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ALGEBRAIC REALIZATION OF UNITARY SYMMETRY

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It is tempting to suppose that the SU_3 group is realized nonlinearly and so the unitary symmetry is broken[2], the hypothetic scalar κ -mesons being Goldstone mesons. Only the good isospin-hypercharge subgroup $SU_2 \times Y$ must be represented linearly ensuring the conservation of the isospin and hypercharge. According to the general theory of nonlinear realizations[3] the covariant derivative of kappaons $\kappa = 1/\sqrt{2} \begin{pmatrix} \kappa_4 - i\kappa_5 \\ \kappa_6 - i\kappa_7 \end{pmatrix}$ is

$$\nabla_\mu \kappa_a = \partial_\mu \kappa_a + O(\kappa^3) \qquad (a = 4, 5, 6, 7) \tag{1}$$

and the covariant derivative of any field ψ is determined by its isospin T and hypercharge Y

$$\nabla_\mu \psi = \partial_\mu \psi + 2i F_\kappa^{-2} \kappa_a \partial_\mu \kappa_b (f_{abi} t_i + f_{ab8} t_8) \psi + O(\kappa^4) , \tag{2}$$

where F_κ is the decay constant (similar to F_π), t_i ($i = 1,2,3$) are isospin matrices appropriate to ψ, $t_8 = \frac{\sqrt{3}}{2} Y$, f_{abi}, f_{ab8} are the SU_3 structure constants and $O(\kappa^3)$, $O(\kappa^4)$ stand for the terms of the 3rd and 4th order in κ's. For the SU_3 invariance it is only necessary that a Lagrangian should conserve the isotopic spin and hypercharge and be constructed of various isomultiplets

ψ , $\nabla_\mu \psi$ and $\nabla_\mu \kappa$. At the first glance such an approach seems needless as the SU_3 classification and all other algebraic consequences of SU_3 fail completely except those connected with its good $SU_2 \times Y$ subgroup. However, for the case of the chiral $SU_2 \times SU_2$ group Weinberg /1/ had shown that algebraic properties will reappear if one imposes the following additional requirement: the forward scattering amplitudes must have a reasonable asymptotic behaviour at high energies. In the present paper we shall consider the algebraic realization of the unitary symmetry SU_3 (ARUS below) following Weinberg's programme /1/.

The form of covariant derivative (2) dictates that a Lagrangian must contain the minimal contact coupling

$$2F_\kappa^{-2}\, \kappa_a\, \partial_\mu\, \kappa_b (f_{abi} V^i_\mu + f_{ab8} V^8_\mu) \quad (a,b = 4,5,6,6;\ i = 1,2,3)\ , \tag{3}$$

where V^i_μ and V^8_μ are conserved vector isotopic and hypercharge currents, $\int V^i_0(x)\, d^3x = T^i$, $\int V^8_0(x)\, d^3x = T^8 = \frac{\sqrt{3}}{2} Y$.

The nonminimal coupling can be written in the form

$$F_\kappa^{-1} V^a_\mu \nabla_\mu \kappa_a \ , \tag{4}$$

where V^a_μ is a phenomenological strangeness changing vector current (there can be also nonminimal couplings containing a higher number of $\nabla_\mu \kappa$ which are nonessential for us). Consider the forward scattering $\kappa + \alpha \to \kappa + \beta$ where α and β are any particles or resonances, neglecting the mass of κ . Let $p_\mu = (-\vec{n}p, p_0)$, $p'_\mu = (-\vec{n}p', p'_0)$, $q_\mu = n_\mu \omega$, $q'_\mu = n_\mu \omega'$ $(|\vec{n}| = n_0 = 1)$ be 4-momenta of α , β , initial and final κ's respectively. The conservation of 4-momentum gives $p + p_0 = p' + p'_0 = E$.

We are studying the behaviour of the invariant amplitude $M_{\beta b, \alpha a}(\omega, \lambda)$ for a given helicity λ as a function of the kappaon energy ω keeping E fixed (a and b are indices of the initial and final κ's). The "strange" current V^a_μ is parametrized according to

$$\langle p'\lambda'\beta |\, n_\mu V^a_\mu(0)\, | p\lambda\alpha \rangle = (2\pi)^{-3}(4p_0 p'_0)^{-1/2} 4E\,\delta_{\lambda\lambda'} (X^a(\lambda))_{\beta\alpha} \ . \tag{5}$$

and this defines the important coupling matrix $X^a(\lambda)$.

Consider the tree Feynman graphs, i.e. graphs without closed loops. Each of the graphs gives a contribution to the amplitude which grows unadmissibly at high energies. Following /1/ we require for the sum of all tree graphs to give the amplitude which grows no faster than would be expected by the Regge-pole theory.

The calculations are similar to those of Weinberg /1/ and we omit them. From the above requirement it follows that

$$[X^a(\lambda), X^b(\lambda)] = i f_{abi} T^i + i f_{ab8} T^8 \ , \tag{6}$$

$$[X^a(\lambda), [X^b(\lambda), m^2]] = -\frac{4}{3} f_{ac8} f_{bd8} [X^c(\lambda), [X^d(\lambda), m^2]] \ , \tag{7}$$

where m^2 is the diagonal mass matrix $m^2_{\gamma\alpha} = m^2_\alpha \delta_{\gamma\alpha}$.

If in addition we assume that for exotic exchanges of the hypercharge ± 2 the intercept $\alpha_{Y=\pm 2}(0) < -1$ (eq. (7) corresponds to $\alpha_{Y=\pm 2}(0) < 0$), which seems to be reasonable /4/ we obtain one more relation:

$$[[X^a(\lambda), m^2], [X^b(\lambda), m^2]] = -\frac{4}{3} f_{ac8} f_{bd8} [[X^c(\lambda), m^2], [X^d(\lambda), m^2]] \ . \tag{8}$$

Taken together with the isospin-hypercharge conservation relations $[T^i, X^a] = i f_{iab} X^b$, $[T^8, X^a] = i f_{8ab} X^b$, relation (6) means that the operators X^a, T^i and T^8 form the SU_3 algebra. Hence, particles included in tree graphs must, for each helicity, furnish irreducible or reduicible representation of the SU_3 group. In other words the eighfold way classification of particles is reestablished. Sum rule (7) states that $m^2 = m^2_{inv} + m^2_8$, where $[X^a, m^2_{inv}] = 0$ and $m^2_8 = \frac{1}{3}[X^c,[X^c, m^2]]$ is the 8th component of the octet (other octet components are $m^2_a = 4/3\, i f_{a8b}[X^b, m^2]$, $m^2_i = \frac{2}{\sqrt{3}} d_{abi}[X^a[X^b, m^2]]$, $a, b = 4,5,6,7$; $i = 1,2,3$). Thus, the squared mass operator has been proved to be the sum of unitary invariant and the 8th component of octet. We have obtained the Gell-Mann-Okubo formula without assuming m^2_8 to be small in comparison with m^2_{inv}. Note the importance of the A_2 trajectory without which the exact SU_3 symmetry would be reestablished for masses.

Relation (8) is satisfied identically for decuplets and for the octet-singlet mixing (8) is written as

$$(m^2_\Xi - m^2_\Sigma)(m^2_\Sigma - m^2_N) = (m^2_\Xi - m^2_\Lambda)(m^2_\Lambda - m^2_N)\cos^2\theta + (m^2_\Xi - m^2_{\Lambda_1})(m^2_{\Lambda_1} - m^2_N)\sin^2\theta \tag{8'}$$

Eliminating the mixing angle θ from (8') and from the Gell-Mann-Okubo formula

$$2m^2_N + 2m^2_\Xi = m^2_\Sigma + 3m^2_\Lambda \cos^2\theta + 3m^2_{\Lambda_1}\sin^2\theta \tag{7'}$$

(which follows from (7)) we obtain the relation

$$2(m^2_\Xi + m^2_N)(m^2_\Lambda + m^2_{\Lambda_1} + 2m^2_\Sigma) = 2(m^2_\Sigma + m^2_N) + 3m^2_\Lambda m^2_{\Lambda_1} + m^2_\Sigma(m^2_\Lambda + m^2_{\Lambda_1} + 3m^2_\Sigma). \tag{9}$$

(For bosons one has to replace m^2_Ξ and m^2_N by m^2_K, m^2_Λ by m^2_η, m^2_Σ by m^2_π).

The necessity of nonets. In the case of the pure octet $\theta = 0$ and we obtain from (7) and (8) (or (7) and (9)) $m^2_\Lambda = m^2_\Sigma$. So a pure octet is possible if the $Y = 0$ states with $T = 0$ and $T = 1$ have the same mass. In this case $m^2_\Xi + m^2_N = 2m^2_\Lambda = 2m^2_\Sigma$ for fermions and $m^2_K = m^2_\eta = m^2_\pi$ for bosons.

Mass Relation (9) is satisfied to a great accuracy for well-established nonets. For the nonet 1^- eq. (9) gives for the mass of the ϕ -meson 1010 MeV (accuracy of about 1%). For the nonet 2^+(K (1420), f (1260), f ′(1514), A_2(1320)) we obtain for the mass of the f ′-meson 1500 MeV (again accuracy is about 1%). On these grounds we shall discuss other possible nonets.

The 0^- nonet. Because of large differences in masses of K , η and π -mesons there must be the ninth 0^- meson. There are two possibilities, E (1420) and X (958) . By using eq. (9) we obtain m_η = 544 MeV (of about 1% accuracy) for E(1420) and m_η = 495 MeV for X (958) . So ARUS prefers E (1420) as the 9th pseudoscalar meson. Note that the broken $SU(6)_W$ symmetry tells the same /5/. The spin-parity of X (958) has not been firmly established till now /5,6/. It can amount either to 0^- or to 2^-. Suppose it is 2^-. In the possible nonet 2^- we can put together with X (958) the resonances π_A(1640) and K_A(1775) /6/. Then the mass of the ninth 2^- meson will be 1835 MeV. The resonance of about the same mass η_A (1830) is referred to in /6/ and ARUS predicts its spin parity to be 2^-.

In a nonet 1^+ containing K_A(1240), D (1285) and A_1(1070) resonances the ninth meson must be very heavy, eq. (9) gives for its mass 2400 MeV. Note that there are indications in favour of the existence of the resonance $\bar{N}\, N_{I=0}$(2380) /6/.

The nonet $1/2^+$. Σ and Λ particles have different masses and because of this ninth baryon $1/2^+$ of the mass about 1310 MeV is predicted (if one takes the mean square of mass in isomultiplets Σ and Ξ). This value of mass is sensitive to small deviations in the masses of N, Ξ, Λ and Σ as the mixing angle is small. Mass relation (9) is well satisfied for the 9th baryon mass in a region 1280-1340 MeV. At present there are some evidence in favour of the existence of the Λ -particle of such a mass, $Y_0^*(1327) \to \Lambda + \gamma$ [7]. In the nonet $1/2^-$ containing $N(1535)$, $\Lambda(1405)$, $\Lambda'(1670)$, $\Sigma(1750)$ resonances for the mass of the Ξ member we obtain two solutions (eq. (9) is quadratic for m_Ξ^2): either $m_\Xi = 1800$ MeV, $|\theta| \approx 18^\circ$, or $m_\Xi = 1710$ MeV, $|\theta| \approx 35^\circ$. By analyzing $\Lambda(1405)$ and $\Lambda'(1670)$ decays Tripp has found the mixing angle $\theta = -18 \pm 3^\circ$ (in accordance with the first solution) but Levi Setti[8] has found $\theta = -36.5 \pm 4^\circ$ (in accordance with the second solution). In the nonet $3/2^-$ containing $N(1520)$, $\Lambda(1690)$, $\Lambda'(1520)$, $\Sigma(1670)$ we again have two solutions for the mass of Ξ either $m_\Xi = 1830$ MeV, or $m_\Xi = 1650$ MeV. Levi Setti[6,8] has included in $3/2^-$ nonet $\Xi(1820)$ in accordance with the first solution. We restrict ourselves to these nonets as others do not contain the sufficient number of established members at present.

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