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EXCHANGE QUASIPOTENTIAL FOR THE HIGH-ENERGY $\pi^{\pm}p$-SCATTERING AND THE PROBLEM OF LINEARITY OF THE REGGE TRAJECTORIES

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EXCHANGE QUASIPOTENTIAL FOR THE HIGH-ENERGY $\pi^{\pm}p$-SCATTERING AND THE PROBLEM OF LINEARITY OF THE REGGE TRAJECTORIES

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§1. Introduction

Recent measurements performed by the IHEP-CERN collaboration at the Serpukhov accelerator gave a number of important results about the behaviour of the total cross sections of the interaction of π^- and K^--mesons and antiprotons with protons and deuterons/1/.

It was established that in the region of laboratory energy $p_L = (25\text{-}65)$ GeV/c:

i) The total cross sections $(\pi^{\pm} p, K^{\pm} p)$ are almost energy independent;

ii) The differences of the total cross sections for interaction of particles and antiparticles with protons deviates appreciably from zero and has no tendency to vanish with increasing energy[x].

Such a behaviour of the total cross sections contradicts the notion that the high-energy particle scattering is defined by the exchange of the finite number of Regge poles, and leads to the necessity of a more detailed theoretical analysis of the asymptotic of the total cross-sections.

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[x] This conclusion was made by extrapolating the pp-, K^+p-interaction total cross section data from the energy region 25 GeV/c, where they are approximately constant, to the region of the Serpukhov energies.

Besides, use has been made of the Glauber correction theory for extracting the total $\pi^+ p$-scattering cross-section from the $\pi^- d$-scattering data as well as of the charge independence principle $\sigma_{\pi^+ p} = \sigma_{\pi^- n}$.

There arises an interesting problem of theoretical interpretation of the observed significant difference of the total cross section for scattering of particles and antiparticles on protons at high energies.

In the present note we use the experimental data obtained at the Serpukhov accelerator to analyse the behaviour of the cross section difference $\Delta\sigma_{\pi p} = \sigma_{\pi^- p} - \sigma_{\pi^+ p}$ and the differential cross sections for the charge-exchange process $\left(\frac{d\sigma}{dt}\right)_{\pi^- p \to \pi^0 n}$. This investigation has been performed on the basis of the quasipotential theory of high-energy scattering, which was developed in papers/2-4/.

As is known, the assumption on the smoothness of the quasipotential for the elastic scattering/5/ makes it possible to reproduce the main regularities of the elastic scattering at high energies/6-8/. The main task of this note is the construction of an "exchange" quasipotential describing pion-nucleon scattering with the exchange of isotopic spin $(I=1)$ in the t-channel.

Some rather general integral representation for the exchange quasipotential, which includes the singular and non-singular quasipotentials is used.

As an example, a more detailed Veneziano-type representation/9-10/ for the exchange quasipotential is considered.

It will be shown, that in the framework of this model the experimental data on the total cross-section difference $\Delta\sigma_{\pi p}$ lead to nonlinear fermion Regge-trajectories with quantum numbers of the πN system[x]/.

The slope of the trajectory in a high-energy region $(p_L = (8 - 65$ GeV/c) is described well by the following empirical formula

$$\alpha_F'(s) = 0{,}8(1+\gamma\cdot s)\ (\mathrm{GeV}/c)^{-2}$$
$$\gamma = 0{,}02\ (\mathrm{GeV}/c)^{-2}\ . \qquad (1.1)$$

[x]/Detailed experimental and theoretical analysis of the problem of linearity of Regge trajectories is given in/11/.

The account of the rescattering effects on the elastic potential gave a correction of about 10% to the values of the parameters in eq. (1.1).

Note, however, that the exact account of the rescattering effects in the region of Serpukhov energies requires the knowledge of the Real parts of the scattering amplitude, polarizations, energy dependence of the diffraction slopes-parameters, etc.

As is seen from eq. (1.1), nonlinearity of the Regge-trajectories can be essential at energies $p_L \gtrsim 1/2m\gamma \approx 25\ \frac{\mathrm{GeV}}{c}$. In the region of nucleon resonances $p_L < 5$ GeV/c the trajectory (1.1) does not practically deviate from a linear one with $\alpha_F' \approx 1\left(\frac{\mathrm{GeV}}{c}\right)^{-2}$. We note the series of a recent works/12,13/ in which the attempts of the explaining of Serpukhov experimental data in the frame-work of complex angular momentum theory was made.

In the assume of such attempts lies that or another choice of parameters, which define the contributions of the cuts in the complex angular momentum plane, connected with the exchange of the several Pomeranchuk-poles, and taking into account the contribution of the multiparticle "streams" of "jets" /12/. Notice, also paper/14/ in which the similar results were obtained for the case of pp and $\bar{p}p$ scattering on the basis of the usual eikonal approach.

It should be stressed that the complex potential, which describes two-particle scattering, takes into account all the inelastic channels, and inclusion of additional many-particle contributions (streams, jets, etc.) can lead to the contradiction, which is near to the "double counting" problem, in FESR/15,16/.

Perhaps, more crucial for the theory is the analysis of the differences of the total cross sections of particle and antiparticle scattering and the differential cross section for the processes of charge-exchange type, in which the elastic scattering effects do not contribute.

§2. Exchange Quasipotential for the $\pi^{\pm}p$ Scattering

The crossing-odd part of the scattering amplitude $T^{(-)}(s,t)$ which determines the difference of the total cross sections of the $\pi^{\pm}$-mesons scattering on the proton and the differential cross section of the charge-exchange process $\pi^- p \to \pi^0 n$:

$$\Delta\sigma_{\pi p} = \sigma_{\pi^- p} - \sigma_{\pi^+ p} = \frac{1}{p\sqrt{s}}\, \mathrm{Jm}\, T^{(-)}(s, t=0) \tag{2.1}$$

$$\left(\frac{d\sigma}{dt}\right)_{\pi^- p \to \pi^0 n} = \frac{1}{\pi} \left| \frac{T^{(-)}(s,t)}{2p\sqrt{s}} \right|^2 \tag{2.2}$$

are connected with the quasipotential scattering amplitudes/17,18/ by:

$$T^{(-)}(s;\vec{p},\vec{k}) = \frac{1}{2}\left[T_{\pi^- p}(s;\vec{p},\vec{k}) - T_{\pi^+ p}(s;\vec{p},\vec{k})\right]$$

$$\bar{T}^{(-)}(s,t) = 32\pi^3\, T^{(-)}(s;\vec{p},\vec{k})\Big|_{\substack{\vec{p}^{\,2}=\vec{k}^{\,2} \\ s=(\omega_p+w_p)^2 \\ t=-(\vec{p}-\vec{k})^2}}, \tag{2.3}$$

where $\omega_p = \sqrt{\mu^2+\vec{p}^{\,2}}$ and $w_p = \sqrt{M^2+\vec{p}^{\,2}}$ – the energies of the pion and nucleon in the c.m.s.

The quasipotential equation for the amplitude $T^{(-)}(s;\vec{p},\vec{k})$ is determined by the exchange quasipotential $V^{(-)}(s,t) = V^{T=1}(s,t)$, describing the pion-nucleon scattering with an exchange of the isotopic spin $T=1$ in the t-channel:

$$T^{(-)} = V^{T=1} + V^{T=1} \times T^{(+)} + V^{(+)} \times T^{(-)}. \tag{2.4}$$

The solution of this equation has the form x/

$$T^{(-)}(s,\vec{p},\vec{k}) = V^{T=1}(s, -(\vec{p}-\vec{k})^2) + \int d\vec{q}\, \frac{\omega_q + w_q}{\omega_q \cdot w_q}\, \frac{V^{T=1}\, V^{(+)}}{(\omega_q + w_q)^2 - s - i0} + \cdots, \tag{2.5}$$

where we omit the terms, which corresponds to the double rescattering effects on the elastic quasipotential $V^{(+)}(s,t)$. As is known, the elastic scattering at small angles has basically diffraction character and can be described by the quasipotential of the type/2/

$$V^{(+)}(s,t) \approx V^{diffr}(s,t) = i\, s\, g_{\pi p}(s)\, e^{a_{\pi p}(s)t}, \tag{2.6}$$

where $g_{\pi p}$ – and $a_{\pi p}$ – are positive, slowly varying functions of the energy. Assume now that the exchange quasipotential $V^{T=1}$ is an analytic function in the complex t-plane with the singularities at $t \geq t_0$ and satisfies the spectral representation/20/

$$V^{T=1}(s,t) = \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\rho(t',s)\,dt'}{t'-t}. \tag{2.7}$$

It can be shown that at $t < t_0$ the quasipotential (2.7) can be rewritten in the following integral form/10/:

$$V^{T=1}(s,t) = \int_0^1 dx\, x^{-1-t}\, f(x,s), \tag{2.8}$$

where

$$f(x,s) = \frac{1}{\pi}\int_{t_0}^{\infty} dt\, x^{t}\, \rho(s,t). \tag{2.9}$$

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x/ In what follows for simplicity we will neglect the spin dependence of the scattering amplitudes. Notice that the quasipotential equation for the two-particle scattering with unequal masses was considered in/19/.

The representation (2.8) includes the singular quasipotentials as well as nonsingular ones.

Eq. (2.9) determines the function $f(x,s)$ which is analytic inside the circle $|x|<1$ with the cut at $-1<x\leq 0$.

The behaviour of the function $f(x,s)$ at the point $x=0$ is determined by the nearest singularities of the quasipotential $V^{T=1}(s,t)$ in the t-plane.

Assume, for example, that the function $f(x,s)$ can be expanded in the neighbourhood of the point $x=0$ in a power series

$$f(x,s)=\sum_{n=0}^{\infty} g_n(s)x^{\gamma+n}\ ;\quad |x|\leq a<1. \tag{2.10}$$

Inserting the series (2.10) in eq. (2.8) and integrating from $x=0$ to $x=a$ we get the following contribution to the quasipotential

$$V^{T=1}(s,t)=\sum_{n=0}^{\infty}\frac{g_n(s)a^{n+\gamma-t}}{\gamma+n-t}+\dots \tag{2.11}$$

The series in eq. (2.11) can be considered as the sum of the poles, which correspond to the particle lying on the linear trajectory $\alpha(t)=-\gamma+t$ /10/ x/.

The asymptotic behaviour of the sum of this series at high energies under an assumption of polynomial growth of the $g_n(s)$ with s, i.e. $g_n(s)\to\beta_n\cdot s^n$, has the Regge-like form

$$V^{T=1}(s,t)\to-\frac{\beta(\alpha_t)}{\sin\pi\alpha_t}(-s)^{\alpha_t}+\dots \tag{2.12}$$

x/ From the condition of integrability of the series (2.10) follows that $-\gamma=\alpha(0)<1$. For the ρ-meson trajectory $1-\gamma=\alpha(0)\approx 1/2$ that corresponds to the branch point of the square root type at $x=0$.

The asymptotic behaviour of the quasipotential (2.8) at high momentum transfers is determined by the behaviour of the function $f(x,s)$ near the point $x=1$.

In particular, for the local quasipotential which has singularities at the origin, e.g.:

$$V^{T=1}_{t\to-\infty}(s,t)\to\frac{c(s)}{t^{1-\epsilon}}\ ;\quad \epsilon>0 \tag{2.13}$$

the function $f(x,s)$ goes to the infinity at $x\to 1$ as

$$f(x,s)\to(1-x)^{-\epsilon}\ ,\quad x=1. \tag{2.14}$$

The connection of the local quasipotential

$$V^{T=1}(s,\vec r)=\int d\vec q\, e^{i\vec q\vec r}\, V^{T=1}(s,-\vec q^{\,2}) \tag{2.15}$$

with the function $f(x,s)$ defines by the expression:

$$V^{T=1}(s,r)=2\pi^{3/2}\int_0^{\infty}\frac{db}{b^2}\, e^{-r^2/4b^2}\, f(e^{-b^2},s). \tag{2.16}$$

From eq. (2.18) it is seen that to the quasipotentials having Gaussian behaviour at large distances

$$V^{T=1}_{r\to\infty}(s,r)\to e^{-r^2/4a} \tag{2.17}$$

there corresponds the function $f(x,s)$, having the singularity of the type $\delta(x-e^{-a})$ or vanishing inside of some interval $e^{-a}\leq x<1$. Such a behaviour contradicts, however, to the dispersion relation (2.7). Below we shall consider a model for the exchange quasipotential $V^{T=1}$, which is based on the Veneziano-type representations for the scattering amplitude $T^{(-)}(ds,t)$.

§3. Veneziano Model and the Problem of the Linearity of Regge Trajectories

In the Veneziano model the invariant amplitude $iT^{(-)}$ for the πp -scattering is represented as a sum of the terms of the following type/9,21/

$$B^{(-)}_{\ell,k}(s,t,u)=\frac{\Gamma[1-a_\rho(t)]\Gamma[\ell-a_F(s)]}{\Gamma[k-a_\rho(t)-a_F(s)]}-(s\to u). \qquad (3.1)$$

Here ℓ , and k - are positive halfintegers, and $a_F(s)$ – the fermion trajectory with the quantum numbers of πN -system (N_α, N_γ, Δ_δ -trajectories).

In what follows we assume that in high energy limit all the fermion trajectories approximately coincide:

$$a_F(s)\underset{s\approx\infty}{\approx} a_{N_\alpha}(s)\approx a_{N_\gamma}(s)\approx a_{\Delta_\delta}(s). \qquad (3.2)$$

The terms of type (3.1) for which $\ell=k=3/2$ will dominate at high energies:

$$B^{(-)}_{3/2,3/2}(s,t,u)\underset{\substack{s\to\infty\\ t=\text{fixed}}}{\to}\Gamma[1-a_\rho(t)][-a_F(s)]^{a_\rho(t)}-(s\to u). \qquad (3.3)$$

We notice the remarkable feature of the asymptotic form (3.3): the energy dependence of the amplitude contained only in $a_F(s)$, which is, generally speaking, the complex nonlinear function of s . Thus the ambiguity, which is connected with the choice of energy scale in the Regge representation is absent in eq. (3.3). The corresponding parameter in (3.3) is the quantity $a'_F(s)$, which depends, generally speaking, on the energy.

Below we assume that the exchange quasipotential $V^{T=1}(s,t)$ in the region of small scattering angles and high energies is defined by the expression of the type eq. (3.3), i.e.

$$V^{T=1}(s,t)\underset{\substack{s\to\infty\\ t=\text{fixed}}}{\to}-g_\rho(t)\Gamma[1-a_\rho(t)][-a_F(s)]^{a_\rho(t)}-(s\to u), \qquad (3.4)$$

where $g_\rho(t)$ - is a smooth function of t .

The simplest quasipotential, behaving as (3.4) can be represented in the integral form (2.8) with the function $f(x,s)$

$$f(x,s)=g_\rho(0)\,a_F(s)\,x^{1/2}\,e^{x a_F(s)}-(s\to u). \qquad (3.5)$$

Indeed, substituting expression (3.5) in eq. (2.8) we find

$$V^{T=1}(s,t)=g_\rho(0)\int_0^1 dx\, x^{1/2-t}\,e^{x a_F(s)}\,a_F(s)-(s\to u)\to$$

$$\underset{\substack{s\to\infty\\ t=\text{fixed}}}{\to} g_\rho(0)\,\Gamma[1/2-t]\,[a_F(s)]^{1/2+t}\,(1+ie^{-i\pi t}) \qquad (3.6)$$

which corresponds to the limiting expression (3.4) with the trajectory $a_\rho(t)=1/2+t$ and constant form-factor $g_\rho(t)=g_\rho(0)$. Quasipotential with form-factor depending on t of the type $g_\rho(t)=g_\rho(0)e^{bt}$ can be constructed by means of the function $f(x,s)$, obtained from (3.5) by substituting $x\to xe^b$. Note that the quasipotential $V^{T=1}(s,t)$, defined by the function of the type (3.5), essentially differs from the Veneziano-like expression (3.1) in the large momentum transfer region which corresponds to small distances. We use below the quasipotential $V^{T=1}(s,t)$, defined by the integral representation (2.8) with $f(x,s)$ like (3.5) for describing the behaviour of the $\pi^{\pm}p$ total cross section and the differential charge-exchange cross section $\pi^- p \to \pi^0 n$ at zero angle.

Using eqs. (2.1), (2.2) and (3.6), neglecting elastic rescattering effects, we obtain:

$$\Delta\sigma_{\pi p} = \frac{c_\rho}{s} \alpha_F^{1/2}(s) \tag{3.7}$$

$$\left(\frac{d\sigma}{dt}(0)\right)_{\pi^- p\to\pi^0 n} = \frac{1}{32\pi}(\Delta\sigma_{\pi p})^2(1+\rho^2) = \frac{c_\rho^2}{16\pi^2}\left[\frac{\alpha_F^{1/2}(s)}{s}\right]^2, \tag{3.8}$$

where ρ is the ratio of real and imaginary parts of the charge-exchange amplitude at zero momentum transfer, and

$$c_\rho = 64\pi^3 g_\rho(0)\Gamma(1/2). \tag{3.9}$$

In the case of linear trajectories $\alpha_F(s)$ the eqs. (3.7) and (3.8) coincide with the corresponding formulae of the Regge analysis. Comparing the new data/1,22/ on the $\pi^{\pm}p$ total cross section differences at high energies with the predictions of the Regge model with linear trajectories, it is possible to explain the existing deviation (cf. Fig. 1) as the appearance the energy dependence in the slope $\alpha_F' = \alpha_F'(s)$.

We used eqs. (3.8) and (3.9) for determining the character of the energy dependence of the trajectories $\alpha_F(s)$, from the experimental data on πp scattering in the region p_L = 8–65 GeV/c. The calculations show that for the ratio $\alpha_F(s)/\alpha_{\ell in}(s)$ the following numerical parametrization

$$\frac{\alpha_F(s)}{\alpha_{\ell in}(s)} = \alpha + \beta\cdot s \tag{3.10}$$

is in good agreement with the experimental data (Fig. 2) for the values of the parameters

$$\alpha = 0{,}8;\quad \beta = 0.015. \tag{3.11}$$

Thus the fermion trajectory can be approximated at high energy by the following empirical formula

$$\alpha_F(s) \approx 0{,}8\, s(1+\gamma s) \tag{3.12}$$
$$s \geq 10,\quad \gamma = 0{,}02\ (\mathrm{GeV/c})^{-2}.$$

In Fig. 3 the experimental data/1,22/ on $\Delta\sigma_{\pi p}$ are plotted together with the curve calculated by means of eq. (3.12) in comparison with the predictions of the Regge pole model (linear fit with $\alpha_\rho(0)$ = 0.54±0.01/23/). Within the experimental errors the two theoretical curves can be considered as describibg the experiment, but the possibility of such an explanation of the data by means of the trajectory (3.12) (which deviates from the linear form) may be essential if such a regime will continue to superhigh energies. It is interesting to note that with the persistence of such a behaviour of the trajectories (3.12) at energies above 70 GeV/c, the asymptotic cross-section difference $\Delta\sigma_{\pi p}$ will be nonzero

$$\Delta\sigma_{\pi p} = \frac{c_\rho}{s}[\alpha_F(s)]^{1/2} \xrightarrow[s\to\infty]{} 0{,}98\ \mathrm{mb}. \tag{3.13}$$

We emphasize, however, that in the case of such a strong nonlinearity, the interpretation of experimental data in terms of Regge poles can be hightly conditional. We note in conclusion that the elastic rescattering corrections can be described by means of eq. (2.5). By taking into account the first corrections instead of c_ρ in eqs. (3.7), (3.8), the quantity $c_\rho(1-\delta)$, appears, where

$$\delta = \frac{4\pi g_{\pi p}(s)}{a_{\pi p}(s) + b_{\pi p}(s)} \tag{3.14}$$

and $b_{\pi p}(s) = b + \alpha'_{\rho} \ln \alpha_F(s)$ is half of the effective slope of the diffraction peak for charge-exchange cross section. Using the experimental data/1,22/ on the sum of the $\pi^{\pm} p$ total cross sections we find that the corrections constitute about 10% and do not change qualitatively the energy dependence of $\alpha_F(s)$. Note, however, that these changes do not exceed the limits of the uncertainties of the theoretical parameter, which are determined by the errors in the Experimental data.

The empirical s -dependence of $\alpha_F(s)$ makes it possible to draw some conclusion about the behaviour of the differential charge-exchange cross section $\pi^- p \to \pi^0 n$ at zero angle and high energy.7

The results of calculations from eq. (3.8) are presented in Fig. 4 (full line), together with the experimental data on $(\frac{d\sigma}{dt})_{ch.ex}(t=0)$ $^{+}/$; the dashed line corresponds to the Regge pole model with linear trajectories/23/. Note that the quasipotential used corresponds to the choice $\rho=1$, which is in agreement with experimental data/25/ (cf. Fig. 5).

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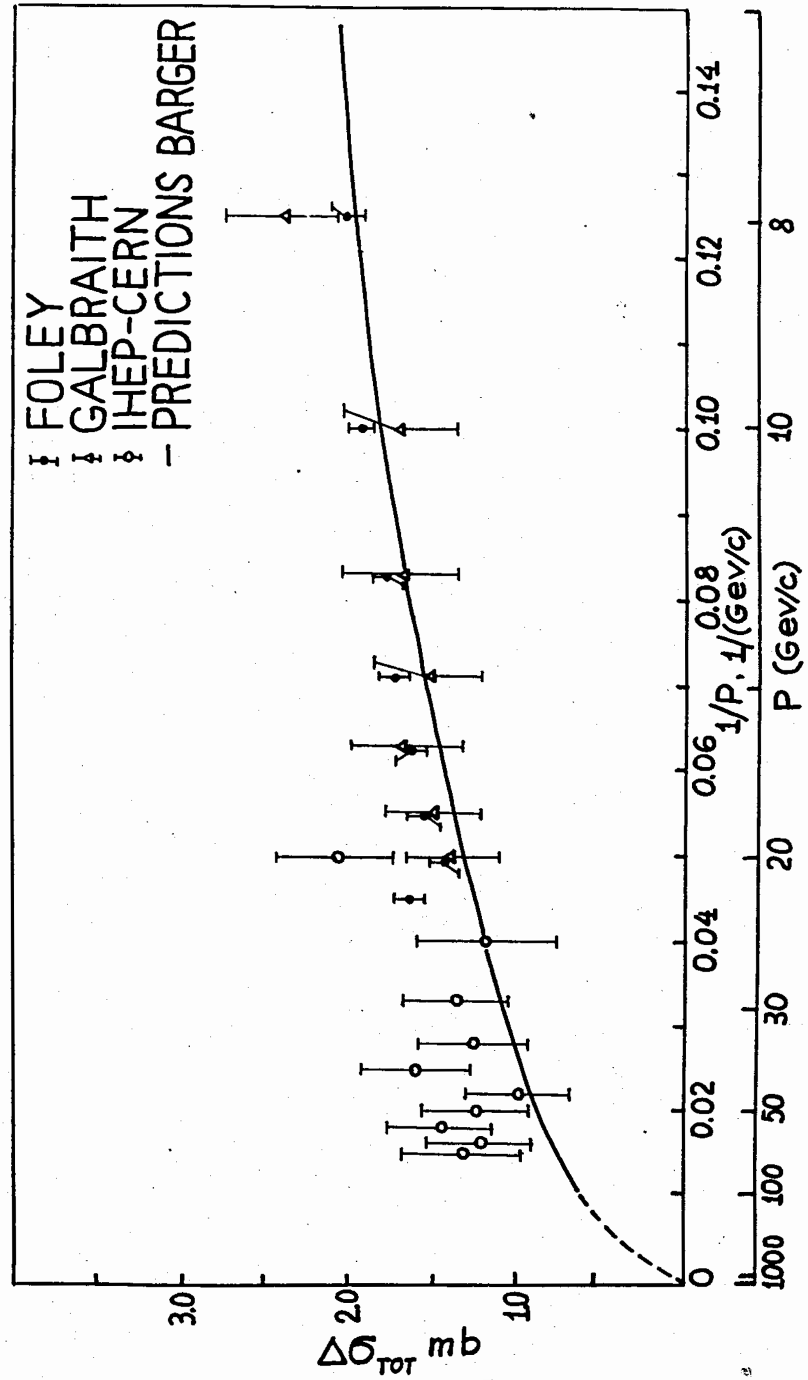

Fig. 1. Experimental data[1,22] on the $\Delta\sigma_{\pi p}$ and the Regge pole model predictions.

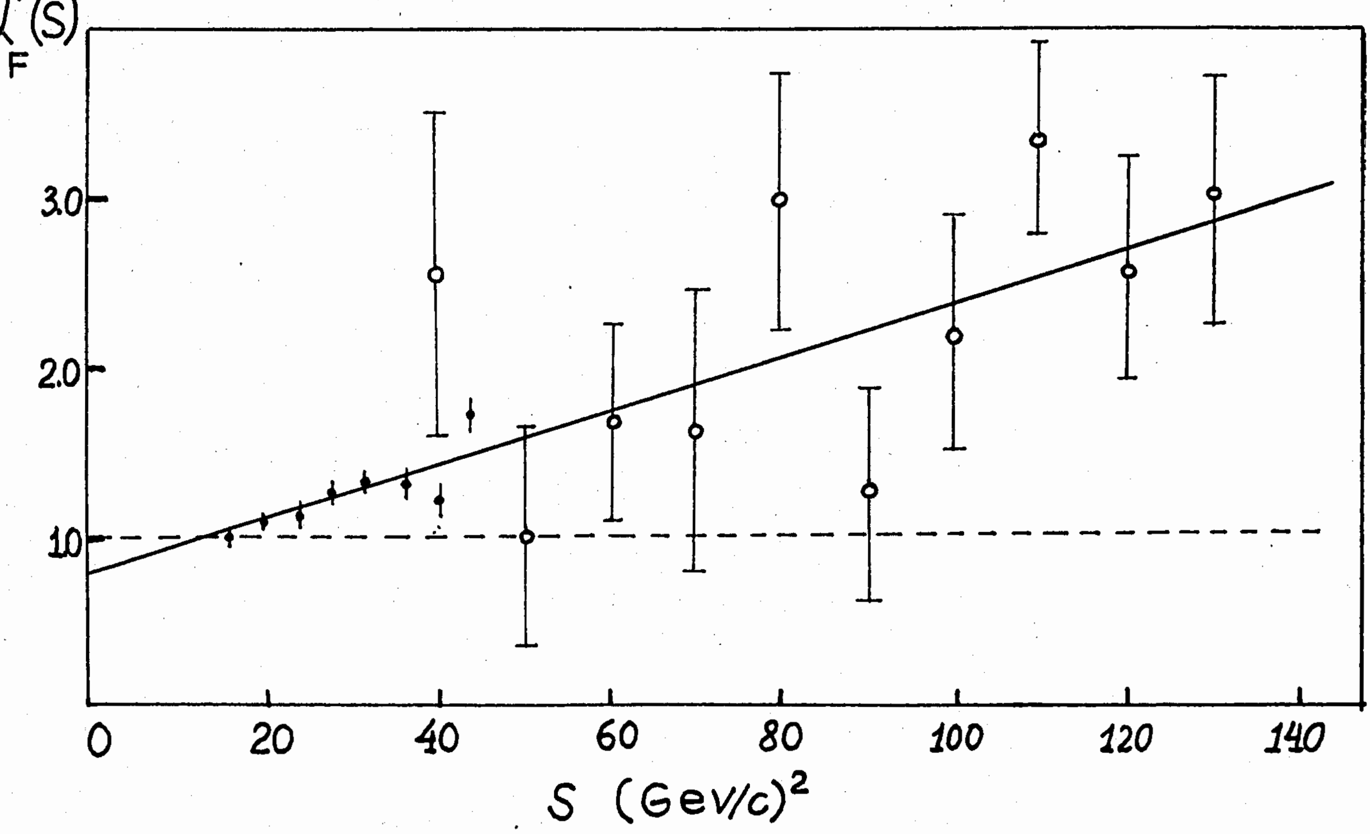

Fig. 2. Energy dependence of the slope of the fermion Regge trajectory.

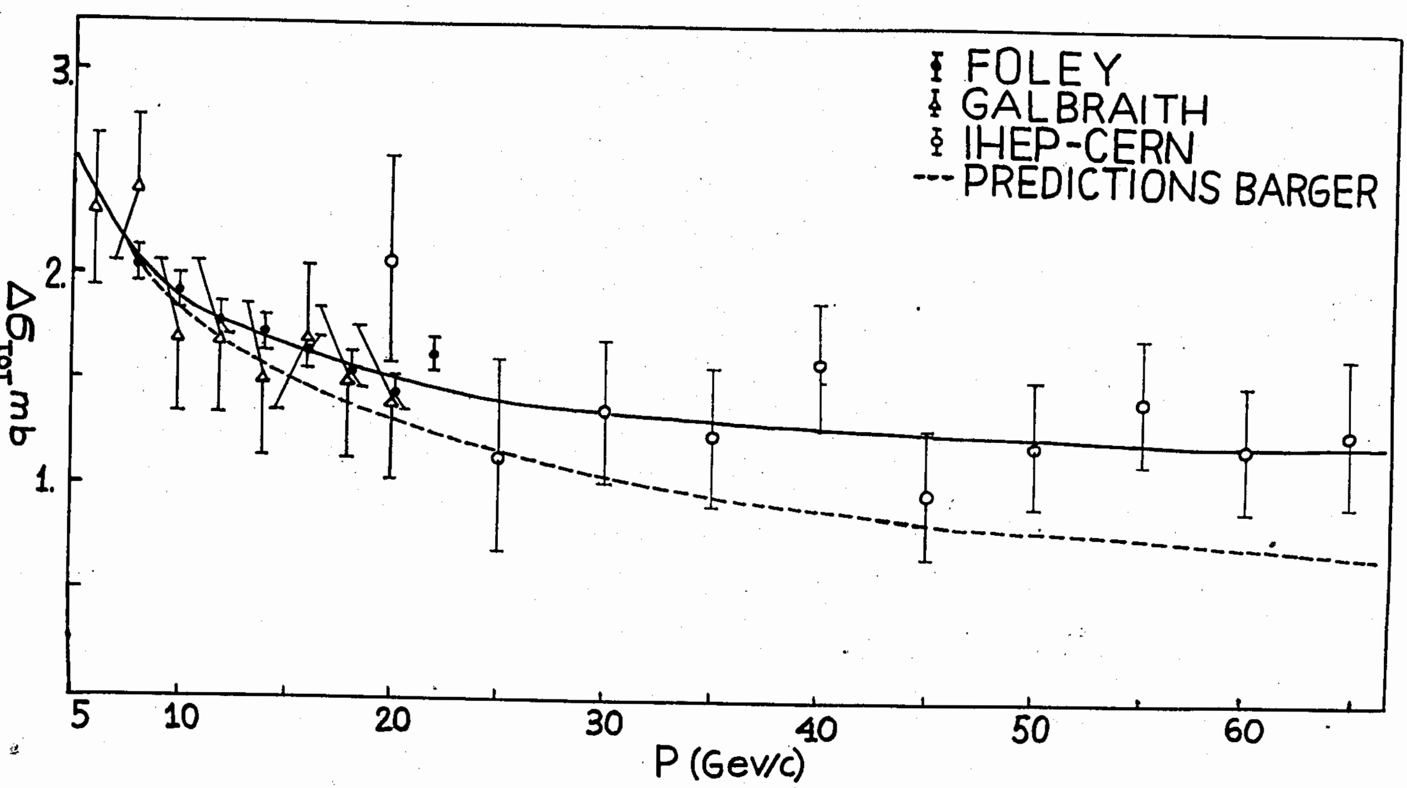

Fig. 3. Total cross section difference ($\Delta\sigma_{\pi^\pm}$) data, theoretical curve (full line), calculated by means eq. (3.12) and the Regge pole predictions (dashed line).

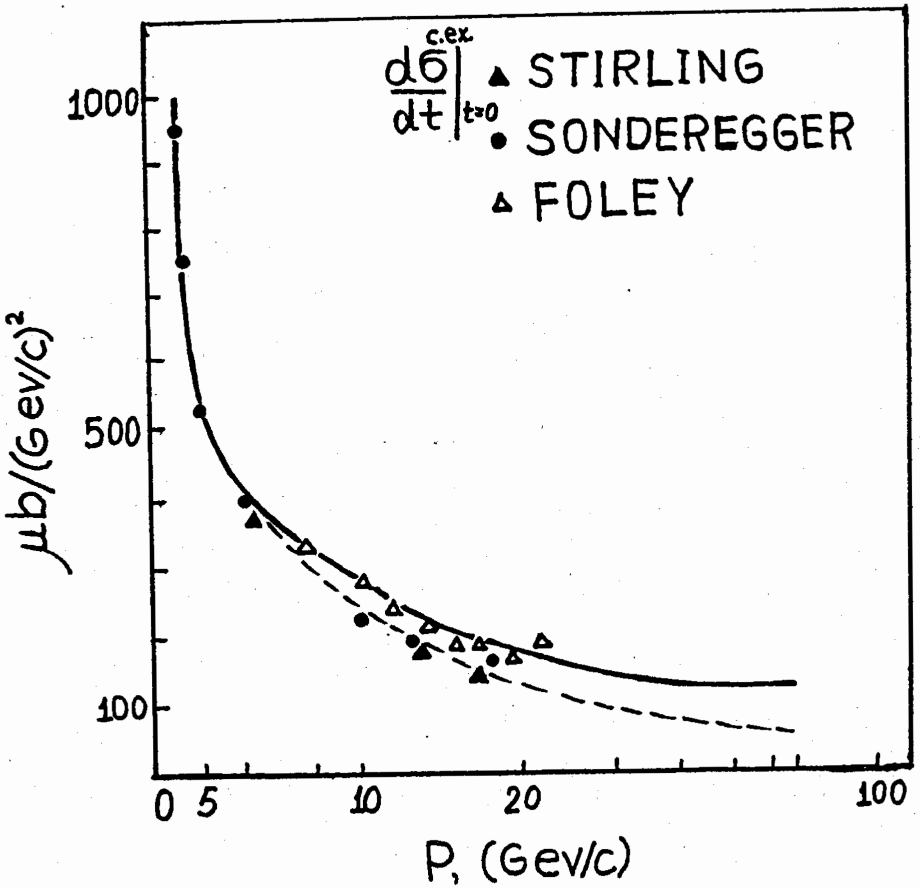

Fig. 4. Differential charge exchange cross section $\pi^- p \to \pi^0 n$ at $t=0$.

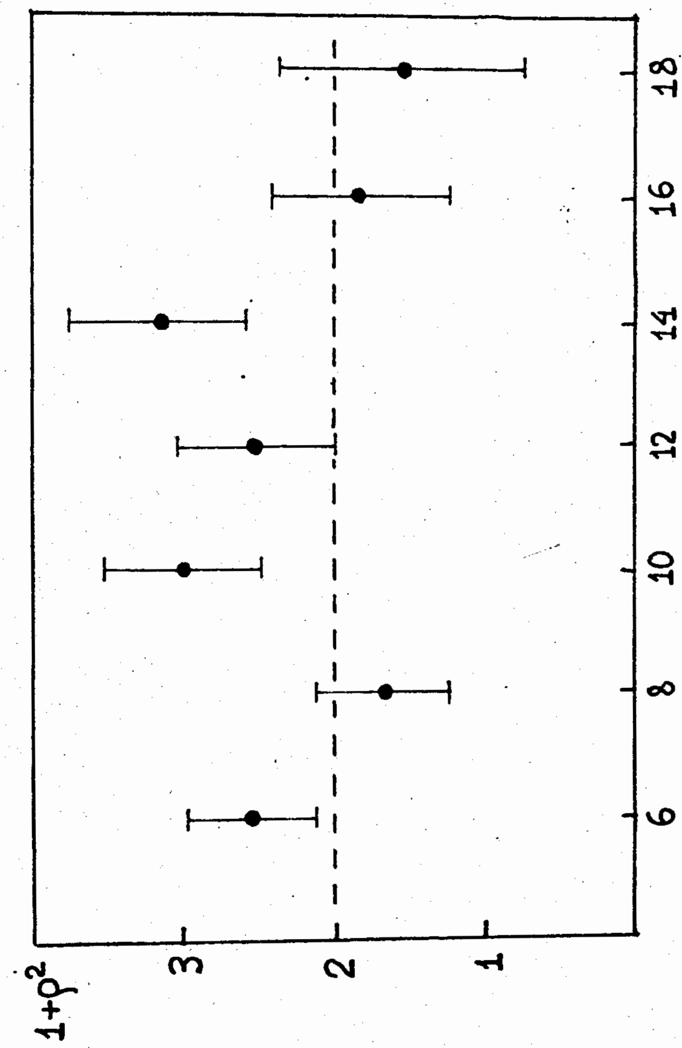

Fig. 5. Experimental data /24,25/ on energy dependence of $\rho = \frac{\mathrm{Re}\,T(0)}{\mathrm{Im}\,T(0)}$ for charge exchange $\pi^- p \to \pi^0 n$.