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ON RELATIVISTIC QUASIPOTENTIAL EQUATIONS IN THE CASE OF ELECTROMAGNETIC INTERACTION

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1. Introduction

The quasipotential approach to the quantum field theory, proposed by Logunov and Tavkhelidze/1/, has been successfully applied to problems of quantum electrodynamics by Faustov/2/ and Faustov and Tuhtyaev/3/. They calculated higher order corrections (up to sixth order in the coupling constant) to the energy levels of positronium and hydrogen atom, constructing the quasipotential with the help of the matrix elements of the scattering matrix on the mass shell. An investigation of the off-mass-shell correction to the quasipotential has been done by Desimirov and Mateev for the cases of electron-positron/4/ and electron-proton/5/ interactions.

A new, manifestedly covariant, version of the quasipotential approach has been proposed by Kadyshevsky/6/. A generalization of this approach to particles with spin 1/2 and pseudoscalar interaction has been done by Kadyshevsky and Mateev/7/.

The present paper is an initial step in the application of the quasipotential approach of Kadyshevsky to systems of particles which interact electromagnetically. It is devoted to two basic scattering processes -electron-positron and Compton scattering. Its aim is to give the necessary tools for investigating electrodynamics, such as diagram rules, equations for the scattering matrix and the wave functions, the spin structure of the quasipotential. We stress on some specific features appearing because of the zero mass of the photon. Some first applications, such as constructing the quasipotential in second order of the coupling constant, are also made.

In Section 2 the diagram technique rules in the case of electromagnetic interaction Hamiltonian are obtained. In Section 3 quasipotential equations for the scattering amplitude and the wave function describing electron-positron and photon-electron systems are constructed. Some specific properties of these equations, connected with the zero mass of the photon are discussed. In Section 4 the spin-structures of the quasipotential in both cases of elctron-positron and photon-electron interactions are obtained. In Section 5 the quasipotential for the electron-positron system has been calculated in lowest order of perturbation theory, and a comparison of its off energy shell terms is made with the off mass shell terms of the corresponding quasipotential in the approach of Logunov and Tavkhelidze.

A useful guide to the notations used in the present paper is given in the Appendix to ref./7/.

2. Diagram Rules in the Vase of Electromagnetic Interaction

In this section we briefly scetch[x/] the way in which the diagram rules corresponding to the electromagnetic interaction Hamiltonian

$$H(x) = e : \bar{\psi}(x)\gamma^{n}\psi(x)A_{n}(x): \tag{1}$$

can be obtained. In eq. (1) $\psi(x)$ are the field operators of the electrons and the positrons, and $A_n(x)$ of the electromagnetic field.

We introduce, after refs./6/ and/7/, the matrix $R(\lambda\kappa, \lambda\kappa')$, where λ is an arbitrary unit time-like four vector and κ and κ' are scalar parameter. $R(\lambda\kappa, \lambda\kappa')$ defines the off-energy shell continuation of the scattering matrix. On the energy shell (which is defined by $\kappa = \kappa' = 0$) we have/6/

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x/ The procedure of developing diagram technique is worked out in detail, on the example of interaction Hamittonian $H(x) = g : \bar{\psi}(x)\gamma_5\psi(x)\phi(x):$ in ref./7/.

$$R(\lambda\kappa,\lambda\kappa') = R(0,0) \equiv T, \tag{2}$$

T being connected with S -matrix in the usual way

$$S = 1 + iT. \tag{3}$$

The matrix $R(\lambda\kappa,\lambda\kappa')$ satisfies the equation (/6/ and the references threin)

$$R(\lambda\kappa,\lambda\kappa') = -\tilde{\tilde{H}}(\lambda\kappa-\lambda\kappa') - \frac{1}{2\pi}\int \tilde{\tilde{H}}(\lambda\kappa-\lambda\kappa_1)\frac{d\kappa_1}{\kappa_1 - i\epsilon} R(\lambda\kappa_1,\lambda\kappa'), \tag{4}$$

where $\tilde{\tilde{H}}(p)$ is the Fourier-transform of the interaction Hamiltonian

$$\tilde{\tilde{H}}(p) = \int e^{-ipx} H(x)dx. \tag{5}$$

After investigation of the first order terms in the coupling constant we get the rules for the graphical description of the particles in the initial and the final states, which are summarized in Table I. Moreover, in any vertex we add an incoming and an outgoing dotted spurion line (or quasiparticle line) which carry four momentum, correspondingly $\lambda\kappa$ and $\lambda\kappa'$ so that a four-momentum conservation holds ref./6,7/.

The graphical representation of the particles in the intermediate states we obtain when consider the structure of the higher order in the coupling constant terms of $R(\lambda\kappa,\lambda\kappa')$. The results are presented in Table II.

The rules of constructing the matrix elements are the same as those in ref./7/, with evident changes in accordance with Tables I and II.

It was demostrated in refs./6,7/ that the most covenient choice of λ is in the direction of the total momentum of the physical incoming or outgoing particles

$$\lambda = \frac{q_1 + q_2}{\sqrt{(q_1 + q_2)^2}} = \frac{p_1 + p_2}{\sqrt{(p_1+p_2)^2}} \tag{6}$$

Table I

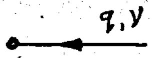
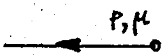
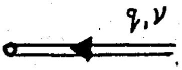
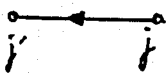
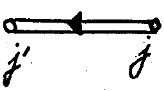
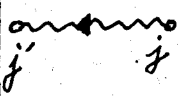
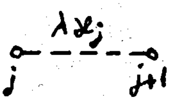
line	particle	state	Factor in the matrix element
	electron	in	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2q_0}}\, u^{\nu}(q)$
	electron	out	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2p_0}}\, \bar{u}^{\mu}(p)$
	positron	in	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2q_0}}\, \bar{v}^{\nu}(q)$
	positron	out	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2p_0}}\, v^{\mu}(p)$
	photon	in	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2k_0}}\left[-g^{\sigma\sigma} e_n^{\sigma}(k)\right]$
	photon	out	$\frac{(2\pi)^{\frac{3}{2}}}{\sqrt{2k_0}}\left[-g^{\sigma\sigma} e_m^{\sigma}(k)\right]$

Table II

line	particle	pairing	factor in the matrix element
	electron	$\psi_\beta(q_{j'})\bar{\psi}_\alpha(p_j)$ $(j'<j)$	$S^{(+)}_{\beta\alpha}(p_j, m)$
	positron	$\bar{\psi}_\alpha(p_j)\psi_\beta(q_{j'})$ $(j'<j)$	$S^{(+)}_{\beta\alpha}(q_j, -m)$
	photon	$A_{n_{j'}}(k_{j'})A_{n_j}(k_j)$ $(j'<j)$	$-g^{n_j n_{j'}}\Delta^{(+)}_0(k_j)$
	quasiparticle		$\frac{1}{2\pi}\,\frac{1}{\varkappa_j - i\varepsilon}$

Particularly, with this choice of λ the matrix and wave function have simplest form.

In the case of electromagnetic interaction one has to be careful when uses λ from eq. (6). The reason can be easily understood if we consider the vertex on Fig. 1, where the particles with momenta q_1 and q_2 are incoming or outgoing. Such vertices give contribution to the exchange (annihilation) type diagrams. In the vertex we have the four-momentum conservation law

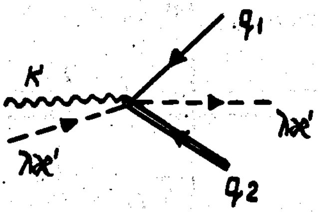


Fig.1.

$$\lambda\kappa_1 + q_1 + q_2 - \lambda\kappa'_1 - k = 0. \qquad (7)$$

The particle momentum q_1 the antiparticle with momentum q_2 and the particle that carries the interaction with momentum k are on the mass shell. Let us consider first the case when
From eq. (7) it follows that the direction of λ can be choosen arbitrary and in particular as in eq. (6). In this latter case we have from eq. (6)

$$\lambda(\kappa_1 - \kappa' + \sqrt{s_q}) = k \qquad (8)$$

and taking intonaccount that $\lambda^2 = 1$:

$$\kappa_1 - \kappa' + \sqrt{s_q} = m_f . \qquad (9)$$

Let now $m_f = 0$, as it is in the case of quantum electrodynamics where the photon is the field which carry the interaction. It follows then eqs. (9) and (8) that $k = 0$, which is impossible. Eq. (7) exhibits the fact that such inconsistency happens only when λ is chosen as in eq. (6), and of course when $k^2 = 0$ x/. There exist two ways

x/ Note that this difficulty does not happen in the case of Compton scattering (in this case the photon is incoming or outgoing particle and does not appear alone in intermediate states).

to overcome this difficulty - either to choose λ different from eq.(6) and leave the photon in the intermeadiate states on the mass shell $k^2=0$, or to choose λ in accordance with (6), but formally to prescribe a mass $m_\gamma \neq 0$ to the photon, i.e. to regard it, throughout the calculation of the matrix elements, off its mass shell. Because of the extreme convenience which creates a choice of λ in the direction of the total momentum of the incoming and outgoing particles we shall choose the second way. At the end of the calculation after passing on the energy-shell $\kappa = \kappa' = 0$ we shall put also $m_\gamma = 0$ to obtain the matrix element of the physical amplitude.

3. Relativistic Quasipotential Equation in the Case of Electron-Positron and Photon-Electron Scattering

The form of the quasipotential equations depends on the spin of the external particles. Therefore, in the case of electron-positron scattering in the centre of mass system, taking into account (6) the quasipotential equation for the scattering matrix is written in the form/7/

$$T^{\mu_1\nu_1}_{\mu_2\nu_2}(\vec{p},\vec{q}) = V^{\mu_1\nu_1}_{\mu_2\nu_2}(\vec{p},\vec{q};E_q) + \frac{1}{(4\pi)^3}\sum_{\rho_1,\rho_2}\int V^{\mu_1\rho_1}_{\mu_2\rho_2}(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\frac{T^{\rho_1\nu_1}_{\rho_2\nu_2}(\vec{k},\vec{q})}{E_k(E_k-E_q-i\epsilon)}, \tag{10}$$

where $E_k=\sqrt{\vec{k}^2+m^2}$. The quasipotential V is the sum of all the irreducible connected corresponding to the interaction Hamiltonian given by eq. (1). The definition of irreducible and connected graph is given in ref./7/.

Eq. (10) can be written in two-component form

$$t_{l_1k_1;l_2k_2}(\vec{p},\vec{q})\,\phi_{k_1}\chi_{k_2} = V_{l_1k_1;l_2k_2}(\vec{p},\vec{q};E_q)\,\phi_{k_1}\chi_{k_2} +$$

$$+\frac{1}{(4\pi)^3}\int V_{i_1 j_1 ; i_2 j_2}(\vec{p},\vec{k};E_q)\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\frac{t_{j_1 k_1 ; j_2 k_2}(k,q)\phi_{k_1}\chi_{k_2}}{E_k(E_k-E_q-i\epsilon)} \qquad (11)$$

If we define the wave function of the electron-positron system, corresponding to the continuous spectrum by

$$\psi_q(p)_{i_1,i_2}=\frac{(2\pi)^3}{m}\delta(p-q)\sqrt{\vec{p}^2+m^2}\;\phi_{i_1}\chi_{i_2}+$$

$$+\frac{t_{i_1 k_1 ; i_2 k_2}(\vec{p},\vec{q})\,\phi_{k_1}\chi_{k_2}}{8mE_p(E_p-E_q-i\epsilon)} \qquad (12)$$

we obtain the quasipotential wave equation

$$E_p(E_p-E_q)\psi_q(p)_{i_1 i_2}=$$

$$=\frac{1}{(4\pi)^3}\int\frac{d\vec{k}}{\sqrt{k^2+m^2}}V_{i_1 j_1 ; i_2 j_2}(p,k;E_q)\psi_a(k)_{j_1 j_2}\;. \qquad (13)$$

In order to evaluate the equation for photon-electron scattering amplitude and wave function we have to introduce some notations. Let us denote the matrix element, corresponding to the set of all irreducible graphs, describing Compton scattering off the energy shell, with

$$V\equiv\frac{(2\pi)^4\,\delta(\lambda\kappa+q+k_1-p-k_2-\lambda\kappa')}{\sqrt{2q_0 2k_{10}\,2p_0 2k_{20}}}V^{\tau\sigma}_{\mu\nu}(\lambda\kappa,p,k_2|q,k_1,\lambda\kappa')\equiv$$

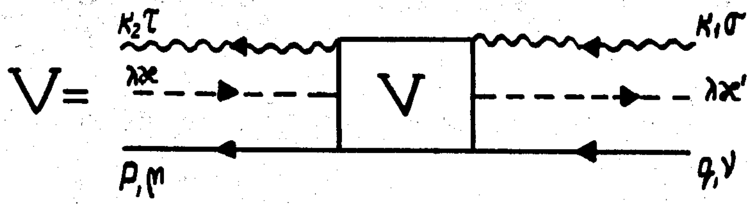
$$\equiv\frac{(2\pi)^4\delta(\lambda\kappa+q+k_1-p-k_2-\lambda\kappa')}{\sqrt{2q_0 2k_{10}\,2p_0\,2k_{20}}}\bar{u}^{\mu}_{\alpha}(p)V_{\alpha\tau;\,\beta\sigma}(\lambda\kappa,p,k_2|q,k_1,\lambda\kappa')u^{\nu}_{\beta}(q) \qquad (14)$$

where the content of the indices σ and τ is

$$V^{\tau\sigma}_{\alpha\beta} = (-g^{\tau\tau} e^{\tau}_{n})\gamma^{n} V_{\alpha\beta} (-g^{\sigma\sigma} e^{\sigma}_{m})\gamma^{m} \tag{15}$$

e_i ($i = 0,1,2,3$) being the polarization vectors of the photons in the initial (e_m) and the final state (e_n) .

Let us represent V graphically with



$V =$ k , τ $\lambda\kappa$ p,μ V k , σ $\lambda\kappa'$ q, ν

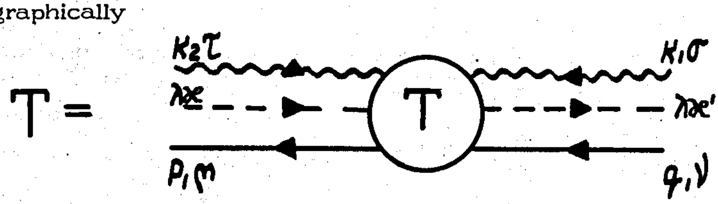
For the Compton scattering amplitude off the energy shell we put:

$$T \equiv \frac{(2\pi)^4 \delta(\lambda\kappa + q + k_1 - p - k_2 - \lambda\kappa')}{\sqrt{2q_0 \, 2k_{10} \, 2p_0 \, 2k_{20}}} T^{\tau\sigma}_{\mu\nu}(\lambda\kappa, p, k_2 \,|\, q, k_1, \lambda\kappa') \equiv$$

$$= \frac{(2\pi)^4 \delta(\lambda\kappa + q + k_1 - p - k_2 - \lambda\kappa')}{\sqrt{2q_0 \, 2k_{10} \, 2p_0 \, 2k_{20}}} \bar{u}^{\mu}_{\alpha}(p) T_{\alpha\beta;\beta\sigma}(\lambda\kappa, p, k_2 | q_1 k_1, \lambda\kappa') u^{\nu}_{\beta}(q) \tag{16}$$

$$T^{\tau\sigma}_{\alpha\beta} = (-g^{\tau\tau} e^{\tau}_{n})\gamma^{n} T_{\alpha\beta} (-g^{\sigma\sigma} e^{\sigma}_{m})\gamma^{m} \tag{17}$$

and graphically

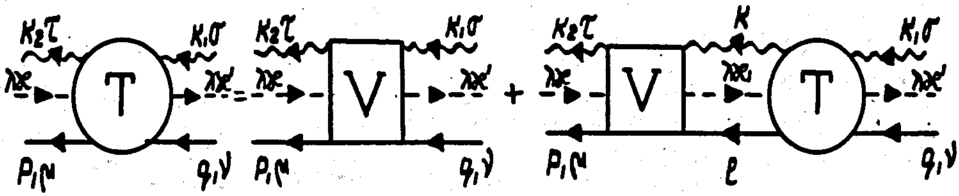


$k_2\,\tau$ $k_1\,\sigma$

$T =$ $\lambda\kappa$ T $\lambda\kappa'$

$p_1\mu$ $q_1\nu$

Then we can write the following graphical equation:



Using the rules of the diagram techniques (see Section 2) we get the integral equation:

$$\frac{(2\pi)^4\,\delta(\lambda\kappa+q+k_1-p-k_2-\lambda\kappa')}{\sqrt{2q_0 2p_0 2k_{10} 2k_{20}}}\,T^{\tau\sigma}_{\mu\nu}(\lambda\kappa,p,k_2|q,k_1,\lambda\kappa')=$$

$$\frac{(2\pi)^4\,\delta(\lambda\kappa+q+k_1-p-k_2-\lambda\kappa')}{\sqrt{2q_0 2p_0 2k_{10} 2k_{20}}}\,V^{\tau\sigma}_{\mu\nu}(\lambda\kappa,p,k_2|q,k_1,\lambda\kappa')+$$

$$+\frac{(2\pi)^{\frac{3}{2}\cdot 4}}{(2\pi)^{\frac{8}{2}}\sqrt{2q_0 2p_0 2k_{10} 2k_{20}}}\int\delta(\lambda\kappa+\ell+k-p-k_2-\lambda\kappa_1)\,\delta(\lambda\kappa+q_1+k_1-\ell-k-\lambda\kappa')\times$$

$$\times\, u^{\mu}_{\alpha}(p)\,V^{\tau,}_{\alpha\gamma}(\lambda\kappa,p,k_2|\ell,k,\lambda\kappa_1)\,\gamma^m\,(-1)\,g^{mn}\,\Delta_0^{(+)}(k)\,S^{(+)}_{\gamma\delta}(\ell,m)\times$$

$$\times\,\frac{1}{2\pi}\,\frac{d\kappa_1}{\kappa_1-i\epsilon}\,\gamma^n\,T^{,\sigma}_{\delta\beta}(\lambda\kappa_1,\ell,k|q,k_1,\lambda\kappa')\,u^{\nu}_{\beta}(\vec{q}).$$

After simple calculations, applying the completness condition for Dirac spinors and the relation:

$$g^{mn}=\sum_{\rho_1,\rho_2} g^{\rho_1\rho_1}\,g^{\rho_2\rho_2}\,e^{\rho_1}_{m}\,e^{\rho_2}_{n} \qquad (19)$$

we obtain

$$T^{r\sigma}_{\mu\nu}(\lambda\kappa, p, k_2 | q, k_1, \lambda\kappa') = V^{r\sigma}_{\mu\nu}(\lambda\kappa, p, k_2 | q, k_1, \lambda\kappa') -$$

$$- \sum_{\rho_1, \rho_2} \frac{1}{(2\pi)^3} \int V^{r\rho_1}_{\mu\rho_2}(\lambda\kappa, p, k_2 | \ell, k, \lambda\kappa_1) \{ \theta(\ell_0)\delta(\ell^2 - m^2) d\ell \, \theta(k_0)\delta(k^2) dk \times \tag{20}$$

$$\times \delta(\lambda\kappa_1 + q + k_1 - \ell - k - \lambda\kappa') \frac{d\kappa_1}{\kappa_1 - i\epsilon} \} T^{\rho_1\sigma}_{\rho_2\nu}(\lambda\kappa_1, \ell, k | q, k_1, \lambda\kappa').$$

Eq. (20) is our quasipotential equation in a covariant form. In the centre of mass system we have:

$$p + k_2 = 0$$
$$q + k_1 = 0$$
$$\ell + k = 0$$

$$s_p = (p + k_2)^2 = (p_0 + k_{20})^2 = (\sqrt{\vec{p}^2 + m^2} + \sqrt{\vec{p}^2})^2 = (E_p + E_{k_2})^2$$

$$s_q = (q + k_1)^2 = (q_0 + k_{10})^2 = (\sqrt{\vec{q}^2 + m^2} + \sqrt{\vec{q}^2})^2 = (E_q + E_{k_1})^2 \tag{21}$$

$$s_k = (k + \ell)^2 = (k_0 + \ell_0)^2 = (\sqrt{\vec{k}^2 + m^2} + \sqrt{\vec{k}^2})^2 = (E_k + E_\ell)^2.$$

If in eq. (20) we pass to CMS, choose λ from eq. (6) and perform the integrations over κ_1 and ℓ (or k) we obtain the quasipotential equation in its most covenient for application form:

$$T^{r\sigma}_{\mu\nu}(\vec{p}, \vec{q}) = V^{r\sigma}_{\mu\nu}(\vec{p}, \vec{q}; E_q, E_k) -$$

$$- \frac{2}{(4\pi)^3} \sum_{\rho_1, \rho_2} V^{r\rho_1}_{\mu\rho_2}(\vec{p}, \vec{k}; E_q, E_k) \frac{d\vec{k}}{\sqrt{\vec{k}^2 + m^2}} \frac{T^{\rho_1\sigma}_{\rho_1\nu}(\vec{k}, \vec{q})}{E_k(\sqrt{s_k} - \sqrt{s_q} - i\epsilon)}. \tag{22}$$

Passing then to two-component spinors

$$\bar{u}_\alpha T_{\alpha\beta} u_\beta = \phi_i^+ t_{ij} \phi_j ,$$
$$\bar{u}_\alpha V_{\alpha\beta} u_\beta = \phi_i^+ v_{ij} \phi_j , \tag{23}$$

we get the quasipotential equation for the Compton scattering in two-component form:

$$t_{ij}(\vec{p},\vec{q})\phi_j = v_{ij}(\vec{p},\vec{q};E_q,E_k)\phi_j - \\ -\frac{2}{(4\pi)^3}\sum_{k=1}^{2} v_{ik}(\vec{p},\vec{k};E_q,E_{k_1})\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}\frac{t_{kj}(\vec{k},\vec{q})\phi_j}{E_k(\sqrt{s_k}-\sqrt{s_q}-i\epsilon)} \tag{24}$$

(for brevity we have omitted the photon polarization indices).

The wave function of the photon-electron system we define as:

$$\phi_q(p)_i = \frac{(2\pi)^3}{m}\delta(p-q)\sqrt{\vec{p}^2+m^2}\phi_i - \frac{t_{ij}(p,q)\ \phi_j}{4m E_k(\sqrt{s_k}-\sqrt{s_q}-i\epsilon)} . \tag{25}$$

With the help of eqs. (25) and (24) we obtain the quasipotential wave equation:

$$E_{k_2}(\sqrt{s_p}-\sqrt{s_q}-i\epsilon)\psi_q(\vec{p})_i = -\frac{12}{(4\pi)^3}\int\frac{d\vec{k}}{\sqrt{\vec{k}^2+m^2}}v_{ij}(\vec{p},\vec{k};E_q,E_k)\psi_q(\vec{k})_j . \tag{26}$$

Compton scattering is a very special case of scattering of particles with different masses, namely when one of them is massless. The nature of this process is, therefore, purely relativistic and it is clear that such notions as reduced mass are meaningless. Consequently the procedure proposed in ref./8/ can not be applied and we do not know a way to interprete the scattering of a massless particle and a particle with mass as a scattering of one effective particle in a quasipotential field. Of course it is highly desirable to investigate in more detail this case and to try to find such an interpretation or to show that it is impossible. The formalism of introducing relativistic r -space developed in ref./9/, can be formally applied on the hyperboloid of the particle with mass (we have done already this when we defined the wave function (25)).

4. The Spin Structure of the Quasipotential

a) Electron-Positron System

The spin structure of the quasipotential in this case is the same as in the case of nucleon-antinucleon system/7/. Therefore the quasipotential can be written in the form:

$$\begin{aligned} V = {} & V_1\, I^{(1)} \otimes I^{(2)} + V_2 \left(I^{(1)} \otimes n\sigma^{(2)} + n\sigma^{(1)} \otimes I^{(2)} \right) + \\ & + V_3\, n\sigma^{(1)} \otimes n\sigma^{(2)} + V_4\, \ell\cdot\sigma^{(1)} \otimes \ell\cdot\sigma^{(2)} + \\ & + V_5\, m\cdot\sigma^{(1)} \otimes m\cdot\sigma^{(2)} + \\ & + V_6 \left(\ell\cdot\sigma^{(1)} \otimes m\cdot\sigma^{(2)} + m\cdot\sigma^{(1)} \otimes \ell\cdot\sigma^{(2)} \right), \end{aligned} \tag{27}$$

where ℓ, m, n is a basis of unit vectors defined by

$$\begin{aligned} \ell &= \frac{1}{A}(p+q) \\ m &= \frac{1}{B}\left[(p+q)\times(p\times q) \right] \\ n &= \frac{1}{C}(p\times q) \end{aligned} \tag{28}$$

A,B,C are normalizing factors and V_i $(i=1,2,3,...6)$ are scalar functions of the invariant variables t_{pq}, s_p+s_q, s_p-s_q and $\kappa+\kappa'$. The first five are even d functions of s_p-s_q and the last odd function of this variable.

b) Compton Scattering

Let us first consider the spin structure of the quasipotential connected with the photon. The quasipotential is a second range tensor as a function of the polarization indices of the initial and the final states of the photon and can be written as:

$$V = V^{\sigma\tau} \tag{29}$$

where we forget for a while about the spinor indices of the electron.

Let now K,L,M and N be a set of orthogonal four-vectors, which we choose in the form:

$$\begin{aligned} &K = k_1 + k_2 \;;\; L = k_2 - k_1 = \lambda(\kappa-\kappa') + q - p \\ &M = p+q - \frac{k}{k^2}(p+q)\,k - \frac{L}{L^2}\,(p+q)\cdot\lambda(\kappa-\kappa') \\ &N^{\lambda} = \epsilon^{\lambda\mu\nu\rho} M_{\mu} L_{\nu} K_{\rho}\,. \end{aligned} \tag{30}$$

Let us introduce:

$$\epsilon^{(1)} = \frac{M}{\sqrt{-M^2}}, \qquad \epsilon^{(2)} = \frac{N}{\sqrt{-N^2}}\,. \tag{31}$$

One can easily check that $\epsilon^{(1)}$ and $\epsilon^{(2)}$ have the following properties:

$$\begin{aligned} &[\epsilon^{(1)}]^2 = [\epsilon^{(2)}]^2 = -1, \quad \epsilon^{(1)}\cdot\epsilon^{(2)} = 0 \\ &\epsilon^{(1)}\cdot k_1 = \epsilon^{(1)}\cdot k_2 = 0, \quad \epsilon^{(2)}\cdot k_1 = \epsilon^{(2)}\cdot k_2 = 0. \end{aligned} \tag{32}$$

Therefore, we can use the vectors $\epsilon^{(1)}$ and $\epsilon^{(2)}$ to describe the polarization properties of the quasipotential.

Taking into account the gauge invariance of the quasipotential, the tensor $V^{\tau\sigma}$ can be decomposed in the usual manner/10/.

$$V^{\tau\sigma} = f_1 \epsilon_\tau^{(1)} \epsilon_\sigma^{(1)} + f_2 \epsilon_\tau^{(2)} \epsilon_\sigma^{(2)} + \\ + f_3 [\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} + \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}] + f_4 [\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} - \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}]. \tag{33}$$

The invariance under space reflections shows us that f_1 and f_2 have to be scalars and f_3 and f_4 pseudoscalars (note that $\epsilon^{(1)}$ is a vector, but $\epsilon^{(2)}$ is an axial). Therefore the spin-structure of f_1, f_2, f_3 and f_4 in the space of the electron spin should be:

$$f_1 = A_1 + B_1 \hat{K} \;;\; f_2 = A_2 + B_2 \hat{K} \\ f_3 = \gamma_5 (A_3 + B_3 \hat{K}) \;;\; f_4 = \gamma_5 (A_4 + B_4 \hat{K}), \tag{34}$$

where $\hat{K} = \gamma^\mu K_\mu$. Then $V^{\tau\sigma}$ is decomposed to eight structures:

$$V^{\tau\sigma} = V_1 I \epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} + V_2 \hat{K} \epsilon_\tau^{(1)} \epsilon_\sigma^{(1)} + V_3 I \epsilon_\tau^{(2)} \epsilon_\sigma^{(2)} + V_4 \hat{K} \epsilon_\tau^{(2)} \epsilon_\sigma^{(2)} + \\ + V_5 \gamma_5 (\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} + \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}) + V_6 \gamma_5 \hat{K} (\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} + \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}) + \\ + V_7 \gamma_5 (\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} - \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}) + V_8 \gamma \hat{K} (\epsilon_\tau^{(1)} \epsilon_\sigma^{(2)} - \epsilon_\tau^{(2)} \epsilon_\sigma^{(1)}). \tag{35}$$

The form factors $V_1, V_2, \ldots V_8$ are scalar functions of the invariant variables $t_{pq} = (p-q)^2$, s_p, s_q, κ and κ' of the form:

$$V_i = V_i\,(t_{pq}\,,\, s_p + s_q\,,\, s_p - s_q\,,\, \kappa + \kappa'\,).$$

$$(\, i = 1, 2, \ldots, 8) \qquad (36)$$

If now we require also time inversion invariance we can obtain some further information about V_i.

Under time reversal we have:

$$(k_0, \vec{k}) \to (k_0, -\vec{k}) \qquad\qquad (L_0, \vec{L}) \to (-L_0, \vec{L})$$

$$(M_0, \vec{M}) \to (M_0, -\vec{M}) \qquad\qquad (N_0, \vec{N}) \to (N_0, -\vec{N}) \qquad (37)$$

$$s_p \to s_q \qquad\qquad s_p - s_q \to -(s_p - s_q).$$

and it is easy to check that the structures which are multiplied by V_1, V_2, V_3, V_4, V_6 and V_7 do not change sign, but these multiplied by V_5 and V_8 do change it. Therefore, in order V to be T-invariant, brcause of (36) and (37), V_1, V_2, V_3, V_4, V_6 and V_7 have to be even functions and V_5 and V_8 odd functions of the difference $s_p - s_q$. On the energy shell $s_p = s_q$ i.e. $s_p - s_q = 0$ and V_5 and V_8 simply vanish and we have, as it should be, only six spin structures of the quasipotential.

5. The Quasipotential in the Second Order of the Coupling Constant

In this section we shall calculate the quasipotential for the electron-positron system in the lowest (second) order of the perturbation theory.

The second order contribution to the quasipotential we obtain as a sum of the irreducible graphs on Fig. 2.

As usualyy we split V into two terms

$$V = V_{dir} + V_{ex} \tag{38}$$

The graphs on the first row of Fig. 2 give the direct interaction $-V_{dir}$, and those on the second row - the exchange interaction $-V_{ex}$. Using the diagram technique rules of Section 2, passing to c.m.s. and taking λ in direction of the total momentum of the incoming particles, we obtain the following expressions:

$$V_{dir} = \frac{(2\pi)^4}{p_0 q_0} \frac{e}{\sqrt{\frac{1}{4}(\kappa-\kappa')^2 + m_\gamma^2 - t}\,[\frac{1}{2}(\kappa+\kappa') + \sqrt{\frac{1}{4}(\kappa-\kappa')^2 + m_\gamma^2 - t}} \times$$

$$\times \phi^+(\mu_1)\chi^+(\mu_2)\{[(p_0+m)(q_0+m) + 4pq + \frac{(pq)^2}{(p_0+m)(q_0+m)} + \frac{p_0+m}{q_0+m}\vec{q}^{\,2} + \frac{q_0+m}{p_0+m}\vec{p}^{\,2}]I^{(1)}\otimes I^{(2)} +$$

$$+ i[3 + \frac{pq}{(p_0+m)(q_0+m)}][\sigma^{(1)}\cdot(p\times q)\otimes I^{(2)} + I^{(1)}\otimes\sigma^{(2)}\times(p\times q)] + \tag{39}$$

$$+[2pq - \frac{p_0+m}{q_0+m}q^2 - \frac{q_0+m}{p_0+m}p^2]\sigma^{(1)}\otimes\sigma^{(2)} -$$

$$-\sigma^{(1)}\cdot p\otimes\sigma^{(2)}\cdot q - \sigma^{(1)}\cdot q\times\sigma^{(2)}\cdot p -$$

$$-\frac{1}{(p_0+m)(q_0+m)}\sigma^{(1)}\cdot(p\times q)\otimes\sigma^{(2)}\cdot(p\times q) + \frac{p_0+m}{q_0+m}\sigma^{(1)}\cdot q\times\sigma^{(2)}\cdot q +$$

$$+\frac{q_0+m}{p_0+m}\sigma^{(1)}\cdot p\otimes\sigma^{(2)}\cdot p\}\phi(\nu_1)\chi(\nu_2)$$

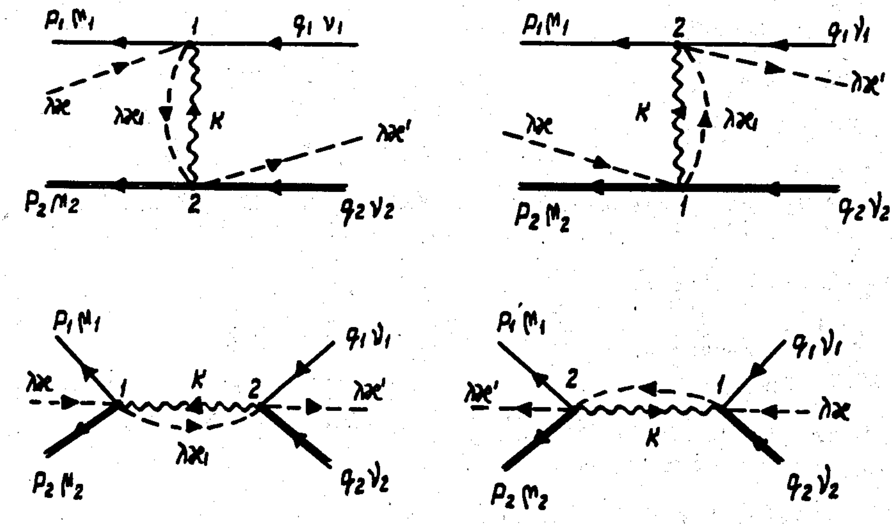


Fig.2.

and

$$V_{ex} = \frac{(2\pi)^4}{p_0 q_0} \frac{e^2}{2m_\gamma} \left[\frac{1}{\kappa + m_\gamma - \sqrt{s_p} - i\epsilon} + \frac{1}{\kappa + m_\gamma + \sqrt{s_p} - i\epsilon} \right] \frac{1}{2} \phi^+(\mu_1) \chi^+(\mu_2) \times$$

$$\times \{ [3 p_0 q_0 - \frac{p_0}{q_0 + m} q^2 - \frac{q_0}{p_0 + m} p^2 + \frac{(p\, q)^2}{(p_0 + m)(q_0 + m)}] I^{(1)} \otimes I^{(2)} +$$

$$+ [p_0 q_0 - \frac{p_0}{q_0 + m} q^2 - \frac{q_0}{p_0 + m} p^2 + \frac{(p\, q)^2}{(p_0 + m)(q_0 + m)}] \sigma^{(1)} \otimes \sigma^{(2)} + \tag{40}$$

$$+ 2 \frac{p_0}{q_0 + m} \sigma^{(1)} \cdot q \otimes \sigma^{(2)} \cdot q + 2 \frac{q_0}{p_0 + m} \sigma^{(1)} \cdot p \otimes \sigma^{(2)} \cdot p -$$

$$- \frac{p\, q}{(p_0 + m)(q_0 + m)} (p \times q) [\sigma^{(1)} \otimes I^{(2)} + I^{(1)} \otimes \sigma^{(2)}] \} \phi(\nu_1) \chi(\nu_2).$$

In low-energy approximation, i.e. keeping terms up to the order of V^2/C^2, or in our units up to $\frac{p^2}{m^2}$ and $\frac{q^2}{m^2}$, taking into account the "energy" conservation law

$$\sqrt{s_p} + \kappa' = \sqrt{s_q} + \kappa \tag{41}$$

and putting $\kappa = 0$, we obtain for the direct and the exchange components of the quasipotential:

$$V_{dir} = -e^2 (2\pi)^4 \phi^+(\mu_1) \chi^+(\mu_2) [\frac{1}{t^2} + 3i \frac{(\sigma^{(1)} \otimes \sigma^{(2)})(p \times q)}{4m^2 t^2} +$$

$$+ \frac{p\, q}{m^2 t^2} + \frac{(\sigma^{(1)} \cdot t)(\sigma^{(2)} \cdot t)}{4m^2 t^2} - \frac{\sigma^{(1)} \cdot \sigma^{(2)}}{4m^2} - \frac{p^2 - q^2}{2m|t|^3} + \frac{(p^2 - q^2)^2}{4m^2 |t|^4}] \phi(\nu_1) \chi(\nu_2), \tag{42}$$

$$V_{ex} = -e^2 (2\pi)^4 \phi^+(\mu_1) \chi^+(\mu_2) \frac{1}{8m^2} [3 + \sigma^{(1)} \cdot \sigma^{(2)}] \phi(\nu_1) \chi(\nu_2), \tag{43}$$

where $t = p - q$. On the energy shell $|p| = |q|$ and we obtain the well known formula for electron-positron interaction (see for instance ref./11/). The off-shell interaction is given by the term:

$$-\frac{p^2 - q^2}{2m|t|^3} + \frac{(p^2 - q^2)^2}{4m^2|t|^4} \qquad (44)$$

It is interesting to compare the off-shell contribution (44) to the quasipdential with the corresponding off-shell contribution to the quasipotential of Logunov and Tavkhelidze, which was calculated in ref./4/. The difference between the two expressions is due to the different off-shell continuation in these cases - in the version of Kadyshevsky the particles are on the mass shell, but of the energy shell, while in the approach of Logunov and Tavkhelidze they are off the mass but on the energy shell. Therefore, all the conclusions made in/4/ about the positronium energy levels are valis also in the present case.

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