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ON CHIRAL $SU_2 \times SU_2$ DYNAMICS FOR A_1, ρ AND π MESONS

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I. Introduction

In recent years the process with A_1, ρ- and π- mesons has been intensively investigated in current algebra. Important information on the $A_1\rho\pi$ system has been derived from the Ward identities for the n-point functions in the tree approximation with some restriction on the momentum dependence of the contact vertices/1/. Nevertheless, such a restriction leads to the solution of the Ward identities which depends on several arbitrary parameters/1/.

The constructive way for the reproduction of the current - algebra results is the effective Lagrangian method based on the $SU_2 \times SU_2$ group/2,3/.

In Section II of the present paper we show that the conventional Lagrangian of the $A_1\rho\pi$ system/2/ contains three- and quadrilinear derivative terms due to the elimination of the nonphysical bilinear $A_1\pi$ coupling. Then one should add all independent terms with three and four field derivatives and this leads to the appearence of the arbitrary parameters in the Lagrangian. Besides, the very presence of such derivative terms is undesirable in the interaction Lagrangian.

In Section III we suggest in this connection the following natural requirement: <u>The effective Lagrangian of the $A_1\rho\pi$ system must contain no more than two field derivatives in each term.</u> This severe restriction determines completely the invariant Lagrangian containing four independent parameters g_ρ, g_A, m_ρ, M_A with the aid of the nonlinear (in pions) substitution for the A_1

field.

In the tree approximation each contaot vertex is a polynomial in 4-momenta, the degree D of which is equal to the number of field derivatives in the appropriate term of the Lagrangian. In Section IV we establish the oonnection of our Lagrangian model with the partial solution of the Ward identities under the condition $D \leq 2$ for any contaot vertex. In the $A_1 \rho \pi$ system it is impossible to strengthen this restriction on the momentum dependence of the contact vertices. Note that in the conventional Lagrangian model[2] $D \lesssim 4$.

In Section V we discuss some consequences for the amplitudes $\rho \to 2\pi$, $A_1 \to \rho\pi$, $A_1 \to 3\pi$, $\pi\pi \to \pi\pi$, $\rho\pi \to \rho\pi$, etc. We derive with necessity the reasonable value

$$\delta = (g_A/g_\rho)^2 \left\{1 - \left[\frac{m_\rho^2}{g_\rho^2 f_\pi^2} - 1\right]^{-2}\right\} - 1 \sim -1 \tag{1.1}$$

for the anomalous magnetio moment of the A_1 meson. The S-wave soattering lengths have been found to be in agreement with those obtained in current algebra[6]. We have estimated also the total width of A_1.

II. Preliminaries

In the present paper we propose a new model for the Lagrangian of the $A_1 \rho \pi$ system which contains no more than bilinear derivative couplings. With this end in view we shall mention here some features of the conventional soheme with A_1, ρ and π mesons (see, e.g., refs.[2,3]).

The nonlinear Lagrangian for veotor, axial-veotor and pion fields V_μ, a_μ and π can be written as

$$\mathcal{L} = -\frac{1}{4}\vec{V}_{\mu\nu}^{\,2} - \frac{1}{4}\vec{a}_{\mu\nu}^{\,2} + \frac{m_0^2}{2}\left(\vec{V}_\mu^{\,2} + \vec{a}_\mu^{\,2}\right) + \frac{d^2}{2}\left(\nabla_\mu \vec{\Phi}\right)^2 \tag{2.1}$$

with the covariant ourls

$$\begin{aligned} \vec{V}_{\mu\nu} &= \partial_\mu \vec{V}_\nu - \partial_\nu \vec{V}_\mu - g\vec{V}_\mu \times \vec{V}_\nu - g\,\vec{a}_\mu \times \vec{a}_\nu, \\ \vec{a}_{\mu\nu} &= \partial_\mu \vec{a}_\nu - \partial_\nu \vec{a}_\mu - g\,\vec{V}_\mu \times \vec{a}_\nu + g\,\vec{V}_\nu \times \vec{a}_\mu \end{aligned} \tag{2.2}$$

and covariant derivative of the pion field

$$\nabla_\mu \vec{\Phi} = \Delta_\mu \vec{\Phi} + [\sigma(1+\sigma)]^{-1}\vec{\Phi}(\vec{\Phi}\cdot\Delta_\mu\vec{\Phi}) \tag{2.3}$$

where

$$\sigma = (1-\vec{\Phi}^2)^{1/2}, \quad \Delta_\mu\vec{\Phi} = \partial_\mu\vec{\Phi} - g\,\vec{V}_\mu\times\vec{\Phi} + g\sigma\vec{a}_\mu \tag{2.3a}$$

In these eqs. $\vec{\Phi} = f_\pi^{-1}\vec{\pi}$ ($f_\pi \simeq 95$ MeV is the pion decay constant) and m_0, g, d are some unrenormalized parameters. This Lagrangian is invariant under the $SU_2 \times SU_2$ group. Moreover, only the mass term for vector and axial-vector fields breaks the invariance under the gauge $SU_2 \times SU_2$ transformations with the coordinate-dependent parameters $\vec{\alpha}, \vec{\beta}$ /2,3/:

$$\begin{aligned} \delta(\vec{V}_\mu \pm \vec{a}_\mu) &= -(\vec{\alpha}\pm\vec{\beta})\times(\vec{V}_\mu \pm \vec{a}_\mu) - g^{-1}\partial_\mu(\vec{\alpha}\pm\vec{\beta}) \\ \delta\vec{\Phi} &= -\vec{\alpha}\times\vec{\Phi} + \vec{\beta}\sigma \end{aligned} \tag{2.4}$$

It follows immediately that the interacting fields $\vec{V}_\mu$ and $\vec{a}_\mu$ transfer spin 1 only[4] (in the limit of zero-mass pions), i.e. the transversality conditions $\partial_\mu \vec{V}_\mu = 0$, $\partial_\mu \vec{a}_\mu = 0$ are satisfied off the mass shell. However, the term $(\nabla_\mu \vec{\Phi})^2$ in eq. (2.1) contains the nonphysical bilinear coupling $\vec{a}_\mu \cdot \partial_\mu \vec{\Phi}$ which is usually eliminated by the following substitution of the axial-vector field:

$$\vec{a}_\mu = \vec{A}_\mu - \frac{g f_\pi^2}{m_0^2} \mathcal{D}_\mu \vec{\Phi} \tag{2.5}$$

Then it is impossible to maintain the condition $\partial_\mu \vec{A}_\mu = 0$ for the new axial-vector field $\vec{A}_\mu$. The introduction of $\mathcal{D}_\mu \vec{\Phi} = \partial_\mu \vec{\Phi} - g \vec{V}_\mu \times \vec{\Phi}$ instead of $\partial_\mu \Phi$ is necessary to ensure transversality for the interacting vector field $\vec{V}_\mu$ [4] since it is important in connection with the vector dominance idea[5].

The crucial point is that substitution (2.5) contributes to $V_{\mu\nu}$ the term $\mathcal{D}_\mu \vec{\Phi} \times \mathcal{D}_\nu \vec{\Phi}$ which is bilinear in the field derivatives. Hence, the conventional effective Lagrangian contains the three- and quadrilinear derivative terms $\partial_\mu \vec{V}_\nu \cdot \mathcal{D}_\mu \vec{\Phi} \times \mathcal{D}_\nu \vec{\Phi}$ $(\vec{A}_\mu \times \mathcal{D}_\nu \vec{\Phi}) \cdot (\mathcal{D}_\mu \vec{\Phi} \times \mathcal{D}_\nu \vec{\Phi})$ and $(\mathcal{D}_\mu \vec{\Phi} \times \mathcal{D}_\nu \vec{\Phi})^2$ generated by $\vec{V}_{\mu\nu}$ in eq.(2.1). It should be said that it is undesirable to have the three- and quadrilinear derivative couplings in the interaction Lagrangian. Nevertheless, if one admits the presence of such couplings, then all possible terms with three and four derivatives should be added and the number of parameters in the theory should increase. Besides, the three- and quadrilinear derivative couplings lead to non-smooth momentum dependence of the amplitudes $\rho \to 2\pi$, $A_1 \to 3\pi$, $\pi\pi \to \pi\pi$ etc.

III. Lagrangian of the $A_1 \rho \pi$ system without three- and quadrilinear derivative couplings.

To avoid the above-mentioned difficulties of the conventional Lagrangian model for the $A_1 \rho \pi$ system we propose the following natural requirement:

Three- and quadrilinear in field derivatives terms must be absent in the effective Lagrangian of the $A_1 \rho \pi$ system.

Now we start to construct the general Lagrangian which satisfies this requirement.

It is convenient to define for this purpose the new covariant quantities

$$\vec{R}_{\mu\nu} = \vec{V}_{\mu\nu} + \frac{c^2}{g} \Delta_\mu \vec{\Phi} \times \Delta_\nu \vec{\Phi} \tag{3.1}$$

$$\vec{S}_{\mu\nu} = \vec{a}_{\mu\nu} + \frac{c^2}{g\sigma} \vec{\Phi} \times (\Delta_\mu \vec{\Phi} \times \Delta_\nu \vec{\Phi}) \tag{3.2}$$

with some constant c. $\vec{R}_{\mu\nu}$ and $\vec{S}_{\mu\nu}$ have the same transformation properties as covariant curls $\vec{V}_{\mu\nu}$ and $\vec{a}_{\mu\nu}$

$$\begin{aligned} \delta^L \vec{R}_{\mu\nu} &= -\vec{\alpha} \times \vec{R}_{\mu\nu} - \vec{\beta} \times \vec{S}_{\mu\nu} \\ \delta^L \vec{S}_{\mu\nu} &= -\vec{\alpha} \times S_{\mu\nu} - \vec{\beta} \times \vec{R}_{\mu\nu} \end{aligned} \tag{3.3}$$

Here eqs.(2.2), (2.3a) and (2.4) have been used.

Let us write down at first the general expression for the invariant Lagrangian of the $A_1 \rho \pi$ system (containing three- and quadrilinear derivative terms):

$$\mathcal{L} = -\frac{Z_\rho}{4}\vec{R}_{\mu\nu}^{\,2} - \frac{Z_A}{4}\vec{S}_{\mu\nu}^{\,2} - \frac{Z_\rho - Z_A}{4}\Big\{(\vec{\Phi}\times\vec{R}_{\mu\nu})^2 - (\Phi\times S_{\mu\nu})^2 - 2\sigma\,\vec{R}_{\mu\nu}\times\vec{\Phi}\cdot\vec{S}_{\mu\nu}\Big\} + h\,[\nabla_\mu\vec{\Phi}\times\nabla_\nu\vec{\Phi}]^2 + K(\nabla_\mu\vec{\Phi}\cdot\nabla_\mu\vec{\Phi})^2 + \frac{m_0^2}{2}(\vec{V}_\mu^{\,2}+\vec{a}_\mu^{\,2}) + \frac{d^2}{2}(\nabla_\mu\vec{\Phi}\cdot\nabla_\mu\vec{\Phi}). \tag{3.4}$$

where Z_ρ, Z_A, m_0, d, h and K are some arbitrary constants.

Our main purpose is the elimination of the three- and quadrilinear derivative terms from the Lagrangian (3.4) together with the elimination of the bilinear $A_1\pi$ coupling. Also the kinetic terms must be correctly normalized. All these requirements can be satisfied only with $h = K = 0$ in a unique way by performing the following substitutions for the vector and axial-vector fields:

$$\vec{V}_\mu = Z_\rho^{-\frac{1}{2}}\vec{\rho}_\mu, \qquad \vec{a}_\mu = Z_A^{-\frac{1}{2}}\left[A_\mu + F(\vec{\Phi}^2)\mathcal{D}_\mu\vec{\Phi}\right] \tag{3.5}$$

We shall use below the renormalized constants:

$$g_\rho = Z_\rho^{-\frac{1}{2}}g, \quad g_A = Z_A^{-\frac{1}{2}}g, \quad m_\rho = Z_\rho^{-\frac{1}{2}}m_0, \quad M_A = Z_A^{-\frac{1}{2}}(m_0^2+d^2g^2)^{\frac{1}{2}} \tag{3.6}$$

To exclude the bilinear $A_\mu\partial_\mu\vec{\Phi}$ coupling we fix in eq.(3.5) the value $F(0) = -g_\rho^2 f_\pi^2/g_A m_\rho^2$ then the function $F(\vec{\Phi}^2)$ is uniquely determined by the requirement that the three- and quadrilinear derivative terms must be absent in the Lagrangian

$$F(\vec{\Phi}^2) = -\frac{C}{g_A(1+C\sigma)} = -\frac{g_\rho^2 f_\pi^2}{g_A m_\rho^2}\left[1 + \frac{1}{2}\left(\frac{g_\rho f_\pi}{m_\rho}\vec{\Phi}\right)^2 + \dots\right] \tag{3.7}$$

where $\sigma = (1-\vec{\Phi}^2)^{1/2}$ and

$$C = \left[\left(\frac{m_\rho}{f_\pi g_\rho}\right)^2 - 1\right]^{-1} \tag{3.8}$$

We have used here the relation /2,3/:

$$f_\pi^{\,2} = \frac{m_\rho^2}{g_\rho^2}\left[1-\left(\frac{m_\rho g_A}{M_A g_\rho}\right)^2\right] \tag{3.9}$$

which follows from the correct normalization of the kinetic term $(1/2)(\partial_\mu \vec{\pi})^2$.

In fact we have determined the nonlinear substitution of A_μ and the value of the constant C to exclude the bilinear derivative terms from $\vec{R}_{\mu\nu}$ (3.1) and $\vec{S}_{\mu\nu}$ (3.2) (see, Appendix), the terms with three and four derivatives being eliminated from eq.(3.4).

Now we can write the final expression for our Lagrangian of the $A_1 \rho \pi$ system which contains nonderivative couplings and the terms with one and two derivatives only:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\vec{\rho}_{\mu\nu}^{\,2} - \frac{1}{4}\vec{A}_{\mu\nu}^{\,2} - \frac{1}{4}B(\vec{\Phi}^2)(\Phi\times\rho_{\mu\nu})^2 + \frac{1}{4}\left(1-\frac{g_A^2}{g_\rho^2}\right)(\vec{\Phi}\times\vec{A}_{\mu\nu})^2 + \\ &+\frac{1}{2}H(\vec{\Phi}^2)(\vec{\rho}_{\mu\nu}\cdot\vec{\Phi}\times\vec{A}_{\mu\nu}) + \frac{1}{2}\left[\frac{f_\pi(1+C)}{1+C\sigma}\right]^2\left[(\mathcal{D}_\mu\vec{\Phi})^2 + \left(\frac{m_\rho g_A}{M_A g_\rho}\right)^2(\vec{\Phi}\cdot\partial_\mu\vec{\Phi})^2\right] \\ &+ g_A f_\pi^2\left(\frac{1+C}{1+C\sigma}\right)\left[(\sigma-1)(\mathcal{D}_\mu\vec{\Phi}\cdot\vec{A}_\mu) + \sigma^{-1}(\vec{A}_\mu\cdot\vec{\Phi})(\vec{\Phi}\cdot\partial_\mu\vec{\Phi})\right] - \\ &-\frac{1}{2}\left(\frac{f_\pi g_\rho M_A}{m_\rho}\right)^2(\vec{\Phi}\times\vec{A}_\mu)^2 + \frac{m_\rho^2}{2}\vec{\rho}_\mu^{\,2} + \frac{M_A^2}{2}\vec{A}_\mu^{\,2} + m_\pi^2 f_\pi^2\sigma\end{aligned} \tag{3.10}$$

where the symmetry-breaking term $m^2 f_\pi^2 \sigma$ leads to PCAC in this model [3]. In eq.(3.10) the following notations have been used:

$$B(\vec{\Phi}^2) = g_A^2 \left[1 + C^{-2}\left(\frac{g_\rho^2}{g_A^2} - 1\right)\right] F^2(\vec{\Phi}^2), \tag{3.11}$$

$$H(\vec{\Phi}^2) = \frac{g_A^2}{g_\rho}\left[1 + C^{-1}\sigma\left(1 - \frac{g_\rho^2}{g_A^2}\right)\right] F(\vec{\Phi}^2), \tag{3.12}$$

$$\vec{\rho}_{\mu\nu} = Z_\rho^{1/2}\vec{R}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho\vec{\rho}_\mu\times\vec{\rho}_\nu - \frac{g_A^2}{g_\rho}(1 - C^2\sigma)\vec{A}_\mu\times\vec{A}_\nu + C g_A/g_\rho\left(\vec{A}_\mu\times\mathcal{D}_\nu\vec{\Phi} + \mathcal{D}_\mu\vec{\Phi}\times\vec{A}_\nu\right), \tag{3.13}$$

$$\vec{A}_{\mu\nu} = Z_A^{\frac{1}{2}}\left[\vec{S}_{\mu\nu} - g_A F(\vec{\Phi}^2)\vec{\Phi}\times\vec{R}_{\mu\nu}\right] = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu - g_\rho\vec{\rho}_\mu\times\vec{A}_\nu + g_\rho\vec{\rho}_\nu\times\vec{A}_\mu - g_A C\vec{\Phi}\times(\vec{A}_\mu\times\vec{A}_\nu) \tag{3.14}$$

IV. Connection between our Model and the "Hard Pion" Method in the Algebra of Currents.

n-point functions derived from the effective Lagrangian of the $A_1\rho\pi$ system are known to be a solution for the Ward identities in $SU_2 \times SU_2$ algebra of currents [1]. In this Section we reformulate our basic requirement for the Lagrangian in terms of the "hard pion" method [1].

The vector and axial-vector currents $\vec{J}_\mu$ and $\vec{J}_{5\mu}$ are obtained from our Lagrangian(3.10) by using the Gell-Levy method (see, e.g. ref. [5]).

$$\vec{J}_\mu = \frac{m_\rho^2}{g_\rho}\vec{\rho}_\mu, \quad \vec{J}_{5\mu} = \frac{m_\rho^2 g_A}{g_\rho^2}\left[\vec{A}_\mu + F(\vec{\Phi}^2)\mathcal{D}_\mu\vec{\Phi}\right] \tag{4.1}$$

Note that these currents obey the equal-time commutation relations of the algebra of fields[5]:

$$\left[J_0^a(\vec{x},t), J_\mu^b(\vec{y},t)\right] = \left[J_{50}^a(\vec{x},t), J_{5\mu}^b(\vec{y},t)\right] = i\varepsilon_{abc} J_\mu^c(\vec{x},t)\delta(\vec{x}-\vec{y}) - i\frac{m_\rho^2}{g_\rho^2}\delta_{ab}\, g_{\mu k}\frac{\partial}{\partial x_k}\delta(\vec{x}-\vec{y}), \tag{4.2}$$

$$\left[J_0^a(\vec{x},t), J_{5\mu}^b(\vec{y},t)\right] = \left[J_{50}^a(\vec{x},t), J_\mu^b(\vec{y},t)\right] = i\varepsilon_{abc} J_{5\mu}^c(\vec{x},t)\delta(\vec{x}-\vec{y}) \tag{4.3}$$

Here $k = 1,2,3$ and $\mu = 0,1,2,3$. The proof of these relations is equivalent to that ref.[5]. We remark here that in all $SU_2\times SU_2$ gauge Lagrangian schemes the corresponding Sohwinger terms have the same simple c-number form if currents are caloulated by the Gell-Mann-Levy method. There are, of course, some model-dependent commutation relations. The strong form of PCAC[3] $\partial_\mu \vec{J}_{5\mu} = m_\pi^2 f_\pi \vec{\pi}$ is present in our model.

In the tree approximation the oontact vertex functions oorrespond to the various contact oouplings in the Lagrangian. The diagram of any n-point function involves contact vertioes connected by propagators without forming loops. Each contact vertex function is a polynomial in 4-momenta, the degree of whioh is equal to the number of field derivatives in the appropriate contaot term. This polynomial degree D characterizes the momentum dependence of a oontaot vertex funotion. In the tree approximation D depends on the number n_g of the ρ-meson and/or A_1-meson lines attached to the vertex irrespectively of the number of pion lines. We have D = 2 for $n_g = 0,2$, D = 1 for n_g = 1,3 and D = 0 for $n_g = 4$.

In the conventional current - algebraic approach to the $A_1 \rho \pi$ system[1,2] there are vertices with D = 3 and D = 4 which correspond to the three- and quadrilinear derivative couplings. In our approach the momentum dependence of the contact vertex functions is minimal and cannot be reduced further. At least two field two field derivatives must be present because the terms ($\nabla_\mu \vec{\Phi}$)^2 , ($\vec{\rho}_{\mu\nu}$)^2 etc. are always present.

<u>Thus, using Lagrangian (3.10) we derive n-point functions which are the minimum momentum dependent solution of Ward identities in $SU_2 \times SU_2$ algebra of currents with c-number Schwinger terms.</u>

Note that in meson-nucleon couplings we can limit ourselves to vertices which are linear in momenta.

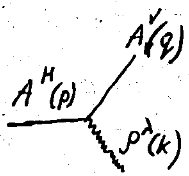
V. <u>Some Consequences of the Model.</u>

Various restrictions on the momentum dependence of amplitudes are often used in the algebra of currents. In our Lagrangian model there are such severe restrictions for processes with A_1, ρ and π mesons. The requirement of the minimum momentum dependence leads to some important consequences for the simplest n-point functions.

First, we present the results for 3-point functions. The $A_1 A_1 \rho$ interaction in Lagrangian (3.10) is described by

$$\mathcal{L}_{AA\rho} = \frac{g_A^2}{g_\rho}(1-C^2)\,\partial_\mu \vec{\rho}_\nu \cdot \vec{A}_\mu \times \vec{A}_\nu + g_\rho(\rho_\mu \times A_\nu - \rho_\nu \times A_\mu)\cdot \partial_\mu A_\nu \qquad (5.1)$$

The corresponding $A_1 A_1 \rho$ vertex $\Gamma_{\mu\nu\lambda}(p,q)$ is linear in momenta (i.e. D = 1)



$$\Gamma_{\mu\nu\lambda}(p,q) = -i g_\rho \big[g_{\mu\nu}(p+q)_\lambda - g_{\nu\lambda} p_\mu - g_{\mu\lambda} q_\nu + \\ + (2+\delta)(k_\mu g_{\nu\lambda} - k_\nu g_{\mu\lambda}) \big] \tag{5.2}$$

Here δ is the anomalous magnetic moment of the A_1 meson.

$$\delta = \frac{g_A^2}{g_\rho^2} \left\{ 1 - \left[\left(\frac{m_\rho}{f_\pi g_\rho} \right)^2 - 1 \right]^{-2} \right\} - 1, \tag{5.3}$$

where eq.(3.7) for C has been used. <u>If the KSFR relation ($2 g_\rho^2 f_\pi^2 = m_\rho^2$) is valid, we obtain $\delta = -1$.</u> This value for is the most preferable (as discussed in details in ref. /6/). In the conventional approach /1,2/ the anomalous magnetic moment of A_1 is not determined. To obtain the value (5.3) for δ it is suffieient to require $D \leqslant 2$ for any 3-point vertex.

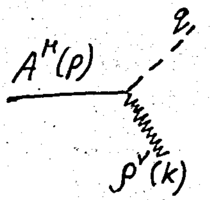
The $A_1 \rho \pi$ coupling in our Lagrangian (3.I0) is described by

$$\mathcal{L}_{A\rho\pi} = \frac{g_A}{g_\rho f_\pi} (\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu) \cdot \Big[C \partial_\nu \vec{\pi} \times \vec{A}_\mu + \Big(1 - \frac{g_\rho^2}{g_A^2 (C+1)} \Big) \partial_\mu \vec{A}_\nu \times \vec{\pi} \tag{5.4}$$

Using the relations /6/

$$g_A = g_\rho, \qquad 2 g_\rho^2 f_\pi^2 = m_\rho^2 \tag{5.5}$$

we obtain for the $A_1 \rho \pi$ vertex



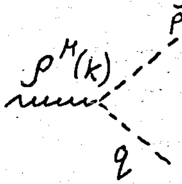
$$\Gamma_{\mu\nu}(p,q)=\frac{1}{f_\pi}\left[k_\mu\left(q_\nu-\frac{1}{2}p_\nu\right)-g_{\mu\nu}\,k\cdot\left(q-\frac{1}{2}p\right)\right] \qquad (5.6)$$

This gives the ratio $G_s/G_D = -m_\rho^2$, where G_s and G_D are the constants which determine S and D-wave decay of $A_1 \to \rho\pi$ With $\delta = -1$ one derives $\Gamma(A_1 \to \rho\pi) \approx 45\,MeV$

The $\rho\pi\pi$ coupling is very simple in this approach

$$\mathcal{L}_{\rho\pi\pi} = -g_\rho\,\rho_\mu \cdot \vec{\pi}\times\partial_\mu\vec{\pi} \qquad (5.7)$$

and the $\rho\pi\pi$ vertex $\Gamma_\mu(p,q)$ is linear in momenta



$$\Gamma_\mu(p,q) = -i\,g_\rho\,(p-q)_\mu \qquad (5.8)$$

while in refs./1,2/ it has D = 3. Eq.(5.8) gives $\Gamma(\rho\to 2\pi)\approx 135\,MeV$

Now we shall obtain expressions for some important 4-point functions.

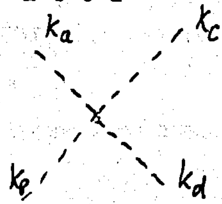
Since the symmetry is broken by the term $m_\pi^2 f_\pi^2\left[1-\left(\frac{\vec{\pi}}{f_\pi}\right)^2\right]^{1/2}$ then in Lagrangian (3.10) the $\pi\pi$ coupling is

$$\mathcal{L}_{\pi\pi}=\frac{1}{2f_\pi^2}\left[(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2-\frac{m_\pi^2}{4}(\vec{\pi}^2)^2\right]+\frac{g_\rho^2}{2m_\rho^2}\left[\vec{\pi}^2(\partial_\mu\vec{\pi})^2-(\vec{\pi}\,\partial_\mu\vec{\pi})^2\right] \qquad (5.9)$$

where the first term is the same as in ref.[3] and the second term is due to the inclusion of the fields ρ_μ and A_μ

Note that in the conventional Lagrangian[2] $\pi\pi$ coupling contains also the additional quadrilinear derivative term $(\partial_\mu \vec{\pi} \times \partial_\nu \vec{\pi})^2$.

From $\mathcal{L}_{\pi\pi}$ we obtain the contact $\pi\pi \to \vec{\pi}\vec{\pi}$ vertex T_{abcd} with $D = 2$



$$T_{abcd} = f_\pi^{-2} \left\{ \delta_{ab}\delta_{cd} \left[s - m_\pi^2 + \frac{f_\pi^2 g_\rho^2}{m_\rho^2}(t+u-2s) \right] + \text{(2 crossed terms)} \right\} \tag{5.10}$$

where $S = (K_a + K_b)^2$, $t = (K_a - K_d)^2$. The matrix element for $\pi\pi$ scattering is the sum of the contact term and the terms with the ρ-meson exchange. In this sum the terms containing g_ρ^2 cancel out at the threshold ($s = 4\, m_\pi^2$, $t = u = 0$) and we obtain for the S-wave scattering lengths:

$$a_0 \simeq 0.20\, m_\pi^{-1}, \quad a_2 = -0.06\, m_\pi^{-1} \tag{5.11}$$

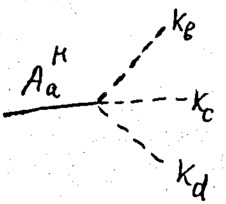
in accordance with those obtained by Weinberg[6] using current algebra, PCAC and smoothness condition which in effect corresponds to our restriction on the number of derivatives in the Lagrangian.

Note that in the conventional Lagrangian model[2] the contact $\pi\pi \to \pi\pi$ vertex has D = 4. Then the $\pi\pi$ scattering lengths cannot be fixed since it is possible to include in the Lagrangian the quadrilinear derivative term K ($\nabla_\mu \vec{\varphi}$)^4 (3.4) (K is an arbitrary constant) which would change a_0 and a_2 .

The direct $A_1 3\pi$ coupling in Lagrangian (3.I0) is

$$\mathcal{L}_{A3\pi} = \frac{g_A}{2f_\pi}\left[2(\vec{\pi}\partial_\mu\vec{\pi})(\vec{A}_\mu\vec{\pi}) - \vec{\pi}^{\,2}(\vec{A}_\mu\partial_\mu\vec{\pi})\right]. \tag{5.12}$$

The corresponding $A_1 3\pi$ vertex $T^{\mu}_{a\,b\,c\,d}$ is linear in momenta



$$T^{\mu}_{abcd} = \frac{i g_A}{f_\pi}\Big[\delta_{ab}\delta_{cd}(k_c+k_d-k_b)^\mu + \delta_{ac}\delta_{bd}(k_b+k_d-k_c)^\mu + \delta_{ad}\delta_{bc}(k_b+k_c-k_d)^\mu\Big], \tag{5.13}$$

while in the conventional approach /1/ it has D = 3.

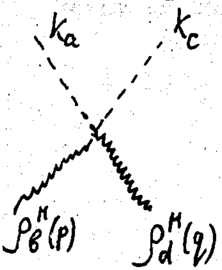
Eq.(5.13) gives for the partial width of the direct $A_1 \to 3\pi$ decay mode $\sim$ 35 MeV if relations (5.5) hold.

It is possible to estimate roughly the total width of A_1 adding the direct $A_1 \to 3\pi$ and $A_1 \to \rho\pi$ decay rates. This gives approximately 80 MeV in agreement with experimental data. More detailed predictions can be obtained from the explicit $A_1 \to 3\pi$ amplitude which contain both the contact term (5.13) and ρ -pole terms.

Relations (5.5) lead to the simple expression for $\rho\pi\rho\pi$ coupling:

$$\mathcal{L}_{\rho\pi\rho\pi} = \frac{g_\rho^2}{2}(\vec{\rho}_\mu \times \vec{\pi})^2 - \frac{g_\rho^2}{8m_\rho^2}\left[(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\times\vec{\pi}\right]^2 \qquad (5.14)$$

whioh gives the contact $\rho\pi\rho\pi$ vertex



$$T^{\mu\nu}_{abcd} = \frac{g_\rho^2}{m_\rho^2}\left(2\delta_{ac}\delta_{bd} - \delta_{ab}\delta_{cd} - \delta_{bc}\delta_{ad}\right)\cdot$$
$$\cdot\left[g^{\mu\nu}m_\rho^2 + \frac{1}{2}\left(p^\nu q^\mu - g^{\mu\nu}p\cdot q\right)\right] \qquad (5:15)$$

In conclusion we would like to emphasize the importance of the restriotion on the momentum dependenoe of the n-point functions in connection with the idea of algebraic realizations of the chiral symmetry /7/. We shall present the analysis of the asymptotio behaviour of tree graphs for the processes in the A_1 system elsewhere.

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Appendix

The elimination of the bilinear derivative terms from the covariants $\vec{R}_{\mu\nu}$ (3.1) and $\vec{S}_{\mu\nu}$ (3.2) leads to the elimination of the three- and quadrilinear derivative couplings from Lagrangian (3.4) at h = K = 0.

Let us introduce a general substitution for the axial-vector field:

$$\vec{a}_\mu = Z_A^{-\frac{1}{2}}\left[\vec{A}_\mu + F(\vec{\Phi}^2)\mathcal{D}_\mu\vec{\Phi} + G(\vec{\Phi}^2)\vec{\Phi}(\vec{\Phi}\partial_\mu\Phi)\right] \tag{A.1}$$

where only the value $F(0) = -g_\rho^2 f_\pi^2/g_A m_\rho^2$ is fixed by the requirement of the exclusion of the $\vec{A}_\mu\partial_\mu\vec{\Phi}$ coupling.

The functions $F(\vec{\Phi}^2)$, $G(\vec{\Phi}^2)$ are determined by the requirement that the bilinear derivative terms be eliminated from $\vec{R}_{\mu\nu}$, $\vec{S}_{\mu\nu}$ which gives the equations:

$$g_A F^2 - C^2(1+g_A\sigma F)^2 = 0,$$

$$\left[F g_A - C^2\sigma(1+g_A\sigma F)\right]G = 0 \tag{A.2}$$

$$g_A(G-2F') - C^2(1+g_A\sigma F)(\sigma^{-1}+g_A F + g_A G\vec{\Phi}^2) = 0$$

Eqs.(A.2) are consistent only with $C = \left[\frac{m_\rho^2}{g_\rho^2 f_\pi^2} - 1\right]^{-1}$ (3.8)

and we have unique solution (3.7): G = 0, $F = -\dfrac{C}{g_A(1+C\sigma)}$

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