m}{|f(0)|} \geq - \gamma \kappa, \quad (23)$$

$$\max_{-4k^2 + t_1 \leq t \leq -t_1} \ln \left| \frac{f_s(t)}{f_s(0)} \right| \geq -\kappa \frac{\alpha}{\delta}. \quad (24)$$

Therefore

$$\max_{-4k^2 + t_1 \leq t \leq -t_1} \left| \frac{F(s,t)}{F(s,0)} \right| \geq e^{-\kappa \frac{\alpha}{\delta}}. \quad (25)$$

Thus the inequality (5) has been proved. Moreover, we found the explicit expression for the constant $\beta(\alpha)$ in Eq. (5):

$$\beta(\alpha) = e^{-\kappa \frac{\alpha}{\delta}}. \quad (26)$$

Suppose that in any interval of the type (3) the magnitude of $F(s,t)$ reaches its maximum at the end point

$$t = -t_1(s).$$

Then we have

$$\left| \frac{F(s, -t_1(s))}{F(s, 0)} \right| \geq \beta(\alpha). \quad (5')$$

It is obvious that our conclusions are true also for the imaginary part $\text{Im} F(s,t)$. If we suppose that at $t = 0$ the

real part of the amplitude is bounded below by some power s^{-n} , then for it we have the same results.

In conclusion we note that the constant a in Eq. (4) can be chosen to be arbitrarily small.

R e f e r e n c e s

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