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COHERENT STATE METHOD IN THE PROBLEMS OF HIGH ENERGY HADRON INTERACTIONS

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COHERENT STATE METHOD IN THE PROBLEMS OF HIGH ENERGY HADRON INTERACTIONS

Recently, in studying high-energy particle scattering the idea has been extensively used that the hadrons in interactions behave as complex systems with many internal degrees of freedom.

It is also assumed that the main regularities of the high energy two-particle elastic scattering are determined by the behaviour of essentially inelastic processes in the two-particle collision.

As was shown in refs.[1-3] the diffraction character of two-particle elastic scattering is naturally explained on the basis of the quasipotential equation for the scattering amplitude in quantum field theory[4,5] in the framework of the hypothesis on the smoothness of the local quasipotential as a function of the relative coordinate of two particles[6].

The corresponding quasipotential describing the high energy elastic scattering at small and large angles turns out to be approximately imaginary and has the Gaussian form which corresponds, in a certain sense, to a nonsingular interaction of two extended objects rapidly decreasing with increasing relative distance between particles[7,8].

The interpretation of such behaviour of the two-particle elastic scattering quasipotential in terms of many-particle inelastic processes is a very important problem of the quasipotential approach.

In this connection we note papers[9-11] where the Gaussian character of the elastic scattering quasipotential was found starting from the idea about the high-energy particle scattering as a random process in which the random quantities are the total momentum and the total number of particles in the intermediate states in the unitary condition[12,13].

For a more detailed knowledge of the dynamics of many-particle processes it may turn out to be advisable to study various models of high-energy hadron interactions.

In the past years a number of models has been suggested in which the hadrons considered as complex systems, in high-energy collision processes are excited and dissociated virtually into constituents (quark-antiquark pairs, "partons"[14,15], "fragments"[16,17] and so on). Some success of these models in the description of the high-energy hadron interactions indicate that the idea about the hadron as a complex system with many internal degrees of freedom is fruitful.

We should emphasize one important property which is, in our opinion, common for all the above considered models: <u>the occurance of a high degree degeneration of hadron states in high energy collisions due to the virtual dissociation of hadrons into a large number of constituents.</u> In the present paper we study a model of high-energy hadron interaction which is based on the assumption about the coherent nature of the excited states of colliding hadrons. In this model the degeneration of the states is efficiently described by a four-dimensional relativistic oscillator which plays the role of some collective variable.

Thus, in the process of collision the hadron states are modelled by the coherent wave functions of a four-dimensional oscillator[x].

The model is essentially equivalent to the account of an effective hadron structure which is due to the production of a cloud of

[x] For the definition and the properties of the coherent states in quantum optics see, for example, refs.[18,19].

real and virtual particles in high-energy interactions by redefinition of the particle coordinate.

We note that for the first time the idea about the hadron excitations of the oscillator type was apparently suggested in refs.[20-23] and then developed in a number of other papers[24,25].

We stress, however, that contrary to the oscillator models describing the mass spectrum of elementary particles we use the notion of the coherent oscillator excitations for describing the hadron degenerated states in high energy strong interaction processes.

In the next section of the paper we determine the wave functions of the hadron coherent states and study their properties. In Sections 3 and 4 the models of the elastic and inelastic high energy hadron interactions are considered.

In Section 5 the coherent state vectors are used for the factorization of the scattering amplitudes satisfying a certain subsidiary integral representation.

The results obtained in this paper are discussed in the Conclusion.

2. Definition and Properties of the Hadron Coherent Wave Function

As is known, the free particle with the momentum p is described by the wave function which is the plane wave

$$\psi_{p,\sigma}(x) = e^{-ipx} \cdot \chi_\sigma(p), \qquad (2.1)$$

where $\chi_\sigma(p)$ is a spin function, which transforms under Lorentz transformations by the Wigner rotation matrices.

The hadron state in processes of interactions at high energies is characterized by the presence of some effective structure.

We postulate that this effective structure can be taken into account by the following redefinition of the particle coordinate:

$$x_\mu \to x_\mu + \rho \cdot D_\mu \cdot \qquad (2.2)$$

Here ρ - is some parameter with dimension of length,

$$D_\mu = a^+_\mu + a_\mu , \tag{2.3}$$

where the operators a_μ and a^+_ν obey the commutator relations of a four-dimensional relativistic oscillator:[x/]

$$[a_\mu, a^+_\nu] = -g_{\mu\nu} . \tag{2.4}$$

Thus we will describe the interacting hadron by the state vector of the form:

$$|\psi_{p,\sigma}(x)> = e^{-ip(x+\rho D)} \cdot \chi_\sigma(p)|0> , \tag{2.5}$$

where the "vacuum" state $|0>$ is defined in a usual way.

$$a_\mu |0> 0. \tag{2.6}$$

The state vectors (2.5) can be normalized by the relativistically invariant way

$$i\int d\vec{x} < \psi_{p',\sigma'}(x)| \frac{\overleftrightarrow{\partial}}{\partial x_0} |\psi_{p,\sigma}(x)> = (2\pi)^3 \, 2p_0 \delta(\vec{p}\,' - \vec{p}) \delta_{\sigma'\sigma} . \tag{2.7}$$

Notice, that the vector[xx/]

$$|\psi_p> \equiv |\psi_p(0)> = e^{-i\rho(pD)} |0>. \tag{2.8}$$

[x/] We use the system of units, where $h = c = 1$, and the metric, where $g_{00} = -g_{xx} = -g_{yy} = -g_{zz} = 1$.

[xx/] In what follows for simplicity we will ignore the complications, which are connected with spins of particles.

describes the so-called coherent state of the four-dimensional relativistic oscillator and is the eigenstate of the operator a_μ :

$$a_\mu |\psi_p> = i\rho p_\mu |\psi_p> . \tag{2.9}$$

Using eq. (2.9) it is easy to show that the expectation values of the operators

$$\hat{\mathcal{P}}_\mu = i/2\rho \, (a^+_\mu - a_\mu) ;$$
$$\hat{Q}_\mu = \rho \, (a^+_\mu + a_\mu); \tag{2.10}$$

which have the meaning of the momentum and the coordinate of the four-dimensional oscillator, in the state, described by the vector (2.8), are equal to p_μ and zero, respectively.

It should be noticed, that the vectors $|\psi_p>$ with different momenta are non-orthogonal, e.g.

$$<\psi_{p'}|\psi_p> = <0|e^{i\rho(qD)}|0> = e^{bt/2} , \tag{2.11}$$

where $b = \rho^2$; $t = q^2 = (p' - p)^2$.

Consider now in more detail the question on the physical meaning of the state described by the vector of the type (2.8).

It is easy to see that the vector (2.8) can be rewritten in the following form

$$|\psi_p> = \frac{1}{\sqrt{\Omega}} e^{-i\rho(pa^+)} |0> ; \quad \frac{1}{\sqrt{\Omega}} = e^{m^2 b/2} . \tag{2.12}$$

Thus, the coherent state of the four-dimensional oscillator described by the vector (2.8), can be considered as a superposition of the states with the arbitrary number of "quants".

It should be stressed, however, that the expansion of the operator exponential in eq. (2.12) gives the sequence of states with indefenite metric.

By analogy with electrodynamics we make now the separation of the oscillator component on the scalar, longitudinal and transverse ones.

More precisely, in the frame, where

$$p = (p_0, p_z, \vec{p}_\perp) \tag{2.13}$$

we will call the operators

$$\begin{aligned} &a_0 - \text{scalar} \\ &a_z - \text{longitudinal} \\ &\vec{a}_\perp - \text{transverse} \end{aligned} \tag{2.14}$$

components of the oscillator, respectively. The state vector (2.12) allows one to separate the vatiables corresponding to the scalar, longitudinal and transverse oscillations:

$$|\psi_p\rangle = \frac{1}{\sqrt{\Omega_\perp}} e^{i\rho(\vec{p}_\perp \vec{a}_\perp^{\,+})} |\psi_L\rangle; \quad \frac{1}{\sqrt{\Omega_\perp}} = e^{-b\vec{p}_\perp^{\,2}/2} \tag{2.15}$$

Let us consider, that the vector

$$|\psi_L\rangle = \frac{1}{\sqrt{\Omega_L}} e^{-i\rho(p_0 a_0^+ - p_z a_z^+)} |0\rangle; \quad \frac{1}{\sqrt{\Omega_L}} = e^{b(p_0^2 - p_z^2)/2} \tag{2.16}$$

describes a "basic" or "normal" state of the hadron moving along the z-axis with the momentum p_z.

The state which is described by the vector (2.15) is considered as an "excited" state of the hadron with the non-zero transverse momentum $\vec{p}_\perp$.

The vector (2.15) can be expanded in the vectors of states with definite numbers of "quants" of the transverse excitation and with the positive norms:

$$|\psi_p\rangle = \frac{1}{\sqrt{\Omega_\perp}} \sum_{n=0}^{\infty} \frac{(i\rho p_\perp)^n}{\sqrt{n!}} |\psi_{L,n}\rangle, \tag{2.17}$$

where

$$|\psi_{L,n}\rangle = \frac{1}{\sqrt{n!}} (a_\perp^+)^n |\psi_L\rangle; \quad a_\perp^+ = \frac{1}{p_\perp}(\vec{p}_\perp \cdot \vec{a}_\perp^{\,+}), \tag{2.18}$$

and

$$\langle \psi_{L,n'} | \psi_{L,n} \rangle = \delta_{n'n}. \tag{2.19}$$

It is seen from the eq. (2.17), that the probability distribution for the states with definite numbers of the transverse "quants" is described by the Poisson formula:

$$P(n) = |\langle \psi_{L,n} | \psi_p \rangle|^2 = e^{-bp_\perp^2} \frac{(bp_\perp^2)^n}{n!}. \tag{2.20}$$

One can see from formula (2.20) that the average number of "quants" in the state (2.17) is proportional to the square of the transverse particle momentum:

$$\bar{n} = \sum_{n=0}^{\infty} n\, P(n) = b\, p_\perp^2. \tag{2.21}$$

§3. A Model of Elastic Hadron Scattering at High Energies

We will describe the elastic hadron scattering at high energies using an operator potential of the form:

$$\hat{V} = i\gamma^2 \cdot \Gamma_\mu^{(1)} \cdot \Gamma_\mu^{(2)} \cdot \delta(x-y), \tag{3.1}$$

where γ -is some dimensionless constant,

$$\begin{aligned} \Gamma_\mu^{(1)} &= i(a_\mu^+ - a_\mu)^{(1)}; \\ \Gamma_\mu^{(2)} &= i(a_\mu^+ - a_\mu)^{(2)}; \end{aligned} \tag{3.2}$$

and the operators corresponding to the oscillators of the type "1" and "2" commute with each other.

One can see, that the potential (3.1) is a bilinear combination of the "creation" and "annihilation" operators and describes correlated pair effects of excitation and absorbtion of the oscillator "quants" of the types "1" and "2".

The Born scattering amplitude is determined by the following expression:x/

$$(2\pi)^4\delta(p'-q'-p-q)T(s,t)= \tag{3.3}$$
$$=\int dx\,dy\langle\psi^{(1)}_{p'}(x);\psi^{(2)}_{q'}(y)|\hat{V}|\psi^{(1)}_{p}(x);\psi^{(2)}_{q}(y)\rangle ,$$

where

$$s=(p+q)^2;\quad t=(p'-p)^2 .$$

From eq. (3.3) it follows

$$T(s,t)=i\gamma^2\langle\psi_{p'}|\Gamma_\mu|\psi_p\rangle\langle\psi_{q'}|\Gamma_\mu|\psi_q\rangle . \tag{3.4}$$

Using the formula

$$\langle\psi_{p'}|\Gamma_\mu|\psi_p\rangle=\rho\,(p'+p)_\mu\, e^{bt/2}\quad;\quad b=\rho^2 \tag{3.5}$$

we get the following expression for the scattering amplitude

$$T(s,t)=ib\gamma^2(s-u)e^{bt} . \tag{3.6}$$

Notice, that the amplitude (3.6) is crossing symmetric under substitution $s\to u$:

$$T(s,t)=T^*(u,t). \tag{3.7}$$

x/ For the case of scattering of two identical particles the initial or finil state vector should be symmetrized in an appropriate way.

Assuming that the Born approximation for the scattering amplitude coincides with the two particle quasipotential, we find that the quasipotential, discribing the elastic two particle scattering at high energies is in our model a pure imaginary and of the Gaussian form as a function of the particle relative coordinate.

For the total cross section of interaction at high energies eq. (3.6) gives

$$\sigma_{tot}=2b\gamma^2 . \tag{3.8}$$

Using the relation between the parameters b and the effective interaction radius R /3/:

$$4b=R^2 \tag{3.9}$$

we find that to the geometrical limit of the total cross section $\sigma_{tot}=2\pi R^2$ there corresponds the following value of interaction constant

$$\gamma^2/4\pi=1. \tag{3.10}$$

Consider now an elastic scattering of two hadrons in the center of mass system.

The high energy limit correponds in this system to the limit of infinitely large particle momenta along one of the axis, for example, the z -axis.

The scattering amplitude is determined in this case by the following limit of the vertex matrix element:

$$\lim_{p_z\to\infty}\frac{1}{2p_0}\langle\psi_{p'}|\Gamma_\mu|\psi_p\rangle=\begin{cases}\rho\, e^{bt/2}; & \mu=0,z ,\\ 0; & \mu=x,y .\end{cases} \tag{3.11}$$

where $t=-\vec{\Delta}_\perp^2=-(\vec{p}'_\perp-\vec{p}_\perp)^2$.

Thus in the limit of the infinite momentum along the z -axis only the scalar and longitudinal "quants" take part in the exchange during two particle scattering.

Notice, that such a behaviour is analogous to the electromagnetic excitation of nucleon in the frame system "$p_z = \infty$" [26].

§4. Generalization to Inelastic Processes

Here we consider possible generalization of the model proposed above to the case of inelastic processes.

First of all, we consider the process of scattering when some additional number of the transverse "quants" of the oscillator type is excited.

The state of a hadron with the momentum p in the presence of the n additional "quants" of transverse excitation will be described by the vector:[x]

$$|\psi_p(x);\nu_n> = e^{-ip(x+\rho D)} \cdot \frac{a^+_{i_1} \dots a^+_{i_2} \dots a^+_{i_n}}{\sqrt{n!}} |0> , \qquad (4.1)$$

where

$$\nu_n = \{ i_1, i_2, \dots, i_n \}; \quad i_k = x, y .$$

The amplitudes of the corresponding processes are determined by the following vertex matrix elements:

$$\lim_{p_z \to \infty} \frac{1}{2p_0} < \psi_p, ; \nu_n | \Gamma_\mu | \psi_p > = \begin{cases} \rho e \dfrac{(-i\rho)^n}{\sqrt{n!}} \Delta_{i_1} \cdot \Delta_{i_2} \dots \Delta_{i_n}; \mu = 0, z \\ 0; \quad \mu = x, y ; \end{cases} \qquad (4.2)$$

where $t = -\vec{\Delta}^2_\perp = -(\vec{p}\,'_\perp - \vec{p}_\perp)^2$.

The differential cross sections of collisions with the excitation of one or both hadrons equal respectively:

[x] Here we consider the frame system where the colliding hadrons have large momenta along z -axis. It can be shown, that in that system the excitation of the non-transverse "quants" can be neglected in the limit of high energies.

$$\left(\frac{d\sigma^*}{dt}\right)_n = \left(\frac{d\sigma}{dt}^{e\ell}\right)_{t=0} e^{-2b\Delta^2_\perp} \frac{(b\Delta^2_\perp)^n}{n!}; \quad n = 1, 2, \dots , \qquad (4.3a)$$

$$\left(\frac{d\sigma^{**}}{dt}\right)_{n,m} = \left(\frac{d\sigma^{e\ell}}{dt}\right)_{t=0} e^{-2b\Delta^2_\perp} \frac{(b\Delta^2_\perp)^{n+m}}{n!\,m!}; n, m = 1, 2, \dots , \qquad (4.3b)$$

where

$$\left(\frac{d\sigma^{e\ell}}{dt}\right)_{t=0} = \frac{\sigma^2_{tot}}{16\pi} = \frac{1}{\pi}(b\gamma^2)^2 .$$

As is seen from eqs. (4.3) the probabilities of the processes with the excitation of a definite number of the transverse "quants" are described by the Poisson formula.

The average number of excited "quarks" on one hadron is proportional to the square of the transverse momentum transfer:

$$(\bar{n})_{\text{one hadron}} = b\Delta^2_\perp = -bt . \qquad (4.4)$$

For the average value of the total number of excitations we have obviously:

$$(\bar{n})_{tot} = 2b\Delta^2_\perp = -2bt . \qquad (4.5)$$

Notice, that the dependence of the differential cross sections of the inelastic processes on the square of momentum transfers $\Delta^2_\perp$. becomes considerably smoother after summing over the number of the "quants".

In particular, taking a sum over the total number of all the "quants" excited in hadron collision, we find that the total differential cross section does not depend at all on the variable t :

$$\left(\frac{d\sigma}{dt}\right)_{tot} = \sum_{n,m=0}^{\infty} \left(\frac{d\sigma^{**}}{dt}\right)_{n,m} = \left(\frac{d\sigma}{dt}\right)^{e\ell}_{t=0} = \text{const} . \qquad (4.6)$$

Such a regularity has probably ruther general grounds[27] and holds in a number of models. In particular an analogous behaviour is observed for the so-called deep inelastic processes of lepton-hadron interactions at high energies and large momentum transfers[26,28].

The differential cross sections of these processes after summing over all the channels have a quite simple asymptotic dependence on kinematic invariants determined by the dimensional analysis and automodelity principle /28/.

Notice, that up to now we deal with some excited states of the four-dimensional oscillator which represent in some sense collective excitations of the system of two colliding hadrons.

There arises an interesting problem of interpretation of the model considered above in the framework of usual quantum field theory.

One of the possible way of solving this problem consists in finding an appropriate representation of the oscillator components in terms of the real particle production and annihilation operators.

In particular in the proceeding description of inelastic hadron collisions with the production of a number of real secondary particles, we will use the following representation of the transverse oscillator components.

$$a_i = \int \frac{d\vec{k}}{k_0} f(\vec{k}) k_i a(\vec{k});$$
$$a_i^+ = \int \frac{d\vec{k}}{k_0} f^*(\vec{k}) k_i a^+(\vec{k}); \qquad (i = x, y) \quad , \tag{4.7}$$

where $a^+(\vec{k})$ and $a(\vec{k})$ are the production and annihilation operators of particles with the space momentum $\vec{k}$, which obey the commutation relations x/

$$[a(\vec{k}), a^+(\vec{k}')] = k_0 \delta(\vec{k}' - \vec{k}). \tag{4.8}$$

x/ For the sake of simplicity we consider here only neutral spinless Bose-particles. A generalization to the case of charged particles and particles with spin can be done in a simple way.

The operators (4.7) give the representation of the commutation relations of the two-dimensional oscillator

$$[a_i , a_j^+] = \delta_{ij} \quad ; \quad (i, j = x, y) \tag{4.9}$$

under the following condition: x/

$$\int \frac{d\vec{k}}{k_0} |f(\vec{k})|^2 k_i k_j = \delta_{ij} \, . \tag{4.10}$$

Obviously, under requirement of the azimuthal symmetry of the function $f(\vec{k})$ in the center of mass system the condition (4.10) can be easily satisfied.

Eqs. (4.7) define the packets of particles with the invariant distribution function of momenta $|f(\vec{k})|^2$, and in accordance with eq. (4.10) the average value of the squared transverse momentum of particles in the packet should be finite.

The vector of the state of the hadron with the momentum p after collision together with a definite number of secondary particles produced in collision is defined as follows:

$$|\psi_p(x); \vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n \rangle = e^{-ip(x+\rho D)} \frac{1}{\sqrt{n!}} \prod_{i=1}^{n} e^{-ik_i x} a^+(\vec{k}_i) |0\rangle \, , \tag{4.11}$$

where the vector $|0\rangle$ corresponds to the vacuum of the real particles and to the "vacuum" of the scalar and longitudinal "quants" of oscillator.

The amplitudes of related processes are determined by the vertex matrix elements of the following type :

x/ Notice, that the function $f(\vec{k})$ depends in a general case on the momenta of colliding hadrons.

$$
\lim_{p_z \to \infty} \frac{1}{2p_0} \langle \psi_{p'}(0); \vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n | \Gamma_\mu | \psi_p \rangle =
$$

$$
= \begin{cases} \rho \, e^{-b\Delta_\perp^2/2} \dfrac{(-i\rho)^n}{\sqrt{n!}} \prod_{i=1}^{n} (\vec{\Delta}_\perp \cdot \vec{k}_\perp)_i \, f^*(\vec{k}_i) & ; \mu = 0, z ; \\ 0; \quad \mu = x, y , \end{cases} \tag{4.12}
$$

where

$$
\vec{\Delta}_\perp^2 = (\vec{p}\,'_\perp - \vec{p}_\perp)^2 .
$$

The differential cross sections of inelastic processes with the production of a definite number of secondary particles by one or both colliding hadrons are equal, respectively:

$$
\left(\frac{d\sigma^*}{dt}\right)_n = \left(\frac{d\sigma^{el}}{dt}\right)_{t=0} \cdot W_n(\Delta); \quad n = 1, 2, \ldots \tag{4.13a}
$$

$$
\left(\frac{d\sigma^{**}}{dt}\right)_{n,m} = \left(\frac{d\sigma^{el}}{dt}\right)_{t=0} \cdot W_n(\Delta) \, W_m(-\Delta) \quad ; \quad n, m = 1, 2, \ldots , \tag{4.13b}
$$

where Δ is the momentum transferred from the initial hadron to the system of the final hadron together with secondary particles, i.e.

$$
\Delta = (p' - p + \sum_{j=1}^{n} k_i) \quad \text{or} \quad = -(q' - q + \sum_{j=1}^{m} k'_j).
$$

The quantity $W_n(\Delta)$ is defined by the expression

$$
W_n(\Delta) = \lim_{p_z \to \infty} \frac{1}{4p_0^2} \cdot \int |\langle \psi_{p'}; \vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n | \Gamma_0 | \psi_p \rangle|^2 \frac{1}{n!} \prod_{i=1}^{n} \frac{d\vec{k}_i}{k_{0_i}} =
$$

$$
= \frac{1}{n!} \int e^{b(\Delta - \sum_{i=1}^{n} k_i)^2} \cdot \prod_{i=1}^{n} \frac{d\vec{k}_i}{k_{oi}} | f(\vec{k}_i) |^2 \cdot b(k_i, \Delta - \sum_{j=1}^{n} k_j)^2 , \tag{4.14}
$$

where the transferred momentum $(\Delta - \sum_{i=1}^{n} k_i)$ in the considered limit of large particle momenta along z -axis is approximately a pure transverse vector. The region of integration in eq. (4.14) is limited by the low of momentum conservation, e.g.:

$$
(p' + \sum_{i=1}^{n} k_i)^2 < s + \Delta^2 . \tag{4.15}
$$

One can see from eq. (4.14) that the distribution functions of the secondary particle momenta are factorized.

Notice, that such a behaviour is suggested in a number of models of the multiparticle production at high energies /12,35/.

Eq. (4.14) allows one to find, in principle, the function $f(\vec{k})$, up to an arbitrary phase, using experimental data on distributions of particle momenta in the processes of the multiparticle production in high-energy hadron collisions.

Consider now an approximation when the total transverse momentum of secondary particles can be neglected when comparing with the transverse component of momentum transfer $\vec{\Delta}_\perp$. In this approximation eq. (4.14) takes a very simple form x/

$$
W_n(\Delta) = e^{-b\Delta_\perp^2} \cdot \frac{1}{n!} \left[\int \frac{d\vec{k}}{k_0} | f(\vec{k}) |^2 \, b(\vec{k} \cdot \vec{\Delta}_\perp)^2 \right]^n . \tag{4.16}
$$

Using the normalization condition (4.10), it is easy to see, that eq. (4.16) coincides with the Poisson formula, with the average number of secondary particles produced by one of the colliding hadrons is determined by the variation of its transverse momentum /17/

$$
\bar{n} = b\Delta_\perp^2 \approx b(\vec{p}\,'_\perp - \vec{p}_\perp)^2 . \tag{4.17}
$$

x/ The region of integration in eq. (4.16) is limited effectively by the condition $k_0 \lesssim p_0$.

The distribution of the secondary particles over the momenta is described by the formula:

$$\frac{d\bar{n}}{dk} = \frac{1}{k_0} |f(\vec{k})|^2 \cdot b(\vec{k} \cdot \vec{\Delta}_\perp)^2 . \tag{4.18}$$

Introducing the new variable $x = \frac{k_z}{p_0}$ /14,15/ and assuming the existence of the limit:

$$\rho(x, k_\perp^2) = \lim_{\substack{p_0 \to \infty \\ x = k_z/p_0 = \text{fixed}}} |f(\vec{k})|^2 \tag{4.19}$$

we rewrite eq. (4.18) in the form:

$$\frac{d\bar{n}}{dx\, d^2 k_\perp} = \frac{1}{x} \rho(x, k_\perp^2) b(\vec{k}_\perp \cdot \vec{\Delta}_\perp)^2 ; \quad 0 < x \leq 1 \tag{4.20}$$

As follows from eq. (4.20) the momentum distribution of secondary particles in the azimuthal plane which is perpendicular to z-axis, is determined by the cosinus of the angle between the transverse particle momentum and transverse component of the momentum transfer $\vec{\Delta}_\perp$:

$$\frac{d\bar{n}}{d\phi} \approx \cos^2 \phi . \tag{4.21}$$

§5. Coherent State Method and the Factorization Problem of Dual Scattering Amplitude

Recently, in description of hadron interactions the scattering amplitudes are widely used which satisfy the so-called duality principle[29]. According to this principle the scattering amplitude is determined in an alternative way either by a sum over all the Regge-poles in t -channel or by a sum over all the resonances in s -channel. A simple example of dual scattering amplitude with equidistant pole singularities in all the channels and the Regge asymptotic behaviour corresponding to linear trajectories in appropriate channels, is given by the Veneziano formula[30].

In paper[31] the generalized Veneziano type representation was dirived on the basis of the finite-energy sum rules for the scattering amplitudes satisfying the duality principle.

An auxiliary integral parametric representation for a scattering amplitude was essentially used which in the t-channel takes the form:

$$F(s,t) = \int_0^1 dx\, x^{-1-\alpha(t)} f(x,s) ; \qquad \alpha(t) = \alpha(0) + \alpha' \cdot t . \tag{5.1}$$

Below we consider the problem of constructing a scattering amplitude satisfying eq. (5.1) by means of the coherent state vectors of the type

$$|\psi_p\rangle = e^{-i\rho(pD)} |0\rangle : \alpha' = \rho^2 . \tag{5.2}$$

It is convenient to make a change of variable $x = e^{-z}$ in eq. (5.1)

$$F(s,t) = \int_0^\infty dz\, e^{z\alpha(t)} \cdot \phi(z,s) . \tag{5.3}$$

where

$$\phi(z,s) = f(e^{-z}, s) .$$

Introduce the operators

$$\hat{S}^{(1)} = \frac{1}{2} [a_\mu^{+2} - a_\mu^2]^{(1)} ; \qquad \hat{S}^{(2)} = \frac{1}{2} [a_\mu^{+2} - a_\mu^2]^{(2)} . \tag{5.4}$$

With the following commutation properties:

$$[\hat{S}^{(1)}, D_\mu^{(1)}] = D_\mu^{(1)} ; \quad [\hat{S}^{(2)}, D_\mu^{(2)}] = D_\mu^{(2)} . \tag{5.5}$$

It can be shown that

$$e^{z\cdot\alpha' t} = \langle \psi_{p'}^{(1)} ; \psi_{q'}^{(2)} | z^{\hat{S}^{(1)} + \hat{S}^{(2)}} | \psi_p^{(1)} ; \psi_q^{(2)} \rangle . \tag{5.6}$$

Using eq. (5.6) we get the representation of the amplitude (5.3) in terms of the coherent state vectors

$$F(s,t) = \langle \psi^{(1)}_{p'} ; \psi^{(2)}_{q'} | \hat{G}(s) | \psi^{(1)}_{p} ; \psi^{(2)}_{q} \rangle \qquad (5.7)$$

$$s = (p+q)^2 ,$$

where

$$\hat{G}(s) = \int_0^\infty dx \; e^{z\alpha(0)} \; z^{\hat{S}^{(1)}+\hat{S}^{(2)}} \; \phi(z,s) . \qquad (5.8)$$

A crossing symmetrical in t and u amplitude can be constructed by the symmetrization of the initial or the final two particle state vector in eq. (5.7).

The crossing transformation $s \to t$ corresponds to the substitution

$$\psi^{(1)}_{p'} \to \psi^{(2)}_{-q}$$

$$\psi^{(2)}_{q} \to \psi^{(1)}_{-p'} , \qquad (5.9)$$

so that

$$F(t,s) = \langle \psi^{(2)}_{-q} ; \psi^{(2)}_{q'} | \hat{G}(t) | \psi^{(1)}_{p} ; \psi^{(1)}_{-p'} \rangle$$

$$t = (p - p')^2 . \qquad (5.10)$$

The other transformations of crossing symmetry are defined in an analogous way.

The amplitudes of the type (5.7) and (5.10) can be written as matrix elements of some local interaction, by analogy with eq. (3.3). In doing so we should change the variables s or t in the argument of the operator $\hat{G}$ on the D'Alemberte operator $\Box = -\partial^2_\mu$.

Notice, that for the Veneziano amplitude the operator has the form

$$\hat{G}(s) = \int_0^\infty dz \; e^{z\alpha(0)} \; z^{\hat{S}^{(1)}+\hat{S}^{(2)}} \; (1 - e^{-z})^{-1-\alpha(s)} . \qquad (5.11)$$

The asymptotic limit of the expression (5.11) is as follows

$$\hat{G}(s) \underset{s\to\infty}{\to} s^{\alpha(0)} \cdot (\ell n\, s)^{\hat{S}^{(1)}+\hat{S}^{(2)}} \; \Gamma(-\alpha(0)) . \qquad (5.12)$$

Thus we have shown that by means of the coherent state vectors it is possible to construct the Veneziano amplitude as well as the more general scattering amplitude satisfying the representation(5.1). As it is seen from eqs. (5.7) and (5.10) the dependence of scattering amplitudes constructed in this way on the variables s and t is factorized.

It is possible that the method proposed here can be useful in studying the general problem of factorization of manyparticle scattering amplitudes satisfying duality principle /32,33/.

6. Conclusion

Here we discuss some features of the model proposed above. First of all, we should stress that the model makes use essentially, besides the coherence hypothesis of hadron states, a special form of the potential operator, describing both the elastic and inelastic processes in high-energy hadron collisions. This form of the potential is the only one which is bilinear in the "creation" and "annihilation" oscillator operators and gives non-vanishing contribution to the total cross section in the limit of high energies.

Propably, this potential gives in the Born approximation a description of the diffraction-like processes at not so large momentum transfers.

In principle, we should take into account the rescattering corrections to the Born approximation which should give in a particular case of elastic scattering the eikonal representation of the scattering amplitude /1-3,35/. Let us discuss now the form of the excited hadron state vector (4.1). The relativistically invariant form of this vector is as follows

$$|\psi^*_p(x); n\rangle = e^{-ip(x+\rho D)} \frac{1}{\sqrt{n!}} \mathcal{E}_{\mu_1\mu_2\ldots\mu_n}(p)\, a^+_{\mu_1} a^+_{\mu_2} \ldots a^+_{\mu_n} |0\rangle , \qquad (5.1)$$

where the coefficient function $\mathcal{E}_{\mu_1\mu_2\ldots\mu_n}(p)$ obeys the condition

1. the total symmetry in the indices μ_i, $i = 1,2,\ldots,n$.
2. the "transversality" condition

$$p_{\mu_i}\,\mathcal{E}_{\mu_1\ldots\mu_i\ldots\mu_n}(p) = 0; \quad i = 1,2,\ldots,n. \tag{5.2}$$

3. the normalization condition

$$(-)^n\,\mathcal{E}^*_{\mu_1\mu_2\ldots\mu_n}\cdot\mathcal{E}^{\mu_1\mu_2\ldots\mu_n} = 1. \tag{5.3}$$

Under these conditions the state vector (5.1) has a positive norm and describes a family of excited hadrons with the spectrum of angular momenta $j \leq n$ and with the squared mass $m_*^2 = p^2$. There exists an equation which the state vector of the type (5.1) obeys and from which it follows that the mass spectrum has an equidistant form[x]. We are going to study this problem elsewhere.

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