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ON THE EXTENDED PARTICLE MODEL IN THE GENERAL THEORY OF RELATIVITY

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Summary

The Einstein-Maxwell field equations allows some specific solutions, these are almost closed Friedmann worlds the metrics of which are connected with the Nordström-Reissner metric through a narrow throat. The classical parameters of this object, the sizes of the throat, its potential and the total mass of the system

$$r_h = \frac{\varepsilon\sqrt{\varkappa}}{c^2} \; ; \; \varphi_h = \frac{c^2}{\sqrt{\varkappa}} \; ; \; m = \frac{\varepsilon}{\sqrt{\varkappa}} \; .$$

are given. These classical systems turn out to be unstable - the pair production effect in strong throat fields polarizes the intial charge of the system and classical systems with different initial charges tend (as $t \to \infty$) to one and the same almost closed system, the so-called "friedmon" with microscopic values of the total charge, sizes and the total mass.

The system ("friedmon") is viewed as the extended particle model in the general relativity theory. It can also be regarded as a specific example of a model with form factor in which the signal propagates with a velocity which is smaller than the velocity of light due to an appropriate change of the metric. In the Euclidean space the form factor leads, as a rule, to the appearance of superlight signals.

In my report I would like to draw attention to the fact that in the general relativity theory there appear peculiar possibilities of introducing the sizes of particles with the help of the form factors.

The difficulties of the nonlocal field theory or, more exactly, of the extended field sources are in that in such a theory the signal can propagate with a velocity larger than the velocity of light thereby violating causality. Yet, the change of the metric near the source characteristic of the general relativity theory decreases the velocity of the signal propagation and thereby may conserve causality. It seems at first sight that the gravitational scales differ by about twenty orders from those with which we deal in elementary particle theory (for example, the characteristic length for the hadron form factors is $\sim 10^{-14}$ cm, and the characteristic gravitational length is $r_g \sim \sqrt{\frac{\hbar \varkappa}{c^3}} \sim 10^{-32}$ cm), and therefore it would seem that the gravitation is of no importance in the elementary particles theory. We consider the problems stated by a model which is described with the appropriate solutions of the system of the Einstein-Maxwell field equations.

This model, more exactly, the system which under real conditions can really be formed by itself is of great interest. In what follows this system will be called friedmon.

I. To the Theory of the Friedmon

As is known, the closed Friedmann world is described with the appropriate Tolman solutions in the case of a spherically symmetric homogeneous and isotropic distribution of dust-like matter at a zero pressure and an appropriate critical matter density[I]. If, however, this system is assumed to be charged by a uniform electric charge then the world metric ceases to be closed at any value of the total nonzero electric charge[2]. If the charge is such that the electrical forces are uncapable of overcoming the gravitational attraction then at a density larger than the critical one the metric of the so-called semiclosed world is formed instead of the closed metric.

In this system there arises a centre of symmetry and the surfaces circumscribed around this centre, at not too high electric charge, are approximately of the form

$$S = a^2 \sin^2\chi \,, \quad \chi < \frac{\pi}{2} \,.$$

As in the case of the absence of the charge, when moving away from the centre ($\chi = 0$), the surfaces increase (when $\chi < \frac{\pi}{2}$) and with further moving away from the centre the surfaces decrease. In the case of the absence of the charge the surface as $\chi = \pi$ reduces to a point; the world becomes closed.

If the system is electrically charged then for χ close to π (and a relativelly small charge) the metric essentially differs from the Friedmann one. Now the spherical surface does not reduce to a point but there arises a certain minimum sphere (throat) through which the metric of the material system is related to the Nordström-Reissner one. The parameters of this throat are obtained automatically when sewing the internal metric of the system with the external Nordström-Reissner one.[2]

It is interesting that the electric potential in the throat is independent of the magnitude of the total electric charge of the system and is [2]

$$\varphi_h = \frac{c^2}{\sqrt{\varkappa}} \,, \tag{I}$$

where c is the velocity of light and $\varkappa$ is the gravitational constant. The matter is that with increasing distance when $\chi > \frac{\pi}{2}$, as we have mentioned earlier, the spherical surfaces decrease, they are a kind of lenses which crowd the electrical strengh lines. The value of the electric potential given by eq.(I), is obviously a maximum large one allowed by in the theory. With increasing charge the electric potential of the throat does not increase while the throat radius increases proportionally to the charge $\mathcal{E}$ namely

$$r_h = \frac{\mathcal{E}\sqrt{\varkappa}}{c^2} \,. \tag{2}$$

The total mass of the system is given by the expression[2]

$$m = \frac{\mathcal{E}}{\sqrt{\varkappa}} \,. \tag{3}$$

When $\mathcal{E} \to 0$, the total mass vanishes as it is the case in the closed Friedmann world. It is remarkable that the total mass of the system is in this case of electrostatic origin. In this case all the parameters r_h, m and $\mathcal{E}$ are, generally speaking, of ultramacroscopic values - characteristics of the "Universe" as a whole.

Now attention should be paid to one very important fact. The sizes of the throat are stable only in the classical consideration. As soon as we take into account the fact, that in such large fields which take place in the throat there arise effects of production of charged particle pairs,the picture becomes essentially different. In fact, the particle of a produced pair with a charge opposite to that of the system strikes the system decrasing its charge. The other component of the pair pushes off from the charge of the system and goes to infinity. Thus, the total charge of the system decreases rapidly, the total mass decreases by Eq. (3) and the external dimensions of the system, its throat, decrease by Eq.(2). The system is striving to be close as completely as possible. The estimations show[2 *)] that this process of pair production decreases the total charge of the system down to the finite value

$$Z_f \leq \frac{1}{\alpha^2 \ln\left(\frac{e/\sqrt{\pi}}{m_e}\right)^2}, \tag{4}$$

here α is the fine structure constant, e is the electric charge of the electron. Since

$$\ln\left(\frac{e/\sqrt{\pi}}{m_e}\right)^2 \sim \frac{1}{\alpha}, \tag{5}$$

then $Z_f \sim 137$.

It is essential to stress the important fact that a)the final charge Z_f of the system is independent of its initial charge; b)the final charge has the microscopic value

$$e_f \sim 137\, e, \tag{6}$$

*) in which the production of only electron-positron pairs is taken into account.

c) the external sizes of the system are microscopic

$$r_f \leq \frac{137\, e\sqrt{\varkappa}}{c^2} \sim 10^{-30}\, cm \qquad (7)$$

and the total mass[***)] is

$$m_f \leq \frac{137\, e}{\sqrt{\varkappa}} \sim 10^{-4}\, gr\,. \qquad (8)$$

The relations (6), (7) and (8) have the form of inequalities. These relations give the upper limits for the final charge and the mass. The real values of them are far lower. So, the electric charge localized in such a small domain must be strongly polarized. If we assume that the given case is associated with the well-known Landau formula which relates the "bare" charge ε with the effective charge e, up to which the former decreases due to the vacuum polarization, then[3]

$$e^2 = \frac{\varepsilon^2}{1 + \nu\, \varepsilon^2 \ln\left(\frac{\Lambda}{m_e}\right)^2}\,, \qquad (9)$$

where m_e is the electron mass, ν is the number of various kinds of pairs, Λ is the maximum value of the parameter connected with the region of the charge localization. When $\nu\varepsilon^2 \ln\left(\frac{\Lambda}{m_e}\right)^2 \gg 1$, Eq.(9) takes the form

$$e^2 \to \frac{1}{\nu \ln\left(\frac{\Lambda}{m_e}\right)^2}\,. \qquad (10)$$

***) Nevertheless the system may contain an arbitrary large number of baryons ("Universe") and have arbitrary large "internal" dimensions, $a(t_0)\chi$, where $a(t_0)$ is the "Universe" radius.

It is interesting that the earlier obtained Eq.(4) contains the same characteristic logarithm as in the Landau formula. But in Eq.(4) one gives a concrete form of the parameter Λ, namely, it turns out that

$$\Lambda = \frac{e}{\sqrt{\varkappa}} \sim 10^{28}\, eV . \qquad (11)$$

In Eq.(4) Λ assumes just the value which is discussed in the Landau's paper in connection with the hypothesis on the possible role of gravitation in elementary particle theory. In our case the role of the parameter Λ is revealed in a natural manner, automatically. It is interesting that Eqs.(4) and (9) are obtained starting from quite different considerations and in different mathematical ways.

According to Eq.(10) the final value of the system charge is also independent of its initial value $\mathcal{E}$ and may be close to the electron charge. We call such an object, such a system, friedmon.

As to the space dimensions of the system, Eq.(7) gives essentially the classic "core" of the object round which a peculiar "atmosphere" consisting of various elementary particles should arise. The situation is that at $e_f \leqslant 137e$ when the particles of the pairs produced cease to "strike" the charged centre the appropriate charges occupy the appropriate orbits about the charged centre. The radii of the first Bohr orbit are estimated as

$$r \sim \frac{\hbar^2}{M e_f e} . \qquad (12)$$

If the bound state has no stable particles heavier than nucleons the most dense "atmosphere" of the friedmon is at a distance $10^{-14}-10^{-13}$ cm from the object core. If this picture is true then between the lengths $10^{-30}-10^{-15}$ cm in the electrostatic friedmon there must be an empty space. Thus, we see that the external fiedmon dimensions must be of the order of the hadron dimensions.

As to the friedmon mass, the relation

$$m_f \leq \frac{e}{\sqrt{\varkappa}} \sim 10^{-6}\, gr \qquad (12)$$

gives only the upper estimate. Moreover, Eq.(12) may be considered as a relation giving the corresponding upper estimate for the mass of the microscopic particle (in general, of elementary particle). If the microscopic particles of such a maximum mass exist ("maximons"), they could play the role of quarks. For these masses the gravitational forces acting between them are sufficient for forming systems with a mass defect comparable with the masses of these particles[4].

We have considered the system which is a source of the electric field. We have called this system electrostatic friedmon. But similar friedmons arise in any other vector fields.

So, the Yang-Mills[5] friedmon will have the mass

$$m_Y^f < \frac{\mu}{\sqrt{\varkappa}} \sim 10^{-5/2} M_p \qquad (13)$$

i.e. mass smaller than ten electronic masses if we take the possible upper estimate of μ from the Yang-Mills paper[6].

The estimation of the ρ-meson friedmon mass gives the upper value of the mass which is larger than that for the electrostatic friedmon:

$$m_{\rho}^{f} \leq \frac{g}{\sqrt{\varkappa}} \sim 10^{-5} \, gr \, , \qquad (I4)$$

where g is the "oharge" of the ρ -meson field.

Non-vector fields, more exaotly the fields in whioh particles and antipartioles are not distinguished (e.g. the scalar field) do not prevent from the formation of the closed metrio.

Moreover, oombined fields, e.g. eleotromagnetio and scalar ones, decrease the total mass of the friedmon sinoe the corresponding expressions for the friedmon mass are[7]

$$m_{e,s}^{f} \leq \sqrt{\frac{e^2 - G^2}{\varkappa}} \, , \qquad (I5)$$

where G - is the soalar field constant.

Thus, in the framework of the general relativity theory systems with microscopio values of the parameters close to the parameters of the so-called elementary particles may be realized. But independently of the latter values, the given system, friedmon, may be oonsidered as a peculiar model for "extended" partiole as an object with a peculiar form factor possible only in the framework of the general relativity theory.

2. Friedmon as the Divergency - Free Phenomenological Model of a Particle.

The external space metrio of an eleotrical friedmon in the Nordström-Reissner coordinates is of the form:

$$ds^2 = \phi \, dt^2 - \phi^{-1} dr^2 - r^2 d\sigma^2, \qquad (I6)$$

here

$$\phi = \left(1 - \frac{r_0}{r}\right)^2 \tag{17}$$

and

$$r_0 = \frac{\sqrt{\varkappa}\, e_0}{c^2} = \frac{\varkappa m_0}{c^2} . \tag{18}$$

Here m_0, e_0 are the total mass and the total electric charge of the system.

The metric assumes a very peculiar form in the isotropic coordinates

$$ds^2 = B^{-1} dt^2 - B\left(dR^2 + R^2 d\sigma^2\right), \tag{19}$$

here

$$B = \left(\frac{R + r_0}{R}\right)^2 . \tag{20}$$

In the isotropic coordinates the equation for the electrostatic potential $\varphi = A_t$, $F_{tr} = -\frac{\partial A_t}{\partial R}$,

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^i}\left(\sqrt{-g}\, \frac{\partial F^{ik}}{\partial x^k}\right) = 0 \tag{21}$$

assumes the form

$$-(R + r_0)^2 \frac{\partial \varphi}{\partial R} = \mathrm{const} = e_0 \tag{22}$$

or

$$\varphi = \frac{e_0}{R + r_0} . \tag{23}$$

Thus, the potential at $R = 0$ is finite

$$\varphi_{R=0} = \frac{e_0}{r_0} = \frac{c^2}{\sqrt{\varkappa}} . \tag{24}$$

The calculation of the total energy in the general relativity theory encounters some difficulties; since the four-dimensional energy-momentum vector is not a well defined quantity. If we take the well known Möller[8] definition, according to which the energy E in the isotropic coordinates is of the form

$$m = \frac{E}{c^2} = \frac{c^2}{8\pi\varkappa} \int_S h R^2 \sin\theta \, d\theta \, d\varphi , \tag{25}$$

as $R = \text{const} \to \infty$,

here
$$h = -\frac{d\phi}{dR}\frac{1}{\phi} = \frac{2 r_0}{R(R+r_0)} ,$$

then
$$m = \frac{E}{c^2} = \frac{c^2 r_0}{\varkappa} = \frac{e_0}{\sqrt{\varkappa}} , \tag{26}$$

in accordance with the above presented theory of friedmon.

It is essential to stress that in the isotropic coordinates the friedmon with metric (19) is a point object. In these coordinates when $R \to 0$ the components of the fundamental tensor $g_{\mu\nu}$ have a singularity, but this singularity does not affect the magnitude of the physical quantities - electric potential and total energy. The isotropic coordinates are convenient for the external description of the friedmon, in particular, its behaviour in the external fields as a point object. It is interesting to note that in the Nordström-Reissner coordinates (16), contrary to the isotropic coordinates (19) the friedmon is not a point one[*]. Here the potential is of the form

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[*] The possibility of the object to be point or extended in the general relativity depending on the coordinates used is indicated in ref.[9]

$\varphi = \frac{e}{r}$, but r changes only from $r=\infty$ to $r=r_0=$ $=\frac{e\sqrt{\varkappa}}{c^2}$ and at the point $r=r_0$ the potential assumes the value $\varphi_{r=r_0} = \frac{c^2}{\sqrt{\varkappa}}$, i.e. the same maximum value which is reached in the isotropic coordinates for $R = 0$.

I would like to stress that the object under consideration (friedmon) itself is a satisfactory in many respects model of the extended charged particle. Since this extended object is described by the solutions of the relativistic equations with local interactions then in this case, by definition, there cannot arise difficulties of the usual nonlocal theory, e.g. causality violation etc.

From a certain point of view, in describing a given "extended" object (friedmon) there arises a form factor, but due to an appropriate change of the metric the signal near the charge source propagates with a velocity smaller than the velocity of light. So, in the isotropic coordinates, according to eq.(19), the velocity of light is

$$\frac{dR}{dt} = \left(\frac{R}{R+r_0}\right)^2 c \qquad (27)$$

and

$$\frac{dR}{dt} \to 0 \qquad \text{as } R \to 0 .$$

Difficulties similar to those in the nonlocal field theory arise also in attempting to construct the appropriate nonlinear field theory free of divergences.[10] Attempts have been made to redefine formally the metric so that to obtain the corresponding velocity of the signal propagation according to the causality principle[II,I2]. One deals there with special illustrating examples, but,in general, in the framework of the modern theory taken ad hoq. The model in question (friedmon) is described completely in the framework of the modern theory. Moreover, it may be stressed that in the given case one implies nonlinear theory since a part of equations (namely, Einstein field equations) is a system of nonlinear equations. In principle, excluding $g_{\mu\nu}$ from the system of Einstein-Maxwell field equations we should obtain for the electromagnetic field a nonlinear equation.

In conclusion I would like to remind that there is another difficulty in attempting to treat the mass of the electric source field from a "purely electromagnetic" point of view. As is known, in this case the total momentum of the electromagnetic field of the charge moving with a constant velocity $\vec{v}$ is

$$\vec{P} = \frac{4}{3} \frac{\vec{v}}{\sqrt{1-\beta^2}} \frac{u_0}{c} \tag{28}$$

The factor 4/3 appears owing to that in calculating the total momentum of the field of a moving charge, in addition to T^{c}_{44} an additional term T^{0}_{11} due to the Maxwell tensions gives a contribution. The factor 4/3 was widely discussed a long time ago and was considered to be a main obstacle in construoting the electromagnetic theory of the partiole mass. It was assumed that, in addition to the electromagnetic forces it is necessary to introduce forces of nonelectromagnetic origin (in the framework of linear theories) which should damp these Maxwell tensions. In the present model this role is played by gravitational forces.

It should also be stressed that in this consideration we are dealing with the classical electromagnetic field theory free of divergences. The corresponding quantum fields theory is not discussed yet.

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О модели протяженной частицы в общей теории относительности

В докладе рассматривается почти замкнутый электрически заряженный фридмановский мир как модель протяженной частицы в общей теории относительности.

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