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THE ACCOUNT OF CORRECTIONS TO THE EIKONAL APPROXIMATION IN THE QUASIPOTENTIAL APPROACH

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1. Introduction

In previous papers[1-3] a relativistic approach to particle scattering at high energies was developed which was based on the quasipotential equation for the scattering amplitude in quantum field theory[4,5].

In the case of scattering of two spinless particles of equal masses the quasipotential equation reads:

$$T(\vec{p}, \vec{k}; E) = V[(\vec{p}-\vec{k})^2; E] + \int \frac{d\vec{q}}{\sqrt{m^2+\vec{q}^{\,2}}} \frac{V[(\vec{p}-\vec{q})^2; E)\, T(\vec{q}, \vec{k}; E)}{\vec{q}^{\,2}+m^2-E^2-i0}, \tag{1.1}$$

where E is the energy, $\vec{p}$ and $\vec{k}$ are the center of mass system relative momenta of the initial and final states, respectively.

An assumption of the nonsingular behaviour of the local quasipotential[6]:

$$V(E; \vec{r}) = \int d\vec{\Delta}\, e^{i\vec{\Delta}\vec{r}}\, V(\vec{\Delta}^2; E) \tag{1.2}$$

leads to the Glauber or eikonal representation for the small angle scattering amplitude[x]:

[x] On the energy shell $\vec{\Delta}_{\perp}=(\vec{p}-\vec{k})_{\perp}$ lies in a plane orthogonal to the vector $(\vec{p}+\vec{k})$.

$$T(\vec{\Delta}^2; E) = \frac{s}{(2\pi)^3} \int d^2\rho\, e^{i\vec{\rho}\vec{\Delta}_\perp} \frac{e^{2i\chi} - 1}{2i} \tag{1.3}$$

with eikonal function

$$\chi = \frac{1}{s} \int_{-\infty}^{\infty} V(E; \vec{r})\, dz; \qquad \vec{r} = (\vec{\rho}; z). \tag{1.4}$$

In deriving the representation (1.3) terms were neglected which decrease as $1/p$ or faster, as compared to the leading contributions at high energy. Such contributions, unessential at ultrahigh energies, may turn out to be important when dealing with various effect in the subasymptotic region (behaviour of real parts of the scattering amplitudes, polarizations, etc.).

In the present work we study the problem of taking into account of the corrections to the elastic scattering amplitude of the relative order $1/p$, considering the pure imaginary quasipotential, which increases as s at high energies[1-3,7].

The problem of the series expansion of the scattering amplitude in terms of inverse powers of momentum has been discussed in a number of papers[8-12] in the framework of the usual potential scattering.

In ref.[13] second Born approximation to the scattering phase was considered and the contribution to the scattering amplitude arising from it was discussed.

2. Quasipotential Equation in Coordinate Space

We shall use in what follows the quasipotential equation for the wave function of two particles in configuration space [14], which has the form of non-local differential equation:

$$(\vec{\nabla}^2 + \vec{p}^2)\,\psi_{\vec{p}}(\vec{r}) = \frac{-1}{\sqrt{m^2-\vec{\nabla}^2}}\, V(E;\vec{r})\,\psi_{\vec{p}}(\vec{r}) \tag{2.1}$$

with quasipotential of the following type :

$$V(E;\vec{r}) = 2ipE\ v(\vec{r}), \tag{2.2}$$

where $v(\vec{r})$ is a smooth positive function of $\vec{r}$, $|\vec{p}| = p$.

We look for the solution of equation (2.1) corresponding to the small angle scattering of high energy particles in the form:

$$\psi_{\vec{p}}(\vec{r}) = e^{ipz}\ F_{\vec{p}}(\vec{r}) \tag{2.3}$$

with boundary condition for the function $F_{\vec{p}}(\vec{r})$:

$$F_{\vec{p}}(\vec{r})\,\big|_{z\to\infty} = 1\ . \tag{2.4}$$

An assumption of smooth behaviour of the quasipotential (2.2) allows the non-local differential equation (2.1) to be reduced to a local one.

Indeed, multiplying equation (2.1) from the left by e^{-ipz} and making use of the operator expansions

$$e^{-ipz}(\vec{\nabla}^2+\vec{p}^{\,2})\,e^{ipz} = 2ip\partial_z + \vec{\nabla}^2 = (\partial_z + ip)^2 + \vec{\nabla}_\perp^2 + p^2 \tag{2.5a}$$

$$e^{-ipz}\frac{1}{\sqrt{m^2-\vec{\nabla}^2}}\,e^{ipz} = \frac{1}{E}\,[1+\frac{i\partial_z}{p} + 0(1/p^2)\,] \tag{2.5b}$$

we are led to the following local differential equation for $F_{\vec{p}}(\vec{r})$ (neglecting terms of order $1/p$):

$$(2ip\partial_z + \vec{\nabla}^2)\;F_{\vec{p}}(\vec{r}) = -2ip(1+\frac{i\partial_z}{p})v(\vec{r})\;F_{\vec{p}}(\vec{r})\;. \tag{2.6}$$

Seeking the solution of eq. (2.6) in the form:

$$F_{\vec{p}}(\vec{r}) = \exp\,[\,-\int_{-\infty}^{z}\chi\;(\vec{\rho}\,,z')\,dz'\,] \tag{2.7}$$

we get the nonlinear integro-differential equation for the function χ:

$$\begin{aligned}2ip\;\chi(\vec{\rho},z) &= 2ip\,(1+\frac{i\partial_z}{p})\;v(\vec{r}) \;+ \\ &+2v(\vec{r})\;\chi(\vec{\rho},z) - \partial_z\chi(\vec{\rho},z) + \chi^2(\vec{\rho},z) - \eta_\perp[\vec{r};\chi]\;,\end{aligned} \tag{2.8}$$

where,

$$\eta_\perp[\vec{r};\chi] = \int_{-\infty}^{z}\vec{\nabla}_\perp^2\;\chi\;(\vec{\rho},z')\;dz' - [\int_{-\infty}^{z}\vec{\nabla}_\perp\;\chi\,(\vec{\rho},z')\,dz'\,]^2. \tag{2.9}$$

Expanding χ in terms of inverse powers of p

$$\chi(\vec{\rho}, z) = \chi^{(0)}(\vec{\rho}, z) + \frac{1}{2ip}\chi^{(1)}(\vec{\rho}, z) + \dots \tag{2.10}$$

we obtain[x/]

$$\chi^{(0)}(\vec{\rho}, z) = v(\vec{r}) \tag{2.11}$$

$$\chi^{(1)}(\vec{\rho}, z) = -3[\partial_z v(\vec{r}) - v^2(\vec{r})] - \eta_\perp[\vec{r}\;;\;v] =$$

$$= -3[\partial_z v(\vec{r}) - v^2(\vec{r})] - \int_{-\infty}^{z} \vec{\nabla}_\perp^2 v(\vec{\rho}, z')dz' + [\int_{-\infty}^{z} \vec{\nabla}_\perp v(\vec{\rho}, z')dz']^2. \tag{2.12}$$

Note that

$$\chi^{(1)}(\vec{\rho}, z)\Big|_{z\to\infty} = \gamma(\vec{\rho}) = -\int_{-\infty}^{\infty} \vec{\nabla}_\perp^2 v(\vec{\rho}, z')dz' + [\int_{-\infty}^{\infty} \vec{\nabla}_\perp v(\vec{\rho}, z')dz']^2 \tag{2.13}$$

and the integral in (2.7) diverges when $z \to \infty$.

[x/] It is interesting to note that in the absence of the relativistic factor $1/\sqrt{m^2 - \vec{\nabla}^2}$ in eq. (2.1) the first term in (2.12) turns out to be three times smaller.

Because of this it is convenient to introduce the quantity

$$\tilde{\tilde{\chi}}^{(1)}(\vec{\rho}, z) = \chi^{(1)}(\vec{\rho}, z) - \theta(z)\,\gamma(\vec{\rho}) \qquad (2.14)$$

so that

$$\int_{-\infty}^{z} \chi^{(1)}(\vec{\rho}, z')\,dz' = \int_{-\infty}^{z} \tilde{\tilde{\chi}}^{(1)}\,dz' + z\,\theta(z)\,\gamma(\vec{\rho}) \;. \qquad (2.15)$$

The integral on the right hand side of eq. (2.15) is convergent when $z \to \infty$.

Thus the solution of eq. (2.6) with an accuracy of $1/p$ is of the form:

$$F_{\vec{p}}(\vec{r}) = \exp\left[-\frac{z\,\theta(z)}{2ip}\,\gamma(\vec{\rho}) - \int_{-\infty}^{z} v(\vec{\rho}, z')\,dz' - \frac{1}{2ip}\int_{-\infty}^{z} \chi^{(1)}(\vec{\rho}, z')\,dz'\right]. \qquad (2.16)$$

The connection of the wave function ψ with the scattering amplitude reads:

$$T(\vec{\Delta}^2; E) = \frac{1}{(2\pi)^3}\int d\vec{r}\;\psi_{\vec{k}}^{(0)*}\,V(E;\vec{r})\,\psi(\vec{r}) \qquad (2.17)$$

$$\vec{\Delta}^2 = (\vec{p} - \vec{k})^2 .$$

Making use of (2.16) we get the following expression for the small angle scattering amplitude at high energy:

$$T(\vec{\Delta}^2; E) = 2ipE\,\frac{1}{(2\pi)^3}\int d\overset{2}{\rho}\,dz\;e^{i\vec{\rho}\vec{\Delta}_{\perp}}\,e^{iz\left[\Delta_z + \frac{\theta(z)}{2p}\gamma(\rho)\right]}$$

$$\times\, v(\vec{r})\;e^{-\int_{-\infty}^{z} v(\vec{\rho}, z')\,dz' - \frac{1}{2ip}\int_{-\infty}^{z} \tilde{\tilde{\chi}}^{(1)}(\vec{\rho}, z')\,dz'} \;. \qquad (2.18)$$

Here $\vec{\Delta}^2 = \vec{\Delta}_\perp^2 + \Delta_z^2$, and on the energy shell $\Delta_z = \frac{\vec{\Delta}_\perp^2}{2p} + 0(1/p^2)$.

Expansion of the integrand in (2.18) in terms of inverse powers of the momentum leads to the following expansion for the scattering amplitude:

$$T(\vec{\Delta}^2; E) = T^{(0)}(\vec{\Delta}^2; E) + \frac{1}{2ip} T^{(1)}(\vec{\Delta}^2; E), \qquad (2.19)$$

where

$$T^{(0)} = -2ipE \frac{1}{(2\pi)^3} \int d^2\rho \, e^{i\vec{\rho}\vec{\Delta}_\perp} \left(e^{-\int_{-\infty}^{\infty} v(\vec{\rho}, z')dz'} - 1 \right) \qquad (2.20)$$

is the eikonal approximation of the scattering amplitude and

$$T^{(1)} = -2ipE \frac{1}{(2\pi)^3} \int d^2\rho \, dz \, e^{i\vec{\rho}\vec{\Delta}_\perp} v(\vec{r}) \, e^{-\int_{-\infty}^{z} v(\vec{\rho}, z')dz'} \times$$

$$\times \left[z\,\theta(z)\,\gamma(\vec{\rho}) + z\,\vec{\Delta}_\perp^2 + \int_{-\infty}^{z} \chi^{(1)}(\vec{\rho}, z')\, dz' \right] \qquad (2.21)$$

is the first correction to it.

Integrating by parts one can reduce expression (2.21) to the form

$$T^{(1)} = 2ipE \frac{1}{(2\pi)^3} \Big\{ \int d^2\rho \, e^{i\vec{\rho}\vec{\Delta}_\perp} \, e^{-\int_{-\infty}^{\infty} v(\vec{\rho}, z')dz'} \, 3 \int v^2(\vec{\rho}, z)\, dz \, +$$

$$+ \int d^2\rho \, e^{i\vec{\rho}\vec{\Delta}_\perp} \int_{-\infty}^{\infty} dz \, \eta_\perp [\vec{r}; v] \left(e^{-\int_{-\infty}^{z} v(\vec{\rho}, z)dz'} - e^{-\int_{-\infty}^{\infty} v(\vec{\rho}, z')dz'} \right)$$

$$- \int d^2\rho \, e^{i\vec{\rho}\vec{\Delta}_\perp} \vec{\Delta}_\perp^2 \int dz \, z\, v(\vec{\rho}, z) \, e^{-\int_{-\infty}^{z} v(\vec{\rho}, z')dz'} \qquad (2.22)$$

where the quantity $\eta_\perp [\vec{r}; v]$ is defined by (2.12).

3. Momentum Space Representation

In this section we study the problem of the type of corrections to the eikonal approximation, using the quasipotential equation (1.1) for the scattering amplitude in the momentum-space representation.

For the sake of simplicity we choose the quasipotential $V(E;\vec{\Delta}^2)$ in the Gaussian form

$$V\,(E\,;\,\vec{\Delta}^2\,) = isg\;e^{at} \qquad ; \qquad t = -\vec{\Delta}^2 \tag{3.1}$$

and restrict ourselves to considering forward scattering.

An expression for the $(n+1)$-th Born approximation of the zero-angle scattering amplitude can be written in the following form:

$$T_{(n+1)} = (isg)^{n+1} \int \prod_{i=1}^{n} d^2\Delta_i \; e^{-a[\vec{\Delta}_1^2 + (\vec{\Delta}_1 - \vec{\Delta}_2)^2 + \ldots + \vec{\Delta}_n^2]} \times$$

$$\times \int \prod_{k=1}^{n} \frac{dq_k}{\sqrt{m^2 + \vec{\Delta}_k^2 + (p+q_k)^2}} \; \frac{e^{-a[q_1^2 + (q_1 - q_2)^2 + \ldots + q_n^2]}}{(p+q_k)^2 + \vec{\Delta}_k^2 - p^2 - i0} \tag{3.2}$$

$\vec{\Delta}_i$ is the transverse and $(p+q_i)$ the longitudinal relative momenta in the i-th intermediate state.

In the high energy limit the kinematical factors entering the expression (3.2) can be represented in the form

$$\frac{1}{\sqrt{m^2+\vec{\Delta}_i^2+(p+q_i)^2}} \qquad \frac{1}{(p+q_i)^2+\vec{\Delta}_i^2-p^2-i0} \approx$$

$$\approx \frac{1}{2p^2}\left(1-\frac{q_i}{p}\right)\left[\frac{1}{q_i+\frac{\vec{\Delta}_i^2}{2p}-i0}-\frac{1}{q_i+2p-\frac{\vec{\Delta}_i^2}{2p}+i0}\right]. \tag{3.3}$$

Representation (1.3) is obtained retaining only the lowest order terms in the expansions (3.3).

Taking into account these factors in the form given by eq. (3.3), the $(n+1)$ -th Born approximation is obtained as a series expansion in powers of $1/p$. The whole amplitude takes the form:

$$T(\vec{\Delta}^2=0; E) = T^{(0)} + \frac{1}{2ip} T^{(1)} + \dots , \tag{3.4}$$

where $T^{(0)}$ is determined by (1.3) and $T^{(1)}$ can be written as

$$T^{(1)} = \frac{3}{2} isg \frac{\pi g}{a^2\sqrt{8\pi a}} \int d^2\rho\, e^{-\frac{\vec{\rho}^2}{4a}} e^{2i\chi} +$$

$$+ isg \frac{1}{4\pi a^2} \int d^2\rho\, e^{-\frac{\vec{\rho}^2}{4a}} \left(1-\frac{\vec{\rho}^2}{4a}\right) \times \tag{3.5}$$

$$\times \int_{-\infty}^{\infty} dz\, z\, V(z) \left[\exp\left(2i\chi \int_{-\infty}^{z} V(z')dz'\right) - \exp\left(2i\chi \int_{z}^{\infty} V(z')\,dz'\right)\right],$$

where

$$2i\chi = -\frac{4\pi^2 g}{a}\, e^{-\frac{\vec{\rho}^{\,2}}{4a}} \qquad (3.6)$$

$$V(z) = \frac{1}{\sqrt{4\pi a}}\, e^{-\frac{z^2}{4a}} \,. \qquad (3.7)$$

In the case of a pure imaginary quasipotential the corrections of relative order $1/p$ to the eikonal approximation are pure real. The minima in their angular distributions are displaced at values of $|t|$, not coinciding with the minima of the leading contributions 1,3). This leads to the so-called "filling of minima" in the elastic scattering differential cross section.

It is worth to note in this connection that for the detailed analysis of experimental data in terms of eikonal type expansions with Born approximation given, for instance, in the form of Regge Pole contributions /15-18/, it is necessary to take into account corrections to the elastic scattering amplitude at least of order $1/p$, since Born terms corresponding to nonzero quantum number exchanges at $t = 0$ are of the same order of smallness as compared to the main contributions.

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Учёт поправок к эйкональному приближению в квазипотенциальном подходе

В работе изучаются поправки к эйкональному приближению в квазипотенциальном подходе относительного порядка $1/p$ по сравнению с главными вкладами при высоких энергиях.

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The Account of Corrections to the Eikonal Approximation in the Quasipotential Approach

The corrections to the eikonal approximation in the quasipotential approach of the relative order $1/p$ as compared to leading contributions at high energies are studied.

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