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W. Tybor, A.N. Zaslavsky<br>E- $\eta$ MIXING AND SOME RADIATIVE DECAY PROCESSES

ААЕОРАТОРМЯ TEO PETMUEKKOИ் МВМКИ

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# E- $\eta$ MIXING AND SOME RADIATIVE DECAY PROCESSES 

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> Е- - -смешивание и радиационные распады мезонов

Обсуждаются следствия $E-\eta$-смешивания в предположении, что девятое место в псевдоскалярном нонете занимает $\mathrm{E}(1420)$-мезон. Приведены значения ширин распада $\mathrm{E} \rightarrow 2 \boldsymbol{\gamma}, \mathrm{E} \rightarrow \rho \gamma, \mathrm{E} \rightarrow \omega \gamma$ и дрь, вычисленные в различных моделях. Наиболее предпочтительное эначение $\Gamma$ ( $\mathrm{E} \rightarrow 2 \gamma$ ) $(200 \sim 300)$ кэв оказывается довольно большим, а ширина распада $\mathrm{E} \rightarrow \rho \gamma$ в -10 Мэв. Обнаружение распада $\mathrm{E} \rightarrow 2 \gamma$ окончательно доказало бы псевдоскалярность Е-мезона, а исследование ширин радиационных распадов Е-мезона помогло бы в выборе правильной модели.

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E - $\eta$ Mixing and Some Radiative Decay Processes
The consequences of $\mathrm{E}-\eta$ mixing under the assumption that the $E(1420)$ meson takes the ninth place in a pseudoscalar nonet are discussed. The width values of $\mathbf{E} \rightarrow 2 \gamma, E \rightarrow \rho \gamma, \mathbf{E} \rightarrow \omega \cdot \gamma$, etc. found in various models are presented. A value $\Gamma(E \rightarrow 2 y)$ -- (200 - 300) KeV turns out to be rather large, the decay width $E \rightarrow \rho \gamma$ being $\sim 10 \mathrm{MeV}$. The determination of the decay $E \rightarrow 2 \gamma$ would prove the pseudoscalarity of the $E$-meson, the investigation of the $E$-meson radiative decay width would help to select reasonable models.

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1. At present there are two main candidates for the ninth. pseudoscalar meson: $X^{0}(960)$ and $E(1420)$ mesons.

In the literature, especially in theoretical papers the $X^{0}(960)$. meson is taken as the ninth pseudoscalar one. As is seen now, this tendency may be irregular, since the conclusion about the pseudoscalarity $X^{0}(960)$ in previous papers was not final. The spin-parity $2^{-}$for the $X^{0}$-meson is possible as well as the spin-parity $0^{-11,2 \mid}$. Available experimental data do not exclude it.

Let us consider the properties of the $\mathrm{E}(1420)$-meson under the condition that this meson is just the 9 th pseudoscalar one.

The effects connected with $X^{0}-\eta$ mixing were considered in detail in a current algebra $/ 3,4 /$, in various models $/ 5,6 /$ by many authors beginning from Dalitz and Sutherland $/ 7 /$.

The consequences of $\mathrm{E}-\eta$ mixing for the radiative decays $\mathrm{E} \rightarrow 2 \gamma, \mathrm{E} \rightarrow \rho \gamma, \mathrm{E} \rightarrow \omega \gamma$, etc, obtained by the method of ref. $/ 7 /$ are discussed in this paper.

The mass formulas (presented in detail in $/ 8 /$ ) which unambiguously select the particle with a mass of $=1.4 \mathrm{GeV}$ as the ninth pseudoscalar meson are considered in $\S 2$.
$. E-\eta$ mixing for decays amplitudes is considered in \& 3. The probabilities of the radiative decays $\mathrm{E}(1420) \rightarrow 2 y, \mathrm{E} \rightarrow \rho \gamma$, $\mathbf{E} \rightarrow \omega \gamma$, etc. obtained in various models are given here in. The result of our calculations, as in the case of $X^{0}-\eta$ mixing, depend on the models. The most preferable value $\Gamma(\mathrm{E} \rightarrow 2 \gamma)-(200-$ - 300) KeV is rather great, and owing to a large phase volume the decay width $E \rightarrow \rho \gamma$ is equal to $\approx 10 \mathrm{MeV}$ according to our estimations. These conditions favour the experimental investigations of the radiative decays of the E(1420) meson.

The searches for the radiative decays are of great importance since the determination of the decay $E \rightarrow i \gamma \gamma$ would definitely prove the pseudoscalarity of the E(1420) meson and would completely exclude another possible value of the spin-parity $1^{+}$.
2. Under general assumptions on the type of $\mathrm{SU}_{\mathrm{w}}(6)$ symmetry breaking the mass formula

$$
\begin{equation*}
\mathrm{m}_{\pi}^{2}+\mathrm{m}_{\eta}^{2}+\mathrm{m}^{2}{ }^{2}=\mathrm{m}_{\rho}^{2}+\mathrm{m}_{\phi}^{2}+\mathrm{m}^{2} \tag{1}
\end{equation*}
$$

was obtaines in ref. $/ 8 /$ i.e. the sum of the nonstrange meson squared masses in pseudoscalar and vector nonets is identical. The formula (1) unambiguously selects the $\mathrm{E}(1420)$ meson (not the $X^{0}$-meson) as the ninth pseudoscalar one. As far as in this case, as mentioned above in §, 1 there is no experimental proof of the pseudoscalarity of the $\mathrm{X}^{0}(960) / 1,2 /$, let us consider the $\mathrm{E}(1420)$ meson as the 9 th member of a pseudoscalar nonet with $\mathrm{E}-\eta$ mixing angle equal to $-6.5^{\circ}$. Under this condition we now discuss the radiative decays of the $E$-meson.
3. Taking into account $\operatorname{SU}(3)$ with octet breaking the matrix elements of the radiative decay of $\pi^{0}, \eta$ and $E(1420)$ mesons are written as follows:

$$
\begin{align*}
& \mathrm{T}(\mathrm{E} \rightarrow 2 \gamma)=\mathrm{M}_{1} \cos \theta_{\mathrm{p}}+\mathrm{M}_{8}(1+\beta) \sin \theta_{\mathrm{p}} \\
& \mathrm{~T}(\eta \rightarrow 2 \gamma)=-\mathrm{M}_{1} \sin \theta_{\mathrm{p}}+\mathrm{M}_{8}(1+\beta) \cos \theta_{\mathrm{p}}  \tag{2}\\
& \mathrm{~T}\left(\pi^{0} \rightarrow 2 \gamma\right)=\sqrt{3} \mathrm{M}_{8},
\end{align*}
$$

where $M_{8}$ and $M_{1}$ are octet and singlet amplitudes; $\beta-$ is the SU(3) breaking contribution, $\theta_{\mathrm{p}}=-6.5^{\circ}$. As is easily seen, there is no relation between matrix elements $T\left(\pi^{0}\right), T(\eta)$ and $T(E)$ since the number of independent amplitudes is equal to three. In order to restrict the number of amplitudes, different assumptions are made, and the results of the calculation depend strongly on them. For example, under different assumptions ${ }^{/ 3-5 /}$ for $\Gamma\left(X^{0} \rightarrow 2 \gamma\right)$ the values, varying in a wide range of (1-50) KeV have been obtained. A similar unambiguity of the result and its dependence on the model obviously remain under the assumption of $\mathrm{E}-\eta$ mixing.
a) In the framework of current algebra and technique of hard pions Riazuddin and Sarker have obtained the following expression for $\beta^{/ 3 /}$ :

$$
\frac{\mathrm{T}\left(\eta_{\mathrm{B}}+2 \gamma\right)}{\mathrm{T}\left(\pi^{0} \rightarrow 2 \gamma\right)}=1+\beta=\frac{\mathrm{f}_{\pi}}{\mathrm{f}_{\eta_{8}}} \frac{\mathrm{~m}_{\omega_{8}}^{2}}{\mathrm{~m}_{\pi}^{2}} \frac{\mathrm{~g}_{\rho}}{\mathrm{g}_{\omega_{8}}} \frac{2 \mathrm{f}_{\pi}^{2}-\mathrm{f}_{\eta_{8}}^{2}}{\mathrm{f}_{\pi}^{2}}
$$

where $f_{\pi}, f_{\eta_{8}}$ are the decay constants of the corresponding particles. The values of the constants in this formula are not well determined, and the authors use two different values of $\mathrm{E}_{\omega_{8}}$. We follow their choice and obtain for the breaking parameter $\beta$ depending on $\mathrm{g}_{\omega_{8}}$ :

$$
\beta=\left\{\begin{array}{l}
0 ; 57 \\
0,3
\end{array} .\right.
$$

Using the experimental data $/ 1 /$ for $\Gamma\left(\pi^{0} \rightarrow 2 \gamma\right)$ and $\Gamma(\eta \rightarrow 2 \gamma)$ we obtain from the formula (2)

$$
\begin{align*}
\Gamma(\mathrm{E} \rightarrow 2 \gamma)= & (155 \pm 80) \mathrm{keV} .  \tag{3}\\
& (260 \pm 100) \mathrm{keV} .
\end{align*}
$$

Large errors in the determination of $\Gamma(E \rightarrow 2 y)$ results from the errors in the measurement of the width of the radiative decays of $\pi^{0}$ and $\eta$-mesons.

Following Dalitz and Sutherland $/ 7 /$, knowing $\Gamma(\mathrm{E} \rightarrow 2 \gamma)$ it is easy to calculate $\Gamma(\mathrm{E} \rightarrow \rho \gamma)$ and $\Gamma(\mathrm{E} \rightarrow \omega \gamma)$ :

$$
\frac{\Gamma(E \rightarrow 2 \gamma)}{\Gamma(E \rightarrow \rho \gamma)}=\frac{\alpha m_{E}^{s}}{9\left(\frac{g_{\rho}^{2}}{4 \pi}\right)}\left(\frac{2 m_{E}}{m_{E}^{2}+m_{\rho}^{2}}\right)^{2}\left(\frac{2 m_{E}}{\left.m_{E}^{2} m^{2}\right)_{\rho}^{3}}\left[\frac{1+\frac{M_{B}(1+\beta)}{M_{1}} \operatorname{tg} \theta_{p}}{1+\frac{2 M_{B}(1+\beta)}{M_{1}} \operatorname{tg} \theta_{p}}\right]_{\mathrm{E}}^{2}\right.
$$

$$
\begin{equation*}
\approx 0,0228\left[\frac{1+\frac{M_{8}(1+\beta)}{M_{1}} \operatorname{tg} \theta_{p}}{1+\frac{2 M_{8}(1+\beta)}{M_{1}} \operatorname{tg} \theta_{p}}\right]^{2} \tag{4}
\end{equation*}
$$

$$
\Gamma(\mathrm{E} \rightarrow \rho \gamma)-1 \begin{align*}
& (6,9 \pm 3,5) \mathrm{MeV} \\
& (11,4 \pm 4,4) \mathrm{MeV} \tag{5}
\end{align*}
$$

$$
\frac{\Gamma(\mathrm{E} \rightarrow \omega \gamma)}{\Gamma(\mathrm{E} \rightarrow \rho \gamma)}=\frac{1}{9}\left(\frac{\mathrm{~m}_{\mathrm{E}}^{2}-\mathrm{m}^{2}}{\mathrm{~m}_{\mathrm{E}}^{2}-\mathrm{m}^{2}} \rho_{\mathrm{E}}^{3}\right)^{3} ; \Gamma(\mathrm{E} \rightarrow \omega \gamma)=1 \quad(0,72 \pm 0,36) \mathrm{MeV}
$$

The $E-\eta$ mixing angle, being equal to $-6.5^{\circ}$ differs from the mixing angle $X^{0}-\eta$. Due to this fact the $\phi \rightarrow \eta \gamma$ and $\omega \rightarrow \eta \gamma$ decay rates are different from those in $|3,4,7|$ calculated under the assumption of $X^{0}-\eta$ mixing and are within the limits of experimental estimates:

$$
\Gamma(\phi \rightarrow \eta \gamma)=\int_{\underset{(175 \pm 15)}{ } \mathrm{keV} .}^{(210 \pm 20) \mathrm{keV} .} \quad \Gamma(\omega \rightarrow \eta \gamma)=\left\{\begin{array}{l}
(3,14 \pm 0,27) \mathrm{keV} \\
(1,90 \pm 0,16) \mathrm{keV} .
\end{array}\right.
$$

The exact $\operatorname{SU}(3)$ symmetry taking into account $E-\eta$ mixing $(\beta=0)$ predicts a somewhat greater value $/ 9 /$ of $\Gamma(E \rightarrow 2 \gamma)$
$\Gamma(\mathrm{E} \rightarrow 2 \gamma)=(425 \pm 140) \mathrm{keV}$.
and, consequently, $\Gamma(E \rightarrow \rho \gamma), \Gamma(E \rightarrow \omega \gamma)$, etc. are also shifted:

$$
\begin{array}{ll}
\Gamma(\mathrm{E} \rightarrow \rho \gamma)=(18,5 \pm 6,5) \mathrm{MeV}, & \Gamma(\phi \rightarrow \eta \gamma)=(120 \pm 10) \mathrm{keV} . \\
\Gamma(\mathrm{E} \rightarrow \omega \gamma)=(1,92 \pm 0,68) \mathrm{MeV} . & \Gamma(\omega \rightarrow \eta \gamma)=(0,42 \pm 0,04) \mathrm{keV} .
\end{array}
$$

b) S. Matsuda and S. Oneda have recently obtained the following relations for the amplitudes $T\left(\eta^{\prime}\right)$ and $T(\eta)$ in their model of SU(3) breaking $/ 5 /$

Assuming that $\eta^{\prime} \nexists X^{0}(960)$ and $\theta_{p}=-10.5^{\circ}$, the authors have obtained

$$
\mathbf{T}_{(\eta \rightarrow 2 y)}=\frac{1}{\sqrt{3}} \mathbf{T}\left(\pi^{0} \rightarrow 2 \gamma\right)(1,5-1,7)
$$

In this case $\Gamma(\eta \rightarrow 2 y)$ is larger by a factor of $(2.5-3)$ than the value predicted by exact $\mathrm{SU}(3)$ i.e. $\Gamma(\eta \rightarrow 2 y) \approx(400-600) \mathrm{eV}$. This is not far from the experimental value $\Gamma(\eta \rightarrow 2 \gamma)=(1,00 \pm 0,22) \mathrm{keV}$. and this is the main result of the work $/ 5 /$. As mentioned in $\S 1$, our assumption consists in that the E(1420) meson is the ninth pseudoscalar one. In this case the formulas (8) give

$$
\begin{aligned}
& \mathrm{T}(\eta \rightarrow 2 \gamma)=\frac{1}{\sqrt{3}} \mathrm{~T}\left(\pi^{0} \rightarrow 2 \gamma\right) \times(1,16-1.18) \\
& \mathrm{T}(\mathrm{E} \rightarrow 2 \gamma)=0.82 \mathrm{~T}\left(\pi^{0} \rightarrow 2 \gamma\right) .
\end{aligned}
$$

We obtain that $\Gamma(\eta \rightarrow 2 y)=(225 \pm 55) \mathrm{eV}$. This is in a sharp disagreement with experiment. Thus, if $\eta^{\prime} \equiv \mathrm{E}(1420)$, the model involved in $/ 5 /$ does not improve the $\mathrm{SU}(3)$ result for $\Gamma(\eta+2 y)$. Taking into account the violation $(X=0.1 / 5 /)$ does not change this statement. In this case a small value of $\Gamma(E \rightarrow 2 \gamma)=(5,2 \pm 1,1) \mathrm{keV}$ is predicted for the decay width $\mathrm{E} \rightarrow 2 \gamma$. This value significantly differs from the result given by the formula (3).
c) In ref. $/ 9 /$ the width of the decay $E \rightarrow 2 \gamma$ is calculated by us in the model of broken $\mathrm{SU}_{\mathrm{w}}(6)$-symmetry with a special type of breaking. An anomalously great value $(-1 \mathrm{MeV})$ for $\Gamma(E \rightarrow 2 \gamma)$ is obtained, the application of the formula (4) gives - 40 MeV for $\Gamma(\mathrm{E} \rightarrow \rho \cdot \gamma)$. The vector dominance does not give predictions with a high accuracy, nevertheless, these results turn out to be overestimated.
4. We have discussed some model for the calculation of $E \rightarrow 2 \gamma$ and $E \rightarrow \rho \gamma$ under the assumption of $E-\eta$ mixing with the mixing angle $\quad \theta_{\mathrm{p}}=-6.5^{\circ}$. The width values obtained in various models are presented in Table. As is seen, the results strongly depend on the model, The most preferable values are
$\Gamma(\mathrm{E} \rightarrow 2 \gamma)-(200-300) \mathrm{keV}$ and $\Gamma(\mathrm{E} \rightarrow \rho \gamma)-10 \mathrm{MeV}$ calculated in the Riazuddin and Sarker's model ${ }^{1 / 3 /}$.

In conclusion we stress that the exprimental determination of the decays $\mathrm{E} \rightarrow 2 \boldsymbol{\gamma}, \mathrm{E} \rightarrow \boldsymbol{\rho} \boldsymbol{\gamma}$ and the measurement of their widths are an important task. The solution of this problem would make clear the matter of $\mathrm{E}(1420)$. meson spin-parity, would be critical for the considered models and would permit to select between them.

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| Model | $\Gamma(\eta-2 y)$ | $\Gamma(E \rightarrow 2 \gamma)$ | $\Gamma(E \rightarrow p y)$ | $\Gamma(E-\omega y)$ |
| :---: | :---: | :---: | :---: | :---: |
| Model ${ }^{3}$ $\left\{\begin{array}{l} 0,57 \\ 0,3 \end{array}\right.$ | (1.0+0.22) keV input | $\left\{\begin{array}{l}(155 \pm 80) \mathrm{keV} \\ (260 \pm 100) \mathrm{keV}\end{array}\right.$ | $\left\{\begin{array}{l}(6,9 \pm 3,5) \mathrm{MeV} \\ (11,4 \pm 4,4) \mathrm{MeV}\end{array}\right.$ | $\left\{\begin{array}{l}(0,72 \pm 0,36) \mathrm{MeV} \\ (1,20 \pm 0,45) \mathrm{MeV}\end{array}\right.$ |
| $\begin{aligned} & \text { Exact } \operatorname{su}(3) \\ & \text { स土th } \\ & \text { E- } \eta \text { mixclng } 9 \end{aligned}$ | $(1.0 \pm 0,22) \mathrm{keV}$ Input | $(425 \pm 140) \mathrm{keV}$ | $(18,5 \pm 6,5) \mathrm{MeV}$ | $(1,92 \pm 0,68) \mathrm{MeV}$ |
| $\begin{aligned} & \text { Model }{ }^{5} \text { with } \\ & \mathrm{E}_{-\eta} \text { mixing } \end{aligned}$ | $(225 \pm 55) \mathrm{eV}$ | $(5.1+1.1) \mathrm{keV}$ | $(225 \pm 50) \mathrm{keV}$ | $(24 \pm 5) \mathrm{ker}$ |
| Model 9 <br> broken SU ${ }_{W}(6)$ | $\begin{gathered} (880 \pm 190) \cdot \mathrm{eV} \\ \text { 1nput } \end{gathered}$ | $(940 \pm 380) \mathrm{keV}$ | $(41 \pm 16) \mathrm{MeV}$ | $(4.3 \pm 1.7) \mathrm{MeV}$ |

