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SCALE INVARIANCE, CURRENT COMMUTATORS AND VECTOR DOMINANCE IN DEEP INELASTIC LEPTON-HADRON INTERACTIONS

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CONTENT

1. Introduction

One of the main problems of elementary particle theory is the investigation of the behaviour of electromagnetic and weak interactions at high energies. The most general process of interaction of a leptonic pair with a hadron system can be represented as follows:

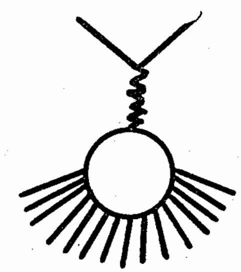
$T_{fi} =$ 

Fig. 1. Matrix element T_{fi} describing an arbitrary electromagnetic or weak process of interaction of a leptonic pair with a system of hadrons.

It factorizes into the leptonic and hadronic parts

$$T_{fi} = c\, L^{\mu}\, H_{\mu} \quad . \tag{1.1}$$

The explicit form of the "coupling constant" c and of the leptonic part L^{μ} is well known. As to the hadronic part the situation is more complicated. Starting from the analogy with electrodynamics where the local currents give a correct description of phenomena it is postulated that there exist operators of local hadron currents namely of electromagnetic $J_{\mu}^{e.m}(x)$ and weak $J_{\mu}^{w}(x)$ ones. These operators have a definite experimental meaning consisting in that their matrix elements are directly connected with observables (cross sections, polarization, etc.). In the most elegant way these quantized currents arise in the Bogolubov's formulation of field theory as a response of the particle system to an external nonquantized perturbation:

$$J_{\mu}(x) = (\Box - m^2)\, A_{\mu} = \frac{1}{i}\, S^{+} \frac{\delta S}{\delta A_{\mu}^{ext}} \Big|_{A_{\mu}^{ext}=0} . \tag{1.2}$$

Thus, the factors entering the matrix element can be represented in the form

$$ {}^{e.m}_{\;w}c = \begin{cases} \dfrac{4\pi a}{q^2} \\ G/\sqrt{2} \end{cases} \quad L = \begin{cases} \bar{u}\gamma^{\mu}u \\ \bar{u}\gamma^{\mu}(1-\gamma)u \end{cases} \quad H = \begin{cases} \langle f|\, J_{\mu}^{e.m}(0)\,|\, i\rangle \\ \langle f|\, J_{\mu}^{W}(0)\,|\, i\rangle . \end{cases} \tag{1.3}$$

In spite of the fact that the explicit form of the hadron part of the matrix element remains unknown it is possible, nevertheless, to

obtain definite information on the hadronic part using the requirements of relativistic covariance, P, C and T transformations and the selection rules resulting from internal SU(2) or SU(3) symmetries. There remains the main theoretical difficulty, i.e. the absence of the quantitative description of the dynamics of strong interactions. This leads to the appearance in the theory of unknown functions, of the so-called structure functions of form factors. The well-known example is the nucleon electromagnetic form factors $G_E(q^2)$ and $G_M(q^2)$ depending on the one Lorentz-invariant variable. Generally speaking, the form factos may depend on several Lorentz-invariant variables. The main task is actually to study theoretically and experimentally these form factors. These investigations may throw light on some fundamental problems such as particle structure, existence of elementary constituents of hadrons (quarks, partons, etc.).

The importance of studying deep inelastic processes was stressed by several authors/1-5/. In works/4/ some general methods were developed for studying deep inelastic strong interaction processes and rigorous estimates for the amplitudes were obtained. These methods can give useful information also in studying the behaviour of form factors of deep inelastic lepton-hadron processes.

We list certain experimentally realizable deep inelastic lepton-hadron processes. Depending on the fact, which particles in Fig. 1 are incoming and which are outgoing, these processes can be divided into three types:

a) annihilation of a lepton pair,

b) scattering of a lepton on a hadron,

c) production of a lepton pair in hadron-hadron collision.

The corresponding matrix elements are presented in Fig.2

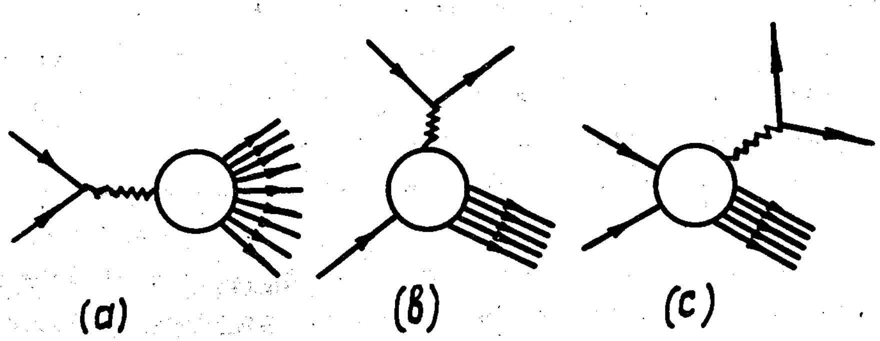


Fig.2. Matrix elements of the processes a, b, c. In processes a and c q^2 is time-like $q^2 > 0$; in process b q^2 is space-like $q^2 < 0$.

The electromagnetic and weak scattering processes corresponding to the diagram 2b have been recently investigated most extensively. It is just the process of deep inelastic scattering of electron on proton

$$e^- + p \to e^- + \text{hadrons} \qquad (1.4)$$

that has been investigated in SLAC experiments. From these experiments it has become evident that the electroproduction process has a very interesting point-like picture. It turns out that the differential cross section $\frac{d\sigma}{dq^2}$ for large q^2 is large and is approximately equal to the Mott cross section for scattering on a structureless point nucleon. This has served as a basis for suggesting and checking a number of theoretical ideas[1-3,5]. A similar point-

like picture has also been observed in the CERN experiments on deep inelastic neutrino-nucleon scattering[7,8]

$$\nu_\mu + N \to \mu^- + \text{hadrons}. \qquad (1.5)$$

The simplest explanation of these facts consists in that with increasing number of channels their total contribution to the form factors depends weakly on q^2. For a point nucleon in the case of neutrino production, on the basis of the simplest diagram we have

$$\sigma_{teor.}(E) = 1.3 \cdot 10^{-38} \text{ sm}^2 ,$$

where E is the neutrino energy in the lab. system, in GeV. The CERN experiments give:

$$\sigma_{exp}(E) = (0.8 \pm 0.2)\, 10^{-38} \text{ sm}^2 = 0.6\, \sigma_{teor}(E)$$

In principle, there is a possibility of studying this process at the neutrino energies up to 50 GeV by means of the accelerator of the Institute of High Energy Physics (Serpukhov).

In the present review we shall investigate in detail the process corresponding to diagram 2c, namely the deep inelastic process of muon pair production in hadron-hadron collisions

$$p + p' \to \mu^+ + \mu^- + \text{hadrons} . \qquad (1.6)$$

Below, in § 2 we give a kinematic analysis of this process. To obtain dynamic information three theoretical schemes will be considered which are based on scale invariance (§3), current commutators (§4) and vector dominance (§5). This consideration is based on the results of the works[9-13], which have recently been performed at Dubna. The process (6) is being studied on the Brookhaven accelerator and some preliminary data are given in ref.[14].

The results obtained in investigating process (1.6) are of great interest by themselves and may be of great value in searching for an intermediate W-meson produced in strong interactions[15-19].

Note that the next stage in studying processes (1.4) and (1.5) consists in singling out any one hadron in the final state. Such processes

$$e^- + p \to e^- + p' + \text{hadrons} \qquad (1.7)$$

$$\nu_\mu + p \to \mu^- + p' + \text{hadrons} \qquad (1.8)$$

have been studied theoretically in the papers by R.F. Kögerler and one of the authors (R.M.)[20].

2. Kinematic Analysis

Let us consider the process of deep inelastic collision of two hadrons a and b which results in the production of a muon pair and of a certain hadron system A :

$$a + b \to \mu^+ + \mu^- + A \; . \qquad (2.1)$$

In the lowest order in electromagnetic interaction the process proceeds through the emission and decay of a virtual photon according to Fig. 3.

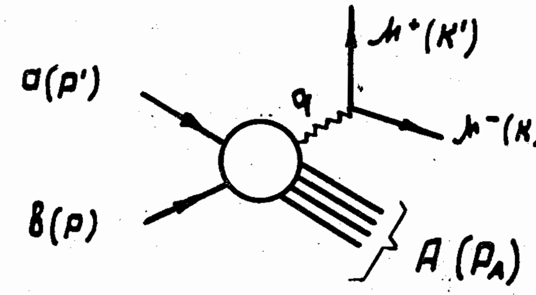


Fig. 3. Kinematics of the lepton pair production.

where the notations of the particle four momenta are given in brackets. The appropriate matrix element of the T-matrix is defined by the expression

$$T_{fi} = \frac{4\pi\alpha}{q^2} j^\mu < A \text{ out} \mid J_\mu(0) \mid p, p' \text{ in} >^c , \qquad (2.2)$$

where $j^\mu = \bar{u}(k)\gamma^\mu v(k')$ is the muon pair electromagnetic current, $J_\mu(x)$ - is the operator of the hadron electromagnetic current, $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ stands for the fine structure constant. The symbol "c" means that it is necessary to take into account only a connected part of the matrix element of the current. If colliding particles are unpolarized and in the final state only the muon pair is detected then the cross section of the process in question can be expressed by means of the following second rank tensor:

$$\rho_{\mu\nu}(p,p',q) = \sum_A (2\pi)^4 \delta(p+p'-q-p_A) <p,p' \text{ in} | J_\mu(0) | A \text{ out}><A \text{ out} | J_\nu(0) | p,p' \text{ in}>^c . \qquad (2.3)$$

Due to the electromagnetic current conservation this tensor must satisfy the gauge invariance condition $q^\mu \rho_{\mu\nu} = \rho_{\mu\nu} q^\nu = 0$ and from hermiticity $\rho_{\mu\nu} = \rho^*_{\nu\mu}$ it follows that the real part of the tensor must be symmetrical and its imaginary part antisymmetrical under replacing $\mu \leftrightarrows \nu$.

It is convenient to decompose the tensor $\rho_{\mu\nu}$ according to the structures corresponding to definite virtual photon polarizations /9,11,21,41/. We determine the directions of the three-dimensional polarization vectors $\vec{\epsilon}^{(T_1)}$, $\vec{\epsilon}^{(T_2)}$ and $\vec{\epsilon}^{(L)}$ in the rest system of a virtual photon $\vec{q} = 0$ i.e. in the c.m.s. of the muon pair according to Fig.4.

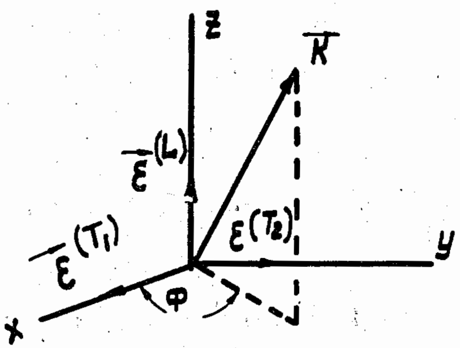


Fig.4. The c.m.s. of the muon pair. The axis z is directed along the momentum $\vec{p}$ and the momentum $\vec{p}\,'$ lies in the production plane xz . The normal to the production plane is directed along the y axis. The axis z is directed along the vector $\vec{p}$ and the vector $\vec{p}\,'$ lies in the xz plane.

Then the corresponding four-dimensional polarization vectors have the form:

$$\epsilon^{(T_1)}_\mu = \frac{1}{\sqrt{-\left(P'^2 - \frac{(PP')^2}{P^2}\right)}} \left(P'_\mu - \frac{PP'}{P^2} P_\mu\right) \qquad (2.4a)$$

$$\epsilon^{(T_2)}_\mu = \frac{1}{\sqrt{q^2 (pp')^2 - q^2 m^2 m'^2}} \epsilon_{\mu\alpha\beta\gamma} p^\alpha p'^\beta q^\gamma \qquad (2.4b)$$

$$\epsilon^{(L)}_\mu = \frac{1}{\sqrt{-P^2}} P_\mu , \qquad (2.4c)$$

where

$$P_\mu = p_\mu - \frac{pq}{q^2} q_\mu , \quad P'_\mu = p'_\mu - \frac{p'q}{q^2} q_\mu . \qquad (2.5)$$

It is not difficult to see that the polarization vectors are orthogonal to the virtual photon momentum q_μ and to one another, their norm is -1:

$$q^\mu \epsilon^{(i)}_\mu = 0 , \quad \epsilon^{(i)}_\mu \epsilon^{(j)\mu} = -\delta_{ij} ; \quad (i,j = T_1, T_2, L) \qquad (2.6)$$

and the completeness conditions

$$\sum_{i=T_1,T_2,L} \epsilon^{(i)}_\mu \epsilon^{(i)}_\nu = -q_{\mu\nu} + \frac{q_\mu q_\mu}{q^2} \qquad (2.7)$$

holds.

Using these vectors we decompose the tensor into five independent structures

$$\rho_{\mu\nu} = \rho_{T_1} \epsilon^{(T_1)}_\mu \epsilon^{(T_1)}_\nu + \rho_{T_2} \epsilon^{(T_2)}_\mu \epsilon^{(T_2)}_\nu + \rho_L \epsilon^{(L)}_\mu \epsilon^{(L)}_\nu +$$
$$+ \rho^{(+)}_{TL} \left(\epsilon^{(T_1)}_\mu \epsilon^{(L)}_\nu + \epsilon^{(T_1)}_\nu \epsilon^{(L)}_\mu\right) + i \rho^{(-)}_{TL} \left(\epsilon^{(T_1)}_\mu \epsilon^{(L)}_\nu - \epsilon^{(T_1)}_\nu \epsilon^{(L)}_\mu\right) . \qquad (2.8)$$

The structure functions or form factors ρ_{T_1}, ρ_{T_2}, ρ_L, $\rho_{TL}^{(+)}$, $\rho_{TL}^{(-)}$ x/ are real functions depending on four independent Lorentz-invariant variables which may be presented, for example, by $s=(p_1+p_2)^2$, q^2, $\nu = pq$, $\Delta^2 = (p'-q)^2 = m'^2+q^2-2\nu'$. We can also fix other invariant variables, e.g. $m_N^2=(p+p'-q)^2$ being the squared effective mass of the hadron system, or the energy transfer $\delta = \frac{1}{m} p(p'-q)$ in the lab. system $\vec{p}=0$.

We note that in the system $\vec{q}=0$ there is a simple relation between the space components of the tensor ρ_{ij} and of the form factor

$$\| \rho \| = \begin{pmatrix} \rho_{xx} & 0 & \rho_{xz} \\ 0 & \rho_{yy} & 0 \\ \rho_{zx} & 0 & \rho_{zz} \end{pmatrix} = \begin{pmatrix} \rho_{T_1} & 0 & \rho_{TL}^{(+)} + i\rho_{TL}^{(-)} \\ 0 & \rho_{T_2} & 0 \\ \rho_{TL}^{(+)} - i\rho_{TL}^{(-)} & 0 & \rho_L \end{pmatrix} \qquad (2.9)$$

x/ In notations of ref./21/ $\rho_{T_1} = F_3$, $\rho_{T_2} = F_2$, $\rho_L = F_3$, $\rho_{TL}^{(+)} = F_4$, $\rho_{TL}^{(-)} = F_5$. These form factors have kinematical singularities. The form factors ρ_1, ρ_2, ρ_3, ρ_4, ρ_5 free of kinematical singularities are determined below, in expansion (2.16).

Up to the normalization this is the density matrix of the virtual photon given in a linear basis x/.

The angular distribution summed over the spins is equal, by definition, to the ratio of the quinary differential cross section

$$\frac{d^5\sigma\,(s,q^2,\Delta^2,\nu,\theta,\Phi)}{dq^2\, d\Delta^2\, d\nu \quad d\Omega}$$

to the triple one

$$\frac{d^3\sigma\,(s,q^2,\Delta^2,\nu)}{dq^2\, d\Delta^2\, d\nu}$$

$$W(\theta,\Phi) \equiv W(\theta,\phi,s,q^2,\Delta^2,\nu) = \frac{d^5\sigma\,(s,q^2,\Delta^2,\nu,\theta,\Phi)}{d^3\sigma\,(s,q^2,\Delta^2,\nu)},$$

x/ Passing from the linear basis to the helicity basis

$$\epsilon_\mu^{(\pm 1)} = \mp \frac{1}{\sqrt{2}} \left(\epsilon_\mu^{(T_1)} \pm i\epsilon_\mu^{(T_2)}\right) \qquad \epsilon_\mu^{(0)} = \epsilon_\mu^{(L)} \qquad (2.10)$$

following Oakes/21/ we get the relation between the form factors and normed matrix element of the density matrix in the helicity basis:

$$\begin{aligned} \rho^{11} &= \rho^{-1-1} = \frac{1}{2\rho}\left(\rho_{T_2}+\rho_L\right) \\ \rho^{00} &= \frac{1}{\rho}\,\rho_L \\ \rho^{1-1} &= \rho^{-11} = \frac{1}{2\rho}\left(\rho_{T_2}-\rho_L\right) \\ \rho^{10} &= \rho^{01*} = -\rho^{-10} = \rho^{0-1*} = -\frac{1}{\sqrt{2}\,\rho}\left(\rho_{TL}^{(+)}+i\rho_{TL}^{(-)}\right), \end{aligned} \qquad (2.11)$$

where

$$\rho = \rho_{xx}+\rho_{yy}+\rho_{zz} = \rho_{T_1}+\rho_{T_2}+\rho_L \qquad (2.12)$$

$$W(\theta,\phi) = \frac{1}{4\pi(1-\frac{v^2}{3})\rho}\left[\rho_{T_1}(1-v^2\sin^2\theta\cos^2\Phi) + \right.$$

$$+ \rho_{T_2}(1-v^2\sin^2\theta\sin^2\Phi) + \rho_L(1-v^2\cos^2\theta) - \tag{2.13}$$

$$\left. - \rho^{(+)}_{TL} v^2 \sin 2\theta \cos\Phi \right],$$

where ρ is given by eq. (2.12); $v = \frac{|\vec{k}|}{E} = \sqrt{\frac{q^2-4m_\mu^2}{q^2}}$ is the velocity of muons in their c.m.s. Studying this angular distribution it is possible to determine ρ_{T_1}, ρ_{T_2}, ρ_L, $\rho^{(+)}_{TL}$ rather than $\rho^{(-)}_{TL}$. The form factor $\rho^{(-)}_{TL}$ can be found by measuring the polarization of one of the muons along the normal to the production plane (along the axis y in Fig.2):

$$\langle S_y \rangle \approx m_\mu E \rho^{(-)}_{TL} . \tag{2.14}$$

Integrating eq. (2.13) over $d\Phi$ or over $d\cos\theta$ we find the distributions only over θ or over Φ

$$W(\theta) = \frac{1}{2\rho(1-\frac{v^2}{3})}\left[(\rho_{T_1}+\rho_{T_2})(1-\frac{v^2}{2}\sin^2\theta) + \rho_L(1-v^2\cos^2\theta)\right] \tag{2.15a}$$

$$W(\Phi) = \frac{1}{2\pi\rho(1-\frac{v^2}{3})}\left[\rho_{T_1}(1-\frac{2}{3}v^2\cos^2\Phi) + \rho_{T_2}(1-\frac{2}{3}v^2\sin^2\Phi) + \rho\right.$$

$$\left. + \rho_L(1-\frac{v^2}{3})\right] \tag{2.15b}$$

respectively.

The form factors $\rho_{T_1} \ldots \rho^{(-)}_{TL}$ have kinematical singularities. The form factors ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, free of kinematical singularities can be determined in the following way [9]:

$$\rho_{\mu\nu} = \rho_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \rho_2 P_\mu P_\nu + \rho_3 P'_\mu P'_\nu + \tag{2.16}$$

$$+ \rho_4(P_\mu P'_\nu + P_\nu P'_\mu) + i\rho_5(P_\mu P'_\nu - P_\nu P'_\mu),$$

where P_μ, P_ν have been determined above in (2.5). It is not difficult to establish the relationship between these two sets of form factors

$$\rho_L = \rho_1 - P^2\rho_2 - \frac{(PP')^2}{P^2}\rho_3 - 2PP'\rho_4 \tag{2.17a}$$

$$\rho_{T_1} = \rho_1 - \frac{(PP')^2 - P^2P'^2}{P^2}\rho_3 \tag{2.17b}$$

$$\rho_{T_2} = \rho_1 \tag{2.17c}$$

$$\rho^{(+)}_{TL} \pm i\rho^{(-)}_{TL} = \frac{PP'}{P^2}\left[P^2P'^2 - (PP')^2\right]^{1/2}\rho_3 + \tag{2.17d}$$

$$+ \left[P^2P'^2 - (PP')^2\right]^{1/2}(\rho_4 \pm i\rho_5) .$$

The triple differential cross section of the process (2.1), when in the final state only one muon pair with definite q^2, Δ^2 and δ is detected and the summation is performed over all possible hadron states, reads

$$\frac{d^3\sigma(s,q^2,\Delta^2,\delta)}{dq^2\, d\Delta^2\, d\delta} = -\frac{\alpha^2}{8\pi^2}\left(1-\frac{q^2-4m_\mu^2}{3q^2}\right)\sqrt{\frac{q^2-4m_\mu^2}{q^2}}\times$$

$$\times\frac{1}{\sqrt{s-(m+m')^2}\sqrt{s-(m'-m)^2}}\,\rho\,(s,\,q^2,\,\Delta^2,\,\delta)\,, \tag{2.18}$$

where $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$, m , m', m_μ are the masses of hadrons a and b and the muon, respectively. By $\rho\,(s\,,q^2,\Delta^2,\delta)$ we denote the following quantity

$$\rho\,(s,q^2,\Delta^2,\delta) = \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\rho_{\mu\nu}(p,p',q) = \rho_{T_1}+\rho_{T_2}+\rho_L\,. \tag{2.19}$$

Notice that the tensor $\rho_{\mu\nu}(p,p',q)$ describes the contents of the hadron "black box" of the Compton effect on two hadrons, presented in Fig.5.

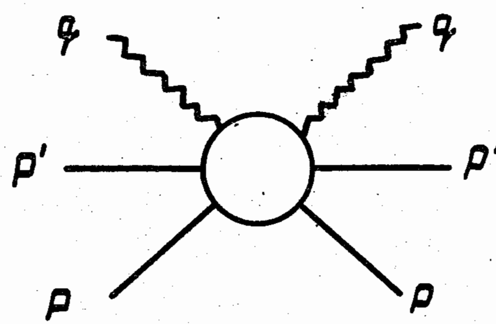


Fig.5. Amplitude of the Compton effect on two hadrons in the forward direction.

The distribution over the squared effective mass of the muon pair is obtained from eq. (2.2) by integrating over $d\,\Delta^2$ and

$d\,\delta$ inside the physical domain. Neglecting the muon mass we get the following formula for di-muon mass spectrum[x/]

$$\frac{d\sigma}{d\,q^2} = -\frac{\alpha}{12\pi^2}\,\frac{m}{\sqrt{s-(m+m')^2}\sqrt{s-(m-m')^2}}\,\frac{1}{q^2}\int_{\Delta^2_{min}}^{\Delta^2_{max}} d\Delta \int_{\delta_{min}}^{\delta_{max}} d\delta\,\rho\,(s,q^2,\Delta^2,\delta). \tag{2.20}$$

For the purpose of applying the vector dominance hypothesis (see below §5) it is convenient to represent the mass spectrum formula in the form

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{3\pi}\,\frac{1}{q^2}\,\sigma^{\gamma^*}(s,\,q^2)\,, \tag{2.21}$$

where

$$\sigma^{\gamma^*}(s,q) = \sigma^{\gamma^*}_{T_2} + \sigma^{\gamma^*}_{T_1} + \sigma^{\gamma^*}_{L} \tag{2.22}$$

is the total cross section of production of a virtual γ^* photon with mass q^2 in the process

$$a + b \to \gamma^* + \text{hadrons}. \tag{2.23}$$

In conclusion of this paragraph we note that there is an interesting kinematical analogy between the reaction considered here and the reaction of inelastic neutrinoproduction. Indeed, if in Appendix to the paper by Adler/2/ we replace the square of the lepton mass $-m_\ell^2$ by our q^2 and the Adler's q^2 by our $-\Delta^2$ then we essentially determine the boundary of the physical domain of the

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x/ The determination of the boundaries of the physical domain is considered in Appendix.

process (1). A detailed determination of this boundary is given in Appendix to the present review. To obtain dynamical information on the form factors below we shall consider three theoretical schemes based on

1. scale invariance,
2. vector dominance,
3. current commutators.

3. Scale Invariance

As has already been mentioned, the SLAC and CERN experiments indicate a point-like character of deep inelastic interactions of leptons with hadrons. Such a behaviour could be understood on the basis of the hypothesis of approximate scale invariance. Let us suppose that in describing deep inelastic processes in which the energy and momentum transfer are large, the dimensional quantities such as masses, "elementary lengths" and others are not essential. Therefore the form factors can depend only upon the variable kinematical invariants.

Out attention to the possibility of the existence of an scale invariant behaviour of the form factors in the problems under consideration has been drawn by N.N. Bogolubov. He pointed out that such a behaviour may be very similar to the so-called automodel solutions of a number of problems of classical hydrodynamics, for instance, of the problem of a strong point-like explosion [23,24]. In finding of automodel or scale invariant solutions of the problems of hydrodynamic it is very useful to employ the methods of the theory of similarity and dimensionality in combination with certain qualitative considerations about the character of physical processes. It is known that the electromagnetic and weak interactions are rather well des-

cribed by means of local electromagnetic and weak currents, while the effect of strong interactions is described by introducing the form factors. It may be expected that at low energies the necessity of taking into account the particle masses would distort the picture of strong interactions while at high energies (and large values of other invariant variables) when the masses of produced particles can be neglected the situation is essentially simplified and, in a certain sense, becomes "hydrodynamical". Qualitatively this hypothesis is supported with the fact that the main singularities of the singular functions of field theory on the light cone are independent of masses (see, e.g. ref. [25]).

In this paragraph we shall try to discuss the principle of approximate scale invariance as applied to the processes of lepton-hadron interactions at high energies and large momentum transfers and shall draw a number of consequences which can be checked experimentally.

We shall assume that the asymptotic behaviour of the form factors of processes involving leptons at high energies and large momentum transfers is due to the dimensionality considerations and the requirement of approximate invariance for scale transformations

$$q \to \lambda q$$
$$p_i \to \lambda p_i ,$$

where q is the momenta transferred from leptons to hadrons, p_i are the momenta of hadrons involved in the reaction.

This assumption essentially means that in the asymptotic limit under consideration when

$$q^2 \to \infty \qquad q p_i \to \infty$$

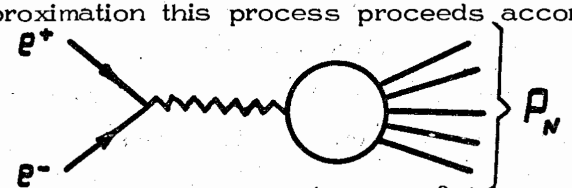
the form factors of the processes involving leptons are defined by the functions of dimensionless ratios $\omega_i = \frac{q^2}{q\,p_i}$ and are approximately independent of the particle masses and other dimensional parameters such as interaction radius, etc. We stress that this principle is unapplicable to the processes of purely strong interactions since in this case the processes appear to depend noticeably on constant dimensional quantities.

Below a number of consequences will be deduced from the scale invariance principle for the processes of annihilation of electron-positron pairs to hadrons, for the electron production process and for the process of production of leptonic pairs in hadron-hadron collisions.

We first consider the simplest deep inelastic process involving leptons, i.e. annihilation of a leptonic pair to hadrons

$$e^+ + e^- \to \text{hadrons.}$$

In the one-photon approximation this process proceeds according to the diagram:



and the total cross section is of the form (for $m_e = 0$)

$$\sigma_{tot.} = \frac{8\pi^2\alpha^2}{q^2}\,\rho(q^2) \quad . \tag{3.1}$$

All the information on the dynamics of the process is contained in the unknown spectral function (form factor) $\rho(q^2)$ which, by definition, is related to the tensor $\rho_{\mu\nu}(q^2)$ by the relation

$$\rho_{\mu\nu}(q) = \int dx\, e^{iqx} \langle 0|\, J_\mu(x)\, J_\nu(0)\, |0\rangle =$$

$$= \sum_N (2\pi)^4 \delta(q - p_N) \langle 0|\, J_\mu(0)\, |N\rangle\langle N|\, J_\nu(0)\, |0\rangle = \tag{3.2}$$

$$= (-g_{\mu\nu} q^2 + q_\mu q_\nu)\, \rho(q^2) \quad .$$

It is easily to calculate the dimensionality of the tensor x/

$$[\rho_{\mu\nu}(q^2)] = [m^2] \quad . \tag{3.3}$$

Hence, it is seen that, as should be expected, $\rho(q^2)$ is dimensionless:

$$[\rho(q^2)] = I \quad . \tag{3.4}$$

Under scale transformations

$$q \to \lambda q \tag{3.5}$$

taking into account the automodelity principle it follows that

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x/ We use the system of units in which the action and the velocity are dimensionless and the mass is chosen as a dimensional quantit We recall that in this system the dimensionality of the current is $[J_\mu] = [m^3]$ the n -particle state vector for a relativistically invariant normalization has the dimension:

$$[\,|p_1, p_2, p_3 \ldots p_n\rangle\,] = [m^{-n}]$$

$$\rho_{\mu\nu}(\lambda q) = \lambda^2 \rho \quad (q)$$
$$\rho.(\lambda^2 q^2) = \rho(q^2) = \text{const} . \tag{3.6}$$

Thus, for large q^2 the total cross section must asymptotically behave in a "point" manner analogously to the case of annihilation of the electron-positron pair to the muon pair

$$e^+ + e^- \to \mu^+ + \mu^- . \tag{3.7}$$

Such a behaviour coincides with the prediction of the algebra of quark currents[26,27]. Using the inverse Fourier transform it is possible to restore the space-time picture and to obtain that the electromagnetic current commutator between the vacuum states is

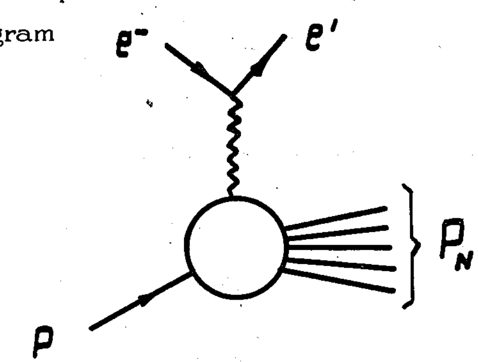
$$<0 | [J_\mu(x), J_\nu(0)] | 0> = \frac{ic}{\pi}(g_{\mu\nu}\Box - \partial_\mu \partial_\nu)\delta(\vec{x}) \; P(\frac{1}{t}) \tag{3.8}$$

where $c = \rho(q) = \text{const.}$ $\Box$ is the d'Alambert operator and P is the symbol of the principal value. Hence, in particular, it follows that the equal time commutator between the time and space components is of the form:

$$<0 | [J_0(\vec{x},0), J_i(0)] | 0> = \lim_{\tau \to 0} \frac{1}{\tau^2} \frac{ic}{\pi} \nabla_i \delta(\vec{x}) , \tag{3.9}$$

i.e. it is equal to the Schwinger term with quadratically divergent c-number coefficients[28].

The electroproduction process in the one-photon approximation is described by the diagram



and the cross section is expressed in a well-known manner (see, e.g.[29]) in terms of the tensor

$$W_{\mu\nu}(p,q) = \sum_N (2\pi)^4 <p | J_\mu(0) | N)^\circ <N | J_\nu(0) | p>^\circ \delta(p+q-p_N) =$$

$$= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) W_1(p,q,q^2) + \tag{3.10}$$

$$+ (p_\mu - \frac{pq}{q^2} q_\mu)(p_\nu - \frac{pq}{q^2} q_\nu) W_2(pq, q^2) .$$

It is easy to make oneself sure that the tensor is dimensionless

$$[W_{\mu\nu}(p,q)] = 1 \tag{3.11}$$

from where it follows that

$$[W_1(q^2, pq)] = 1$$
$$[W_2(q^2, pq)] = [m^{-2}] . \tag{3.12}$$

From the scale invariance it follows that under scale transformation

$$q \to \lambda q \tag{3.13}$$
$$p \to \lambda p$$

the form factors W_1 and W_2 must obey the following requirements:

$$W_1(\lambda^2 q^2, \lambda^2 pq) = W_1(q^2, pq)$$
$$\lambda^2 W_2(\lambda^2 q^2, \lambda^2 pq) = W_2(q^2, pq) \qquad (3.14)$$

These requirements can be satisfied by putting

$$W_1(q^2, pq) = F_1\left(\frac{q^2}{pq}\right), \quad W_2(q^2, pq) = \frac{1}{q^2} F_2\left(\frac{q^2}{pq}\right) . \qquad (3.15)$$

Such a universal dependence of the form factor W_2 on one dimensionless variable q^2/pq has, in fact, been observed in the experiments on the Stanford accelerator[6]. In ref.[22] some theoretical arguments in favour of this dependence are given. Note that at present ,many attempts are being made to understand qualitatively such a point behaviour of the electroproduction form factors at large momentum transfers by constructing appropriate models (see, e.g. refs.[30-32]) . Now we go over to the consideration of the process of muon pair production in strong interaction. The dimension of the tensor $\rho_{\mu\nu}$ and, consequently, of the form factors ρ_i are

$$[\rho_{\mu\nu}] = [\rho_i] = [m^{-2}] \qquad i = T_1, T_2, L, TL^{(+)}, TL^{(-)} \qquad (3.16)$$

Using the scale invariance requirement and taking (3.2) into account we get

$$\rho_{\mu\nu}(\lambda p, \lambda p', \lambda q) = \lambda^{-2} \rho_{\mu\nu}(p, p', q)$$
$$\rho_i(\lambda^2 s, \lambda^2 q^2, \lambda^2 \Delta^2, \lambda^2 \delta) = \lambda^{-2} \rho_i(s, q^2, \Delta^2, \delta) . \qquad (3.17)$$

Hence, it follows that the form factors for large values of the invariants must be of the form

$$\rho_i(s, q^2, \Delta^2, \delta) = \frac{1}{q^2} F_i(\alpha, \beta, \omega) , \qquad (3.18)$$

where α, β, ω are dimensionless variables composed from the ratios of the invariants s, q^2, Δ, δ .

The automodelity principle can also be used in analysing the behaviour of the form factors of weak processes. Of great interest is the experimental check of the consequences of the approximate scale invariance principle for the process of deep inelastic neutrino-nucleon scattering. We note that the automodel or self-like character of the form factors of electromagnetic or weak processes makes it possible in the asymptotic domain, firstly, to decrease by unity the number of independent variables and, secondly, knowing the form factors for one set of invariants to predict their value for another one provided that their ratios remain fixed.

In our opinion, the greatest interest would be excited by the experimental confirmation of the form factor behaviour predicted by the scale invariance principle up to certain large values of the invariants. Some deflection from these predictions would mean that a certain dimensional factor, e.g. "elementary length" or some other which violates scale invariance at supersmall distances is coming in play.

In this paragraph we have considered the possibility of realizing "maximum automodelity " i.e. automodelity with respect to all the variables. It is quite possible that in reality a "partial automodelity" can be realized i.e. when the automodelity takes place only with respect to certain singled out variables rather than with respect to all of them. Further, of course, it would be very interesting to understand the mechanism of violation of the scale invariance principle and to develop a method of calculation of the corrections to this approximations. Obviously, this process is closely connected with the idea of spontaneous breakdown of the conformal symmetry up to the symmetry of the Poincarè group[51].

The conformal symmetry is one of possible physically interesting generalization of the Poincarè symmetry. We recall briefly the main information on the conformal group which contains as one of the transformations the scale transformation of space-time. This 15-parameter group includes the following transformations[45,49,50]:

1. Space-time translations

$$x'^{\mu} = x^{\mu} + a^{\mu} \qquad (4 \text{ parameters}) \tag{3.19}$$

2. Homogeneous Lorentz transformations

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}, \; \Lambda^{\mu}_{\nu} \subset 0(3,1) \qquad (6 \text{ parameters}). \tag{3.20}$$

3. Special conformal transformations

$$x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2} \qquad (4 \text{ parameters}). \tag{3.21}$$

4. Scale transformations

$$x'^{\mu} = \rho x^{\mu}, \; \rho > 0 \qquad (1 \text{ parameter}). \tag{3.22}$$

According to the Noether's theorem, to these transformations correspond local currents, in particular, the currents of special conformal transformations $C^{\mu\nu}$ and of scale transformations are expressed in terms of the energy-momentum tensor $\theta^{\mu\nu}(x)$ as follows:.

$$C^{\mu\nu} = \theta^{\mu a}(2x^{\nu} x_a - g^{\nu}_{a} x^2) \tag{3.23}$$

$$S^{\mu} = \theta^{\mu a} x_a . \tag{3.24}$$

The generators of transformations are expressed in a usual way in terms of space integrals of the currents zeroth components

$$C^{\mu} = \int (2\theta^{\mu a} x^0 x_a - \theta^{\mu 0} x^2)\, d\vec{x} \tag{3.25}$$

$$S = \int \theta^{0a} x_a \, d\vec{x} . \tag{3.26}$$

It may be shown[45,49,50] that in a wide class of Lagrange field theories the current divergencies (3.23) and (3.24) are connected by the relation:

$$\partial_{\mu} C^{\mu\nu} = 2x^{\nu} \partial_{\mu} S^{\mu} = x^{\nu} \theta^{\mu}_{\mu} . \tag{3.27}$$

The vanishing of the current divergencies corresponds to the conservation of "charges" (3.25) and (3.26). It is seen from (3.27) that in this case from scale invariance it follows an invariance with respect to the full conformal group, and thus, the violation of conformal symmetry occurs in a "minimal" way due to the violation of scale invariance. In the case when Lagrangian does not depend on masses and other dimensional constants $\theta^{\mu}_{\mu} = 0$ which leads, as was suggested above, to scale invariance. The problem of a possible spontaneous violation of this symmetry is being extensively discussed in literature in connection with violation of chiral symmetry[46-49].

4. Current Commutators and Asymptotic Sum Rules

Let us consider the Fourier transform of the matrix element of the electromagnetic current commutator between two-particle "in" states[9,11]:

$$R_{\mu\nu}(p,p',q) = \int dx\, e^{-iqx} \langle p,p'\ in\, |[J_\mu(x), J_\nu(0)]|\, p,p'\ in \rangle^{c} = \tag{4.1}$$

$$= r_{\mu\nu}(p,p',q) - r_{\nu\mu}(p,p',-q) \ .$$

The particles are supposed to be unpolarized. The symbol "c" means that the weakly connected part of the matrix element is taken.

Consider in more detail the quantity $r_{\mu\nu}$. Using the condition of completeness of the "out" state vectors we obtain

$$r_{\mu\nu}(p,p',q) = \int dx\, e^{-iqx} \langle p,p'\ in\, |\, J_\mu(x)\, J_\nu(0)\, |\, p,p'\ in \rangle^{c} = \tag{4.2}$$

$$= \sum_A{}^{c} (2\pi)^4 \delta(p'+p-q-p_A) \langle p,p'\ in\, |J_\mu(0)|\, A\ out\rangle \langle A\ out\, |J_\nu(0)|\, p,p'\ in\rangle ,$$

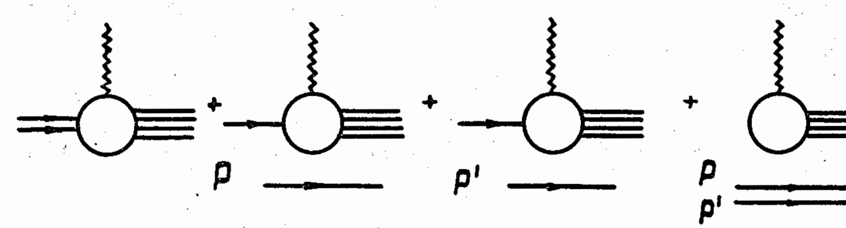
where "c" under the sign of the sum means that only the connected matrix elements of the two-current product are chosen.

We single out from this sum the completely connected part which corresponds to the quantity $\rho_{\mu\nu}$ x/

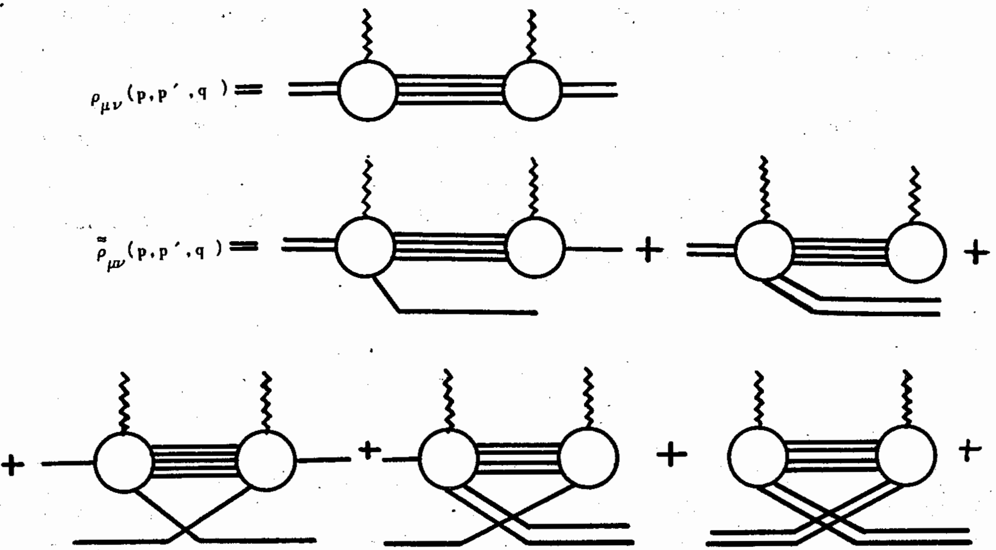
$$r_{\mu\nu}(p,p',q) = \rho_{\mu\nu}(p,p',q) + \tilde{\tilde{\rho}}_{\mu\nu}(p,p',q) \ , \tag{4.3}$$

where $\tilde{\tilde{\rho}}_{\mu\nu}$ denotes the contribution of 15 z -diagrams. This division can be represented graphycally as:

x/ It is known that if the state $\langle A\ out|$ contains particle p or p' then the current matrix element $\langle A\ out|\, J_\mu(0)|\, p,p'\ in\rangle$ will contain disconnected parts corresponding to a free propagation of these particles. Graphically the division of the matrix element into connected and disconnected parts can be represented as follows:



The first term is a completely connected part and enters the expression for the physical cross section; the remaining three are disconnected parts, they lead to the appearance of the so-called semi-connected z -diagrams.



\+ diagrams obtained by symmetrizing the initial and final states.

From the law of momentum conservation and the spectrality condition it follows that for $q^2 > 0$

$$\rho_{\mu\nu}(p,p',q) = \theta(\nu)\,\theta\left((\sqrt{s} - \sqrt{q})^2 - m_N^2\right)\rho_{\mu\nu}(p,p',q) \quad (4.4)$$

$$\tilde{\tilde{\rho}}_{\mu\nu}(p,p',q) = \theta(-\nu)\,\theta\left(m_N^2 - (\sqrt{s} + \sqrt{q^2})^2\right)\rho_{\mu\nu}(p,p',q) \quad (4.5)$$

Thus, in the physical domain the contribution of $\tilde{\tilde{\rho}}_{\mu\nu}(p,p',q)$ is exactly equal to zero.

However, in deriving the sum rules (see below) both the physical and unphysical domains are used and then nonzero contribution can be given by z-diagrams from the second part of the commutator $\tilde{\tilde{\rho}}\,(p,p',-q)$. Below it will be shown that under ordinary assumptions commonly used in the derivation of the sum rules by means of the current algebra the contribution of these diagrams tends to zero at $s \to \infty$.

We shall show that the problem of the behaviour of the form factors of the process of muon pair production at high colliding hadron energies and large virtual photon energies and masses when

$$s,\ q^2,\ \nu \to \infty$$

and the following ratios

$$a = \frac{p'q}{pq} = \frac{m'^2 + q^2 - \Delta^2}{2\nu}, \qquad \omega = \frac{q^2}{2\nu} \quad (4.6)$$

remaining fixed, can be reduced to the study of the equal time commutation relations between the space components of the operator of the hadron electromagnetic current and its time derivative.

The use of the equal time commutation relations is considerably simplified in the c.m.s. of the muon pair, where $q = \{q_0, 0\}$

In this system the decomposition of the tensor $\rho_{ij}(\vec{p},\vec{p}',q_0)$, $i,j = x,y,z$ takes the form:

$$\rho_{ij}(\vec{p},\vec{p}',q_0) = \rho_{T_1}\delta_{ix}\delta_{jx} + \rho_{T_2}\delta_{iy}\delta_{jy} + \rho_L\delta_{iz}\delta_{jz} + \rho_{TL}^{(+)}(\delta_{ix}\delta_{jz} + \delta_{iz}\delta_{jx}) + i\rho_{TL}^{(-)}(\delta_{ix}\delta_{jz} - \delta_{jx}\delta_{iz}) \quad (4.7)$$

It is obvious that R_{ij}, r_{ij} and $\tilde{\tilde{\rho}}_{ij}$ can be decomposed in a similar fashion into five structures:

$$R_{ij}(\vec{p},\vec{p}',q_0) = R_{T_1}\delta_{ix}\delta_{jx} + \ldots \quad (4.8a)$$

$$r_{ij}(\vec{p},\vec{p}',q_0) = r_{T_1}\delta_{ix}\delta_{jx} + \ldots \quad (4.8b)$$

$$\tilde{\tilde{\rho}}_{ij}(\vec{p},\vec{p}',q_0) = \tilde{\rho}_{T_1}\delta_{ix}\delta_{jx} + \ldots \quad (4.8c)$$

in this case

$$R_i(\vec{p},\vec{p}\,';q_0)=r_i(\vec{p},\vec{p}\,';q_0)-r_i(\vec{p},\vec{p}\,';q_0)=$$
$$=\epsilon(q_0)\rho_i(\vec{p},\vec{p}\,',|q_0|)+\epsilon(-q_0)\tilde{\rho}_i(\vec{p},\vec{p}\,',-|q_0|), \tag{4.9}$$

where

$$\epsilon(q_0)=\pm 1,\ q_0\gtrless 0,\quad i=T_1,T_2,L,TL^{(+)}$$
$$R^{(-)}_{TL}(\vec{p},\vec{p}\,';q)=r^{(-)}_{TL}(\vec{p},\vec{p}\,';q_0)+r^{(-)}_{TL}(\vec{p},\vec{p}\,';-q_0)=$$
$$=\rho^{(-)}_{TL}(\vec{p},\vec{p}\,';q_0)+\tilde{\rho}^{(-)}_{TL}(\vec{p},\vec{p}\,';-|q_0|). \tag{4.10}$$

It is seen that the quantities R_{T_1}, R_{T_2}, R_L, $R^{(+)}_{TL}$ are odd and $R^{(-)}_{TL}$ even functions of q_0.

Integrating (4.1) over dq_0 and $q_0 dq_0$ it is possible to obtain a series of relations

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq_0 R_{ij}(\vec{p},\vec{p}\,';q_0)=iB_{ij}(\vec{p},\vec{p}\,') \tag{4.11}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} q_0 dq_0 R_{ij}(\vec{p},\vec{p}\,';q_0)=C_{ij}(\vec{p},\vec{p}\,'), \tag{4.12}$$

etc.

Here

$$B_{ij}(\vec{p},\vec{p}\,')=-i\int d\vec{x}\langle p,p'\,\mathrm{in}\,|[J_i(\vec{x},0),J_j(0)]|\,p,p',\mathrm{in}\rangle^c \tag{4.13}$$

$$C_{ij}(\vec{p},\vec{p}\,')=-i\int d\vec{x}\langle p,p'\,\mathrm{in}\,|[\dot{J}_i(\vec{x},0),J_j(0)]|\,p,p'\,\mathrm{in}\rangle^c\ . \tag{4.14}$$

From these relations we keep only those which are not trivial from the parity considerations

$$\frac{1}{\pi}\int_0^\infty dq_0 R^{(-)}_{TL}(\vec{p},\vec{p}\,',q_0)=B_{xz}(\vec{p},\vec{p}\,')-B_{zx}(\vec{p},\vec{p}\,') \tag{4.15a}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R_{T_1}(\vec{p},\vec{p}\,',q_0)=C_{xx}(\vec{p},\vec{p}\,') \tag{4.15b}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R_{T_2}(\vec{p},\vec{p}\,';q_0)=C_{yy}(\vec{p},\vec{p}\,') \tag{4.15c}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R_L(\vec{p},\vec{p}\,';q_0)=C_{zz}(\vec{p},\vec{p}\,') \tag{4.15d}$$

$$\frac{1}{\pi}\int_0^\infty dq_0 q_0 R^{(+)}_{TL}(\vec{p},\vec{p}\,',q_0)=C_{xz}(\vec{p},\vec{p}\,')+C_{zx}(\vec{p},\vec{p}\,'). \tag{4.15e}$$

Note that in the system $\vec{q}=0$ chosen the invariant variables from which the form factors depend upon are of the following form:

$$s=m^2+m'^2+2(p_0p_0'-\vec{p}\vec{p}\,'),\ q^2=q_0^2,\ \nu=p_0q_0,$$
$$a=p_0'/p_0\ . \tag{4.16}$$

Hence it follows that when integrating over dq_0 the variables s and a are fixed and $q^2=\nu^2/p_0^2$, i.e. in the plane (q^2,ν) the integration in (3.14) is performed along the parabola.

Similar sum rules for arbitrary fixed moments $\vec{p}$ and $\vec{p}\,'$ contain contributions from the spectral functions $\tilde{\rho}$ of the corresponding z-diagrams. As is seen, from condition (5) in the limit $s\to\infty$ the contributions of z-diagrams are defined by the intermediate states of hadrons A with infinitely heavy effective masses m_A. According to the commonly accepted ideology of the current algebra method we shall assume that the z-diagram contributions vanish at $s\to\infty$. This supposition is found to be valid in the case when it is possible to change the order of the transition to the limit $s\to\infty$ and the integration in eqs. (4.15). In fact, for the sum rule, e.g. (15a) the contribution of the z-diagrams is determined by

$$\frac{1}{\pi}\int_0^\infty \rho^{(-)}_{TL}(\vec{p},\vec{p}\,',-|q_0|)\,dq_0 = -\frac{1}{\pi}\int_s^\infty \frac{dm_N^2}{2E_N}\rho^{(-)}_{TL}(\vec{p},\vec{p}\,',-|q_0|)\,. \qquad (4.17)$$

Passing to the limit $s\to\infty$ under the sign of the integral for fixed m_A^2 and taking into account (4.5) we find that the z-diagram contribution in the sum rules vanishes in this limit.

In the c.m.s. of the leptonic pair the transition to the limit $s\to\infty$ is performed under the condition

$$p_0\to\infty\,,\quad p_0'\to\infty\,.$$

We shall assume that

$$\alpha = p_0'/p_0 \quad \text{is fixed;} \qquad \beta = \frac{p_z'}{p_z} \quad \text{is fixed;}$$
$$\omega = q_0/2p_0 \quad \text{is fixed.} \qquad (4.18)$$

The fixing of β in the invariant from implies that

$$\frac{s}{\nu} = \frac{\alpha(1-\beta)}{\omega} \quad \text{is fixed.} \qquad (4.19)$$

Now we suppose that there exist the limits

$$B_{ij}(\alpha,\beta) = \lim_{\substack{p_0,p'\to\infty\\ \alpha,\beta \text{ is fixed}}} p_0 B_{ij}(\vec{p},\vec{p}\,') \qquad (4.20)$$

$$C_{ij}(\alpha,\beta) = \lim_{\substack{p_0,p_0'\to\infty\\ \alpha,\beta \text{ is fixed}}} C_{ij}(p,p')\,, \qquad (4.21)$$

where the left-hand side tensors are dimensionless quantities.

Now we make in the sum rules a transition to the limit $s\to\infty$, $\nu\to\infty$, $q^2\to\infty$ under the condition that α, ω and β are fixed. In this limit, as has already been mentioned the z-diagrams contributions drop out and the form factors have the following automodel behaviour

$$\rho_i(s,q^2,\alpha,\nu)\,\omega = \frac{\omega^2}{q^2}F_i(\alpha,\beta,\omega)\quad i = T_1, T_2, L, TL^{(+)}, TL^{(-)}. \qquad (4.22)$$

Next, going over in (4.15) to the integration over $d\omega$ we get finally the following sum rules connecting the limiting automodel values of the form factors with the current matrix elements

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\; F^{(-)}_{TL}(\alpha,\beta,\omega) = B_{xz}(\alpha,\beta) - B_{zx}(\alpha,\beta) \qquad (4.23a)$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\,\omega\, F_{T_1}(\alpha,\beta,\omega) = C_{xx}(\alpha,\beta) \qquad (4.23b)$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\,\omega\, F_{T_2}(\alpha,\beta,\omega) = C_{yy}(\alpha,\beta) \qquad (4.23c)$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\,\omega\, F_{L}(\alpha,\beta,\omega) = C_{zz}(\alpha,\beta) \qquad (4.23d)$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\,\omega\, F^{(+)}_{TL}(\alpha,\beta,\omega) = C_{xz}(\alpha,\beta) + C_{zx}(\alpha,\beta), \qquad (4.23e)$$

where

$$\omega_0 = \frac{p_0 + p_0'}{2p_0} = \frac{1}{2}(1+\alpha). \qquad (4.24)$$

The right parts of these equalities depend upon the choice of an appropriate model for the current and therefore may serve as a criterion for the choice of some or other model.

In the model of quarks interacting by the exchange of a vector meson ("gluon" model) and in the vector field model the commutators are of the form:

$$[J_i(\vec{x},0), J_j(0)] = \begin{cases} 2i\,\delta(x)\,\epsilon_{ijk}\,\psi^+(0)\,\sigma_k Q^2 \psi(0) & |\text{quarks}| \\ 0 & |\text{fields}| \end{cases} \quad \begin{matrix}(4.25)\\ (4.26)\end{matrix}$$

$$[\dot{J}_i(\vec{x},0), J_j(0)] = \begin{cases} -\delta(\vec{x})\psi^+(0)\{ i(\alpha_i \partial_j + \alpha_j \partial_i - 2\vec{\alpha}\vec{\partial}\delta_{ij}) - \\ -2 g(\alpha_i B_j + \alpha_j B_i - 2\vec{\alpha}\vec{B}\delta_{ij}) + 4M\delta_{ij}\} Q^2\psi(0) & |\text{quarks}| \\ \delta(\vec{x})\, C_{ab} J_i^a(0) J_j^b(0) + C & |\text{fields}|, \end{cases} \quad \begin{matrix}(4.27)\\ (4.28)\end{matrix}$$

where

$$Q^2 = \frac{2}{9} + \frac{1}{3} Q, \quad Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}. \tag{4.29}$$

Taking into account (4.25) and (4.26) from the sum rules for a polarization form factor we get

$$\int_0^{\omega_0} d\omega\, F_{TL}^{(-)}(\alpha, \beta, \omega) = \begin{cases} \text{const} & \text{for quark model} \\ 0 & \text{algebra of fields} \end{cases} \tag{4.30}$$

It may also be shown from the sum rule that the quark model predicts a larger value of the transverse form factors F_{T_1} and F_{T_2} compared with the longitudinal ones F_L.

Consequently, the sum rules obtained here can be used for the choice of some or other model. In the case of ordinary electroproduction similar sum rules have been obtained in ref. [33] (see also [22,34]). For the process of electroproduction with one singled out hadron in the final states analogous sum rules have been considered in ref. [20].

5. Vector Dominance and Di-Muon Mass Spectrum

According to the vector dominance hypothesis (see e.g. reviews [35,36]) the process of di-muon production proceeds through the emission of a virtual vector meson which turns into a virtual photon then decaying to a muon pair as is shown in Fig.6.

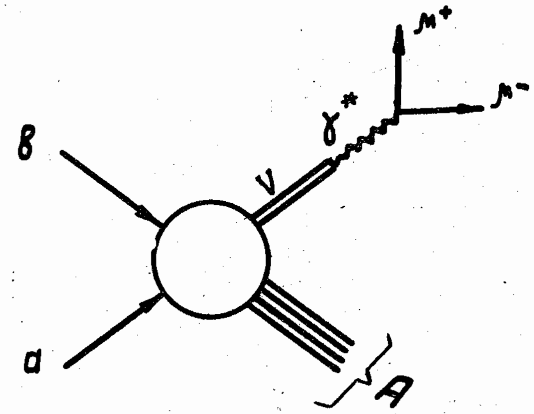


Fig.6. Illustration to the vector dominance model

It may be expected that the use of the vector dominance hypothesis will lead to a correct description of the process under consideration since here q^2 is time-like.

We determine the density matrix of a virtual vector meson V ($V = \rho^0$, ω or Φ) produced in the reaction

$$a + b \to V + \text{hadrons} \tag{5.1}$$

according to

$$W_{\mu\nu}(p,p',q) = \sum_A (2\pi)^4 \delta(p+p'-q-p_A) \langle p,p' \text{in} | J_\mu^{(V)}(0) | A \text{ out} \rangle^c \langle A \text{ out} | J_\nu^{(V)}(0) | p,p' \text{in} \rangle^c \tag{5.2}$$

where $J_\mu^{(V)}(x) = (\Box^2 - m_V^2) V_\mu(x)$ is the density of the V-meson current.

Making use of the "current-field" identity

$$J_\mu(x) = -\sum_V \frac{m_V^2}{2\gamma_V} V_\mu(x) = -\left(\frac{m_\rho^2}{2\gamma_\rho} \rho^0_\mu(x) + \frac{m_\omega^2}{2\gamma_\omega} \omega_\mu(x) + \frac{m_\Phi^2}{2\gamma_\Phi} \Phi_\mu(x) \right) \tag{5.3}$$

we get the following connection between the density matrices of the virtual photon and vector mesons

$$\rho_{\mu\nu}(p,p',q) = \sum_V \left(\frac{m_V^2}{2\gamma_V} \right)^2 \frac{1}{(m_V^2 - q^2)^2} W^{(V)}_{\mu\nu}(p,p',q) + \tag{5.4}$$

+ interference terms.

The relation (5.4) allows to express form factors ρ_{T_1}, ρ_{T_2}, ρ_L, $\rho_T^{(\pm)}$ giving a complete description of the process of dimuon production in terms of the corresponding V-meson form factors x/.

x/ Recall that the contribution to the cross section is given only by the form factors ρ_{T_1}, ρ_{T_2}, ρ_L; the form factor $\rho_{TL}^{(+)}$ can be determined from the angular distribution of the muon pair and $\rho_{TL}^{(-)}$ by measuring the polarization of one of the muons (see §2).

With the aim to employ the hypothesis of vector dominance it is convenient to represent the formula for the mass spectrum in the form /12/:

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{2\pi} \frac{1}{q^2} \left(1 - \frac{q^2 - 4m_\mu^2}{3q^2}\right) \sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}\, \sigma^{\gamma^*}(s,q^2), \tag{5.5}$$

where

$$\sigma^{\gamma^*}(s,q^2) = \sigma^{\gamma^*}_{T_1} + \sigma^{\gamma^*}_{T_2} + \sigma^{\gamma^*}_L \tag{5.6}$$

is the total production cross section for a virtual γ^* photon with mass q^2 in the process

$$a + b \to \gamma^* + \text{hadrons}. \tag{5.7}$$

According to the vector dominance hypothesis it is connected with the total cross section for production of real vector mesons in process (5.1) as follows:

$$\sigma^{\gamma^*}(s,q^2) = \frac{\alpha}{4}\left[\left(\frac{m_\rho^2}{m_\rho^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\rho^2} \sigma^\rho(s) + \left(\frac{m_\omega^2}{m_\omega^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\omega^2} \sigma^\omega(s) + \left(\frac{m_\Phi^2}{m_\Phi^2 - q^2}\right)^2 \frac{4\pi}{\gamma_\Phi^2} \sigma^\Phi(s)\right] + \text{interference terms}. \tag{5.8}$$

Inserting this approximate value for σ^{γ^*} into (5.5) and neglecting the muon mass ($m_\mu = 0$) we get, assuming the contribution of the interference terms to be small, the following expression for the muon pair mass spectrum:

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{12\pi} \sum_{V=\rho^0,\omega,\Phi} \left(\frac{m_V^2}{m_V^2 - q^2}\right)^2 \frac{4\pi}{\gamma_V^2} \sigma^V(s). \tag{5.9}$$

It is known that the Φ -meson is weakly produced in hadron-hadron collisions. Keeping therefore only the contribution of ρ^0 and ω mesons and assuming $m_\rho \approx m_\omega$, $\gamma_\rho^2 : \gamma_\omega^2 = 1:9$, $\gamma^2/4\pi = 0,5$ we bring (5.9) to the form ($m_{\mu\mu} \equiv \sqrt{q^2}$).

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2\cdot 10^{-6}}{m_{\mu\mu}(m_{\mu\mu}^2 - 0,6)^2}\left[\sigma^{\rho}(s) + \frac{1}{9}\sigma^{\omega}(s)\right]\frac{sm^2}{GeV} \qquad (5.10)$$

or for large $m_{\mu\mu}$

$$\frac{d\sigma}{dm_{\mu\mu}} = \frac{2\cdot 10^{-6}}{m_{\mu\mu}^5}\left[\sigma^{\rho}(s) + \frac{1}{9}\sigma^{\omega}(s)\right]\frac{sm^2}{GeV}. \qquad (5.11)$$

We apply eqs. (5.10) or (5.11) to the analysis of the process of production of a muon pair in hadron-hadron collisions.

a) Proton-Proton Collisions

The production of the ρ^0 meson in the reaction $p+p\to p+p+\rho^0$ was not observed in all the interval up to $p_{lab} = 28,5\ \frac{GeV}{c}$.

In this same interval the ω meson production cross sections in the reaction $p+p\to p+p+\omega$ are equal to /37/.

p_{lab}	$5\ \frac{GeV}{c}$	$10\ \frac{GeV}{c}$	$28,5\ \frac{GeV}{c}$
σ^ω	$140\pm 20\ \mu b$	$60\ \mu b$	$50\pm 10\ \mu b$

This fact is in agreement with the analysis based on the double Regge-pole model/38/. The analysis of the six-prong reaction

$$pp\to pp\pi^+\pi^+\pi^-\pi^-$$

shows that about 24% of the events proceed through the production of ρ^0 meson which corresponds to the cross section of 90 μb /39/. The cross section for the eight-prong process $pp\to pp\pi^+\pi^+\pi^+\pi^-\pi^-\pi^-$ is 20μb.

Assuming that here also about 1/4 of the events proceeds through the ρ^0 production the corresponding cross section is estimated to be about 5 μb . Thus, it may be assumed that the total cross section for ρ^0 production in pp collisions for p_{lab} = 28.5 GeV/c is about 100

$$\sigma^{pp\to\rho+..} = 100\,\mu b \ . \qquad (5.12)$$

The contribution of ω in (5.10) may be neglected due to the coefficient 1/9 . If we assume that $\sigma^\omega \approx \sigma^\rho = 100\,\mu b$ then for the mass spectrum of a di-muon produced in pp collisions with p_{lab} = 28.5 GeV/c we finally obtain from (5.10) or (5.11) the following expression

$$\frac{d\sigma^{pp}}{dm_{\mu\mu}} = \frac{2,2\cdot 10^{-34}}{m_{\mu\mu}(m_{\mu\mu}^2 - 0,6)^2}\frac{sm^2}{GeV} \approx \frac{2,2\cdot 10^{-34}}{m_{\mu\mu}^5}\frac{sm^2}{GeV} \qquad (5.13)$$

The corresponding curve is given in Fig.7.

b) Pion-Proton Collisions

Let us consider the case of π^+p collisions ($a=\pi^+$, $b=p$) Basing on the analysis performed in ref./40/ it may be concluded that the ρ -meson production cross section in the process $\pi^+ + p \to \rho^0$ + hadrons is larger or about equal to 1840 μb for p_{lab} = 8.5 GeV/c

$$\sigma^{\pi^+p\to\rho^0+..} \geq 1840\,\mu b \qquad (5.14)$$

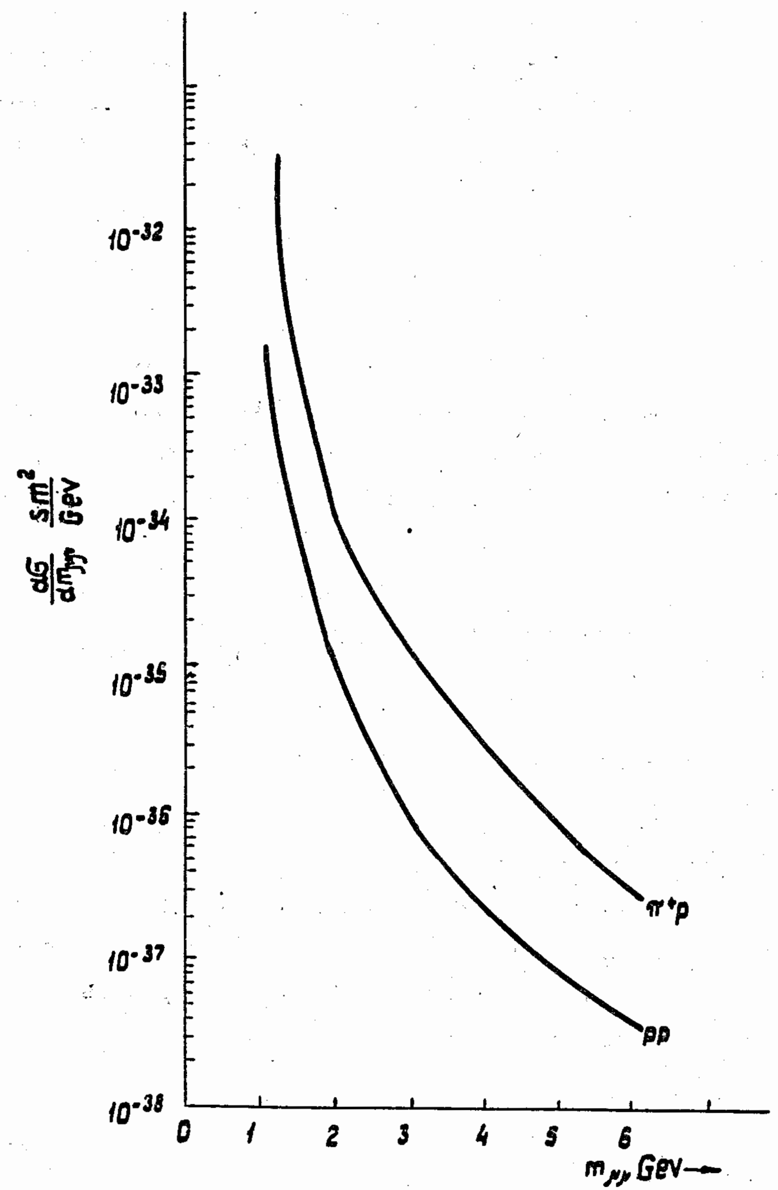


Fig.7. Predictions of the vector dominance model for the mass spectrum of a di-muon produced in proton-proton (for p_{lab} = 28.5 GeV/c) and π^+-proton (for p_{lab} = = 8.5 GeV/c) collisions according formulas (5.13) and (5.16).

and the ω meson production cross section in the process $\pi^+p \to \omega$ + hadrons is

$$\sigma^{\pi^+ p \to \omega + \ldots} \geq 200\,\mu b \qquad (5.15)$$

From here and from eqs. (5.10) or (5.11) we get the following approximate estimate (from below) for the mass spectrum of a di-muon produced in the π^+p collisions with the pion lab. momentum 8,5 GeV/c

$$\frac{d\sigma^{\pi^+ p}}{dm_{\mu\mu}} = \frac{3{,}7\cdot 10^{-33}}{m_{\mu\mu}(m_{\mu\mu}^2-0{,}61)^2}\,\frac{sm^2}{GeV} \approx \frac{3{,}7\cdot 10^{-33}}{m_{\mu\mu}^5}\,\frac{sm^2}{GeV}. \qquad (5.16)$$

6. Estimation of the Lower Limit for the Mass Spectrum

To obtain the asymptotic estimate for the di-muon mass spectrum we consider the hadron part of the matrix element of the muon pair production process when $|\vec{p}\,'| \to \infty$. Then up to the terms $0(\frac{1}{|\vec{p}\,'|})$ the matrix element is

$$\langle A\,out|\,J_\mu^{c.m.}(0)\,|\,p,p',in\rangle^c \xrightarrow[|\vec{p}\,'|\to\infty]{} \frac{p'_\mu}{E'}\langle A\,out|\,J_0^{c.m.}(0)\,|\,p,p',in\rangle^c + 0\Big(\frac{1}{|\vec{p}\,'|}\Big). \qquad (6.1)$$

This means that the di-muon production process is mainly defined by the $J_0(0)$ component of the electromagnetic current i.e. has the "Coulombic" character.

Next using the Bjorken limit, i.e. the expansion of the T - product into a series of the equal time commutators and keeping only the first term of this asymptotic series we obtain the following approximate connection with the matrix element of hadron-hadron scattering off the energy shell

$$\langle A\,\text{out}\,|\,J_0^{c.m.}(0)\,|\,p,p'\,\text{in}\rangle^c = -i\int dx\, e^{iqx} \langle A\,\text{out}\,|\,T(J_0^{c.m.}(x)\,J^{(a)}(0)\,|\,p\rangle^c =$$

$$\xrightarrow[q^2\to\infty]{} \frac{1}{\sqrt{q^2}}\int d\vec{x}\, e^{-i\vec{q}\vec{x}} \langle A\,\text{out}\,|\,[J_0^{c.m.}(0),\, J^{(a)}(0)]\,|\,p\rangle^c = \tag{6.2}$$

$$= \frac{1}{\sqrt{q^2}} \langle A\,\text{out}\,|\,J^{(a)}(0)\,|\,p\rangle^c \quad + \text{ the contribution of quasi-local terms,}$$

where $J^{(a)}(x)$ is the hadron current carrying the four-momentum Δ.

Using eqs. (6.1) and (6.2) we can obtain the following approximate expression for form factor ρ defining the di-muon production process:

$$\rho\,(s, q^2, \Delta^2, \delta) \approx \frac{4m\sqrt{\delta^2 - \Delta^2}}{q^2}\,\sigma_{ab}\,(\delta, \Delta^2). \tag{6.3}$$

The quantity $\sigma_{ab}(\delta\Delta^2)$ entering here is the analytic continuation of the total cross section of interaction of hadrons a and b throughout the unphysical domain where the square of the hadron mass is negative and equal to Δ^2, δ being the unphysical hadron energy in the lab. system.

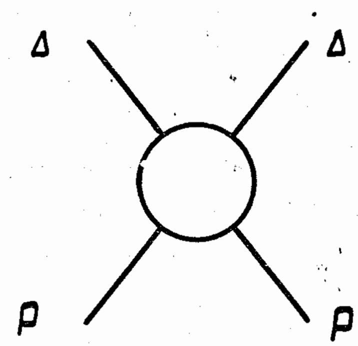


Fig. 8. The amplitude of scattering of a hadron with four-momentum Δ and unphysical mass-square Δ^2 by hadron b with mass $p^2 = m^2$. In the lab. system $(\vec{p} = 0)$ the unphysical hadron energy is $\delta = \frac{1}{m}\,p\Delta$.

In this approximation we find for the triple differential cross section the following expression (neglecting m' and m_μ)

$$\frac{d^3\sigma}{dq^2\,d\Delta^2\,d\delta} = \frac{\alpha^2}{3\pi}\,\frac{m^2}{s^2 q^4}\sqrt{\delta^2 - \Delta^2}\;\sigma_{ab}\,(\delta\Delta^2) \tag{6.4}$$

or for the mass spectrum

$$\frac{d\sigma}{dq^2} = \frac{\alpha^2}{3\pi^2}\,\frac{m^2}{s^2 q^4}\int_{q^2-s}^{0} d\Delta^2 \int_{-\frac{\Delta^2}{2m}}^{\epsilon\frac{\Delta^2}{\Delta^2-q^2}+\frac{\Delta^2-q^2}{4\epsilon}} d\delta\;\sqrt{\delta^2 - \Delta^2}\;\sigma_{ab}\,(\delta, \Delta^2). \tag{6.5}$$

If from the axiomatic field theory or from the analytic S-matrix theory [42,43] we obtain a restriction on $\sigma_{ab}(\delta, \Delta^2)$ out the mass shell then eq. (6.5) will give a restriction on the mass spectrum. It is known that for simpler case of the electromagnetic form factor $F(t)$ from the field theory and the analytic S-matrix theory it follows an exponential restriction for the lower boundary of decrease of the form factor. By analogy it may be expected that

$$\sigma_{ab}(\delta, \Delta^2) \geq \sigma_{ab}^{ph} e^{-a\sqrt{-\Delta^2}}, \tag{6.6}$$

where σ_{ab} is the total cross section of interaction of real particles and a is a certain constant. Then from eq. (6.5) it follows the low estimate for the mass spectrum provided that $s \gg q^2 \gg \frac{1}{a^2}$

$$\frac{d\sigma}{dq^2} \geq 20a^2 \frac{\sigma_{ab}}{q^8 a^6}. \tag{6.7}$$

Another method of estimation of the mass spectrum had been considered in a recent paper/44/.

The works reviewed in this report have been discussed repeatedly at seminars at the JINR Laboratory of Theoretical Physics (Dubna), at the Institute of High Energy Physics (Serpukhov) and the Institute of Theoretical Physics (Kiev). The authors express their sincere thanks to the members of these seminars for helpful comments.

APPENDIX

Determination of the Boundaries of the Physical Domain in the Process of Di-Muon Production

The law of four-momentum conservation is of the form:

$$p' + p = q + p_N. \tag{A.1}$$

Introduce the vector $\Delta = p' - q$ then

$$p + \Delta = p_N. \tag{A.2}$$

From where

$$\Delta^2 = m_N^2 - m^2 - 2m\delta,$$

where

$$\delta = \frac{1}{m} p\Delta = (\epsilon - q_0).$$

The case $m_N \equiv m$ corresponds to the case of elastic scattering. Then Δ^2 and δ are connected unambiguously, i.e. they are not independent variables:

$$\delta = -\frac{\Delta^2}{2m}.$$

This is minimal $\delta = \delta_{min}$, since q_0 in this case maximal. Let us consider the case when a virtual photon flies in the backward direction in the lab. system. It is clear that for fixed invariants it will get a minimum energy $(q_0)_{min}$ and then

$$\delta_{max} = \epsilon - (q_0)_{min}. \tag{A.3}$$

We find $(q_0)_{min}$ from the equality

$$\Delta^2 = m'^2 + q^2 - 2\epsilon (q_0)_{min} - 2\sqrt{\epsilon^2 - m'^2}\sqrt{(q_0)^2_{min} - q^2}. \tag{A.4}$$

We put $m' \equiv 0$ then solving this equation we find

$$(q_0)_{min} = \frac{q^2 - \Delta^2}{4\epsilon} + \frac{\epsilon q^2}{q^2 - \Delta^2} \tag{A.5}$$

$$\delta_{max} = \epsilon - (q_0)_{min} = \epsilon\left(1 - \frac{q^2}{q^2 - \Delta^2} - \frac{q^2 - \Delta^2}{4\epsilon}\right) = \epsilon^* + \frac{\Delta^2}{\epsilon^*}, \tag{A.6}$$

where

$$\epsilon^* = \epsilon \frac{\Delta^2}{\Delta^2 - q^2}.$$

Thus, in the physical region

$$-\frac{\Delta^2}{2m} \leq \delta \leq \epsilon^* + \frac{\Delta^2}{4\epsilon^*} \qquad (A.7)$$

Now we shall find the physical domain Δ^2 for fixed s and q^2. It is determined from the condition

$$\delta_{min} = \delta_{max} \qquad (A.8)$$

from where we find

$$\Delta^{2(-)} \leq \Delta^2 \leq \Delta^{2(+)} \qquad (A.9)$$

where

$$\Delta^{2(\pm)} = \frac{q^2\epsilon + q^2 m - 2m\epsilon^2 \pm \epsilon\sqrt{4m^2\epsilon^2 + q^4 - 4q^2\epsilon m - 4q^2 m}}{2\epsilon + m}.$$

We note that there is an interesting analogy between the reaction under consideration and the inelastic neutrinoproduction reaction. Namely, if in the Appendix to the Adler's paper /2/ we replace the square of the lepton mass by our q^2 and the Adler's q^2 by our $-\Delta^2$ then we essentially reduce both problems to each other.

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