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THE CLOSED FRIEDMANN WORLD METRIC PERTURBED BY ELECTRIC CHARGE

(On theory of electromagnetic "friedmons")

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Summary

A generalization of the well-known Tolman problem to the case of an electrically charged dust-like matter of the central-symmetrical system is considered.

In Sec. I the first integrals of the appropriate system of Einstein-Maxwell equations are found.

In Sec. 2 the problem is formulated in such a special form that when the total charge of the system tends to zero the closed Friedmann world metrics arises. This system is considered at the initial moment, namely, at the moment of maximum expansion. For a small charge the external and internal solutions are sewed together.

For any arbitrary small electric charge the metrics is found to be unclosed. The metrics of a near Friedmann part of the world turns through a narrow throat (for a small charge) into the Nordström-Raissner metrics with parameters $\sqrt{\kappa}\ m_0 = e_0$.

The expression for the electric potential in the throat $\phi_h = \frac{c^2}{\sqrt{\kappa}}$ is independent of the electric charge. With increasing charge the radius of the throat $r_h = \frac{e_0\sqrt{\kappa}}{c^2}$ grows. The state of the throat in classical description is essentially unstable from the point of view of quantum physics. The production of pairs of various kinds in large electric fields of the throat makes the latter polarized up to an effective charge $Z < 137$, independently of the initial arbitrary large charge of a material system.

1. Generalization of the Tolman Solution to the Case of Electrically Charged Dust-Like Matter

The solution of the Einstein equations for the case of a central-symmetric gravitation field in the comoving coordinates system for dust-like matter (pressure $p = 0$) was found by R.Tolman/1/.

For a number of problems it is interesting to generalize the Tolman solution to the case of an electrically charged dust-like matter.

As is known, the closed Friedmann world is described by particular solutions of the Tolman problem.

It is also known that for charged matter the world metrics cannot be closed even in the case if the matter density exceeds the critical one.

The question arises as to how the closed Friedmann world metrics is deformed under the action of, e.g. weak perturbation induced by an electric charge. The answer to this question must be obtained by solving the system of the Einstein-Maxwell equations

$$G_i^k \equiv R_i^k - \frac{1}{2}\delta_i^k R = \frac{8\pi\kappa}{c^4}(T_i^k + E_i^k), \qquad (1)$$

$$F_{ik}^{\;\;ik} \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^k} (\sqrt{-g}\, F^{ik}) = -\frac{4\pi}{c} j^i , \tag{2}$$

$$\frac{\partial F_{ik}}{\partial x^{\ell}} + \frac{\partial F_{\ell k}}{\partial x^{i}} + \frac{\partial F_{\ell i}}{\partial x^{k}} = 0 . \tag{3}$$

The energy tensor in the right-hand side of eq. (1) is chosen in the form

$$T_i^{\,k} + E_i^{\,k} = \begin{pmatrix} \epsilon + \frac{\Lambda}{8\pi} & 0 & 0 & 0 \\ 0 & \frac{\Lambda}{8\pi} & 0 & 0 \\ 0 & 0 & -\frac{\Lambda}{8\pi} & 0 \\ 0 & 0 & 0 & -\frac{\Lambda}{8\pi} \end{pmatrix} , \tag{4}$$

where

$$\Lambda = \frac{e^2}{r^4} \equiv - F_{01} F^{01} \tag{5}$$

arises as a result of solving the Maxwell equations under the condition that the system is spherically symmetric. Here

$$T_i^{\,k} = \delta_0^{\,k} \delta_i^{\,0}\, \epsilon \tag{6}$$

is the matter tensor in the comoving corrdinate system $(x^1 = q)$.

The detailed form of eq. (1) is

$$-\frac{8\pi\kappa}{c^4}(T_1^{\,1} + E_1^{\,1}) = \frac{1}{2} e^{-\lambda} \left(\frac{\mu'^2}{2} + \mu'\nu'\right) - e^{-\nu}\left(\ddot{\mu} - \frac{1}{2}\dot{\mu}\dot{\nu} + \frac{3}{4}\dot{\mu}^2\right) - e^{-\mu} = -\frac{\kappa}{c^4}\Lambda \equiv -\tilde{\tilde{\Lambda}}, \tag{I}$$

$$-\frac{8\pi\kappa}{c^4}(T_2^{\,2} + E_2^{\,2}) = \frac{1}{4} e^{-\lambda} (2\nu'' + \nu'^2 + 2\mu'' + \mu'^2 - \mu'\lambda' - \nu'\lambda' + \mu'\nu') +$$

$$+ \frac{1}{4} e^{-\nu} (\dot{\lambda}\dot{\nu} + \dot{\mu}\dot{\nu} - \dot{\lambda}\dot{\mu} - 2\ddot{\lambda} - \dot{\lambda}^2 - 2\ddot{\mu} - \dot{\mu}^2) = \frac{\kappa}{c^4} \Lambda \equiv \tilde{\tilde{\Lambda}} , \tag{II}$$

$$\frac{8\pi\kappa}{c^4}(T_0^{\,0} + E_0^{\,0}) = -e^{-\lambda}\left(\mu'' + \frac{3}{4}\mu'^2 - \frac{\mu'\lambda'}{2}\right) + \frac{1}{2} e^{-\nu}\left(\dot{\lambda}\dot{\mu} + \frac{\dot{\mu}^2}{2}\right) + \tag{III}$$

$$+ e^{-\mu} = \frac{8\pi\kappa}{c^4}\epsilon + \frac{\kappa}{c^4}\Lambda \equiv \tilde{\tilde{\epsilon}} + \tilde{\tilde{\Lambda}} ,$$

$$\frac{8\pi\kappa}{c^4}(T_0^{\,1} + E_0^{\,1}) = \frac{1}{2} e^{-\lambda} (2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - \nu'\dot{\mu}) = 0 . \tag{IV}$$

Here the metrics is chosen as

$$ds^2 = e^{\nu} (dx^0)^2 - e^{\lambda} (dx^1)^2 - e^{\mu}\, d\sigma^2 ,$$

$$d\sigma^2 \equiv (dx^2)^2 + (\sin x^2)^2 (dx^3)^2 . \tag{7}$$

The point denotes the differentiation with respect to x^0, the dash is the differentiation with respect to q.

Using conservation laws it is easy to get /1/

$$\dot{\tilde{\tilde{\epsilon}}} = -\tilde{\tilde{\epsilon}} \left(\frac{\dot{\lambda}}{2} + \dot{\mu}\right) , \tag{V}$$

$$2 \frac{e'}{e} \tilde{\tilde{\Lambda}} = \frac{1}{2} \nu' \tilde{\tilde{\epsilon}} . \tag{VI}$$

In our case the comoving system is not synchronous: $\nu \neq 0$. From (V) integrating over x^0, we get the relation

$$\tilde{\tilde{\epsilon}} = 2 \frac{\kappa}{c^4} \frac{C(q)}{r^2} e^{-\frac{\lambda}{2}}, \tag{8}$$

where

$$r^2 \equiv e^{\mu} .$$

Eq. (VI) yields

$$\nu' = \frac{2 e e'}{r\ C(q)} e^{-\frac{\lambda}{2}}. \tag{9}$$

Eq. (IV) can be rewritten in the form

$$2(\ln r')^{\dot{}} - \dot{\lambda} - \frac{\nu' \dot{r}}{r'} = 0 . \tag{10}$$

Integrating eq. (10) over x^0 we get

$$\ln(r')^2 = \lambda + \int \frac{\nu' \dot{r}}{r'} dx^0 + \ln(1 + \tilde{\tilde{f}}), \tag{11}$$

where $\tilde{\tilde{f}} = \tilde{\tilde{f}}(q)$, $1 + \tilde{\tilde{f}} \geq 0$.

Denoting by ϕ the expression

$$\phi = \int \frac{\nu' \dot{r}}{r'} dx^0 = \frac{2 e e'}{C(q)} \int \frac{e^{\frac{\lambda}{2}} \dot{r}}{r' r^2} dx^0 , \tag{12}$$

eq. (11) can now be rewritten in the form

$$e^{\lambda} = \frac{r'^2}{1 + f} e^{-\phi}. \tag{13}$$

The expression for ϕ can be obtained in the following way: Using (13) eq. (12) can be rewritten in the form of the integral equation for ϕ

$$\phi = \frac{2 e e'}{C(q)\sqrt{1 + \tilde{\tilde{f}}}} \int \frac{e^{-\frac{\phi}{2}} \dot{r}}{r^2} dx^0 . \tag{14}$$

Hence for ϕ we get the differential equation

$$\dot{\phi} = \tilde{\tilde{\delta}}(q) e^{-\frac{\phi}{2}} \frac{\dot{r}}{r^2}, \tag{15}$$

where

$$\tilde{\tilde{\delta}}(q) = \frac{2 e e'}{C(q)\sqrt{1 + \tilde{\tilde{f}}}}, \tag{16}$$

from it

$$2 e^{\frac{\phi}{2}} = - \frac{\tilde{\tilde{\delta}}}{r} + 2 \psi(q) . \tag{17}$$

Inserting (17) into (13) we get

$$e^{\lambda} = \frac{r'^2}{\left(\sqrt{1 + \tilde{\tilde{f}}}\,\psi - \frac{e e'}{C(q) r}\right)^2}, \tag{18}$$

or denoting

$$\sqrt{1 + \tilde{\tilde{f}}}\, \psi \equiv \sqrt{1 + f}, \tag{19}$$

$$\delta(q) \equiv \frac{2 e e'}{\sqrt{1 + f}\ C(q)}, \tag{20}$$

we get finally

$$e^{\lambda} = \frac{r'^2}{1 + f} \frac{1}{\left(1 - \frac{\delta}{2r}\right)^2}. \tag{21}$$

Eq. (1) can be rewritten as

$$e^{-\lambda}(r'^2 + r' r \nu') - e^{-\nu}(2\ddot r r + \dot r^2 - \dot r r \dot\nu) - 1 = -\frac{\kappa}{c^4}\frac{e^2}{r^2}. \tag{22}$$

It is easy to check that

$$e^{-\lambda}(r'^2 + r' r \nu') \equiv (1+f)\left(1 - \frac{\delta^2}{4r^2}\right),$$

$$e^{-\nu}(2\ddot r r + \dot r^2 - \dot r r \dot\nu) \equiv \frac{1}{\dot r}\left(e^{-\nu}\dot r^2 r\right)^{\cdot}.$$

Integrating (22) over x^0 we obtain

$$e^{-\nu}\dot r^2 = f + \frac{2m(q)}{r} - \frac{1}{r^2}\left(\frac{\kappa}{c^4}e^2 - \frac{\delta^2(1+f)}{4}\right), \tag{23}$$

where $m(q)$ is the integration constant over x^0. Eq. III should be rewritten as

$$-e^{-\lambda}(2r''r + r'^2 - r'r\lambda') + e^{-\lambda}(\dot\lambda \dot r r + r \quad) + 1 = (\tilde\Lambda + \tilde\epsilon)\, r^2. \tag{24}$$

Making oneself sure that

$$e^{-\lambda}(2r''r + r'^2 - r'r\lambda') \equiv \frac{(e^{-\lambda}r'^2 r)'}{r'},$$

$$e^{-\nu}(\dot\lambda \dot r r + \dot r^2) \equiv \frac{(e^{-\nu}\dot r^2 r)'}{r'},$$

inserting into the obtained expressions (21) and (23) and denoting by

$$m_1(q) \equiv \frac{c^2}{\kappa}\left[m(q) + \frac{\delta(1+f)}{2}\right], \tag{25}$$

we get the relation

$$m_1'(q) = \frac{1}{c^2}C(q)\sqrt{1+f}. \tag{26}$$

Eq. (11) gives no new relations, it is a consequence of the equations used by us.

The first integrals of eqs. (I) and (III) contain three unknown functions

$$f(q),\ m(q) \quad \text{and} \quad e(q). \tag{27}$$

These functions concretize the problem – they must be given by initial conditions.

As the space-like hypersurface Σ which must specify initial conditions we choose the surface $x^0 = 0$. The relation

$$e^{\lambda(0,q)} = \frac{r'^2(0,q)}{1+f(q)}\,\frac{1}{\left(1 - \frac{\delta(q)}{2r(0,q)}\right)^2} \tag{28}$$

is compatible with the condition (IV). Eq. (III) on the surface Σ can be written in the form

$$(e^{-\nu}\dot r^2 r)' - (e^{-\lambda}r'^2 r)' + r' = 2\frac{\kappa}{c^4}C(q)e^{-\frac{\lambda}{2}}r' + \frac{\kappa}{c^4}\frac{e^2}{r^2}r'. \tag{29}$$

We choose as the q canonical coordinate the distance from the centre at the initial time moment $(e^{\lambda(0,q)} = 1)$, then the relation (28) becomes the definition

$$\sqrt{1+f} = r'(0,q) + \frac{ee'}{C(q)\,r(0,q)}. \tag{30}$$

Further we specialize our problem mainly for the case in which there arises a closed Friedmann world when the electric charge of the system in question tends to zero.

2. Friedmann World Deformed by the Presence of Electric Charge

a) Internal solution

In what follows we will try to specify the unknown functions $f(q)$, $m(q)$ and $e(q)$ so that at $e(q) \to 0$ the closed Friedmann world metrics will be obtained. Since, in the closed world the total electric charge is zero then it is clear a priori that the metrics of the world under consideration even for small electric charge must be incompletely closed and the Friedmann metrics deformed by the charge must have outside matter a Nordström-Raissner "Aussenwelt". Our problem is to find, at least, particular examples for which it is possible to describe in a continious manner all the space of such a world.

Thus, we expect that the internal solution close to the Friedmann solution for the closed world must pass through the throat to the well-known external Nordström-Raissner solution.

Therefore for the internal solution we try to formulate the initial conditions (to the $x^0=0$ moment of maximum expansion of the system) are the closest to the Friedmann ones.

Namely, let

1° for $x^0=0$ all the space belong to the R-domain/2/,

2° initial velocities of all the particle be zero,

3° the energy density at the initial moment be independent of q

$$T^0_0 + E^0_0 = \epsilon_0 = \text{const}.$$

Below we will show that the problem with such conditions in the case of electrically charged dust has a solution, i.e. there exists such a function $C(q)$ or $M(q)$ which is compatible with the given conditions.

With the given initial conditions eq. (29) is rewritten in the form

$$1 - \frac{(r'^2 r)'}{r'} = 8\pi \frac{\kappa}{c^4} \epsilon_0 r^2 . \tag{31}$$

Denote

$$\frac{8\pi\kappa}{c^4}\epsilon_0 \equiv \frac{3}{4a_0^2} . \tag{32}$$

Integrating eq. (31) we get

$$1 - \frac{r^2}{4a_0^2} = r'^2 , \tag{33}$$

or

$$r = 2a_0 \sin\frac{q}{2a_0} , \tag{34}$$

and expression (29) can now be rewritten as follows

$$2\frac{\kappa}{c^4} C(q) + \frac{\kappa}{c^4}\frac{e^2}{r^2} = 3\sin^2\frac{q}{2a_0} . \tag{35}$$

Next we specify the distribution of the charge.

Let all particles of dust of the system have the same ratio of the charge to the mass, equal to β.

We denote

$$\frac{1}{c^2}\int_0^q C(q)\,dq = M(q)\,, \tag{36}$$

then the new condition is written as

$$e(q) = \beta\, M(q)\,. \tag{37}$$

Now the equality (35) takes on the form of the equation for finding $M(q)$

$$2\bar{M}'(q) + \bar{\beta}^2\,\frac{\bar{M}^2(q)}{4a_0^2\sin^2\frac{q}{2a_0}} = 3\sin^2\frac{q}{2q_0}\,, \tag{38}$$

where

$$\bar{M} = \frac{\kappa}{c^2}M\,,\quad \bar{\beta} = \frac{\beta}{\sqrt{\kappa}}\,.$$

By means of substitution it can be checked that eq. (38) is satisfied by the following expression

$$\bar{M} = \frac{4a_0}{\bar{\beta}^2}\sin\chi\,(b\,\mathrm{ctg}\,b\chi\cdot\sin\chi - \cos\chi)\,, \tag{39}$$

where

$$\chi \equiv \frac{q}{2a_0}\,,\qquad b \equiv \sqrt{1-\frac{3}{4}\bar{\beta}^2}\,.$$

It is easy to check that for $\bar{\beta}\to 0$ $\bar{M}$ transforms into

$$\bar{M}_0(q) = \frac{3}{2}a_0\left(\chi - \frac{\sin 2\chi}{2}\right)\,, \tag{40}$$

i.e. into the expression for the "internal mass" /3/ in the non-charged Friedmann world x/.

Further using (9) it is possible to get

$$e^{\nu(0,q)} = \left(\frac{\sin b\chi}{b\sin\chi}\right)^4\,. \tag{41}$$

With these remarks we finish to consider the internal solution at the initial time moment namely at the moment of maximum expansion of the system.

In the next paragraphs the solution in vacuum (in the regions where $\tilde{\tilde{\epsilon}} = 0$) and the problem of sewing of the internal and external solutions are analysed.

b) External Nordström-Reissner solution

As is known the geometry of the space outside the spherically symmetric mass m_0 with electric charge e_0 is described by the Nordström-Reissner metrics

$$ds^2 = \Phi(r)\,dt^2 - \frac{dr^2}{\Phi(r)} - r^2 d\sigma^2\,, \tag{42}$$

where

$$\Phi(r) = 1 - \frac{2\kappa m_0}{c^2 r} + \frac{\kappa e_0^2}{c^4 r^2}\,. \tag{43}$$

In the problem considered one should distinguish among three cases

x/ The total mass taking into account the gravitational mass defect in the closed world is zero /1/.

$$1^0. \quad \sqrt{\kappa}\, m_0 > e_0 ,$$
$$2^0. \quad \sqrt{\kappa}\, m_0 = e_0 , \qquad (44)$$
$$3^0. \quad \sqrt{\kappa}\, m_0 < e_0 .$$

In the first case the metrics is characterized by two pseudo-singularities of the Schwarzschild type

$$\Phi(r_1) = \Phi(r_2) = 0 .$$

For $r_2 < r < r_1$ the coordinate r has a time-like character. Using the Kruskal type coordinate it is possible to describe the space-time for this case /4/. A test particle placed in $r = r_1$ for $x_0 = 0$ for the time $T = \frac{\pi \kappa m_0}{c^3}$ reaches $r = r_2$, stops and returns to r_1.

At the initial moment (moment of time symmetry) the space geometry has the form of a "wormhole" (the Einstein- Rosen bridges). Its throat pulses with a period $2T$ and never is closed (contrary to the Schwarzschild case). The complete closing of the throat is prevented by field electric lines going through it x/ to the Euclidean infinity.

The second case (2^0) differs from the first one by that the T-domain is absent. At the point

$$r_h = \frac{\kappa m_0}{c^2} = \frac{e_0 \sqrt{\kappa}}{c^2}$$

x/ For $e_0 \to 0$ $\Phi(r) = 1 - \frac{2\kappa m_0}{c^2 r}$ the Schwarzschild solution may be interpreted as the external solution on for a semi-closed world /3,5/. In ref. /5/ the Kruskal metric has its physical interpretation.

Φ has a zero of the second order

$$\Phi(r) = \left(1 - \frac{r_h}{r}\right)^2 . \qquad (45)$$

As it follows from further analysis in this case the geometry at the moment of time symmetry may be of two kinds:

α^0 - "wormhole",

and

β^0 - geometry with monotonous change of r in particular, realized in the Papapetrou model (static model of charged dust with $\bar{\beta} = \frac{e}{\sqrt{\kappa} M} = 1$).

When a semi-closed charged world is implied the external solution satisfying the Euclidean condition at infinity is of the type α^0.

Case 3^0 ($e_0 > \sqrt{\kappa}\, m_0$) turns into case 2^0(β^0) (with decreasing e_0). Here singularities are absent all the space being of the R -type. In this case semi-closed worlds (with the Euclidean condition at infinity) are not realized. The limit $\bar{\beta} = 1$ yields the everywhere static system (Papapetrou model).

We are interested in the problem of sewing the external Nordström-Raissner solution with the internal solution describing a near closed world, i.e. the world the metrics of which, as $e_0 \to 0$ would transform to the metrics of the closed Friedmann world.

For $e_0 \neq 0$ our problem is to continue as much as possible the internal Friedmann solution (to decrease the sizes of the throat) to a degree allowed by the presence of the electric field. From this point of view it is advisable to consider the deformation of the Friedmann metrics by a weak electric charge $\bar{\beta} \ll 1$.

From all the cases considered, only case $2^0(a^0)$ satisfies our conditions. All the remaining cases do not lead to the closed world when $e_0 \to 0$.

c) Sewing of the internal and external solutions

In order that it will be convenient to sew the boundary conditions we transform (7) to a form close to (42), namely, to

$$ds^2 = \beta\, dt^2 - a\, dr^2 - r^2 d\sigma^2 , \tag{46}$$

where as x^1 (or q-coordinate) we have chosen a coordinate the square of which is the coefficient of $d\sigma^2$.

The transformation $dx^1 = \dfrac{dr - \dot{r}\, dx^0}{r'}$, $dx^0 = dx^0$ transforms the first two terms of (7) as follows

$$e^{\nu}(dx^0)^2 - e^{\lambda}(dx^1)^2 = \{[e^{\nu} - e^{\lambda}(\frac{\dot{r}}{r'})^2]^{1/2}\, dx^0 + e^{\lambda}\frac{\dot{r}}{r'^2}[e^{\nu} - e^{\lambda}(\frac{\dot{r}}{r'})^2]^{-1/2} dr\}^2 - \\ - (r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu})^{-1} dr^2 . \tag{47}$$

The expression in the brackets can be transformed by means of the integrating multiplier $\tilde{\tilde{\mu}}(t,r)$ to the form

$$[e^{\nu} - e^{\lambda}(\frac{\dot{r}}{r'})]^{1/2}\, dx^0 + e^{\lambda}\frac{\dot{r}}{r'^2}[e^{\nu} - e^{\lambda}(\frac{\dot{r}}{r'})^2]^{-1/2} dr = \frac{1}{\tilde{\tilde{\mu}}(t,r)} dt ,$$

that contains in the right-hand side the total differential.

For further consideration of importance is the expression for

$$a = \frac{1}{r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu}} . \tag{48}$$

From the conditions of sewing the internal and external solutions on the surface Σ:

$$a^{in} = a^{out} \,\Big|_{\Sigma} , \quad r^{in} = r^{out} \,\Big|_{\Sigma} , \tag{49}$$

we get

$$(r'^2 e^{-\lambda} - \dot{r}^2 e^{-\nu})\Big|_{q=q_0} = 1 - \frac{2\kappa m_0}{c^2 r_0} + \frac{\kappa e_0^2}{c^4 r_0^2} . \tag{50}$$

Applying (21), (22) and (25) we find

$$m_1(q_0) = m_0 , \quad e(q_0) = e_0 . \tag{51}$$

We have not received as yet answer to the main question, namely for what q_0 we are forced to sew the internal and external solutions if we want to continue the Friedmann world up to its maximum possible closeness with minimum size of the throat at the moment of time symmetry. In case 3^0 the sewing of a semi-closed world with a space Euclidean at infinity is impossible. Consequently, on the boundary $\sqrt{\kappa}\; m_1(q_0) \geq e(q_0)$ and the desired q_0 is found from the equation

$$\sqrt{\kappa}\, m_1(q_0) = e(q_0) . \tag{52}$$

The condition (52) can be written in the form[x/]

$$r_0'^2 = (1 - \frac{r_h^2}{r_0})^2 , \tag{53}$$

where $r_h = \frac{\sqrt{\kappa}\, e_0}{c^2} = \frac{\kappa\, m_0}{c^2}$, and r_0 and r_0' are the values of r and r' on the boundary of the matter. The condition (53) for a semi-closed world ($r' < 0$) in the case of a Euclidean space at infinity leads to the relation $r_h < r_0$ that corresponds to the presence of "wormhole" i.e. to the case 2^0 (a^0).

Thus, for the semi-closed world with Euclidean metrics at infinity condition (52) can be rewritten as

$$r_0' = \frac{r_h}{r_0} - 1 . \tag{54}$$

Let us investigate in more detail the model of a weakly charged world $\bar\beta \ll 1$ (or $\pi\bar\beta \ll 1$).

In this case

$$\bar M = \frac{3}{2} a_0 (\chi_0 - \sin\chi_0 \cdot \cos\chi_0) + 0(\bar\beta^2) ,$$

$$r_h = \frac{3}{2}\bar\beta\, a_0 (\chi_0 - \sin\chi_0 \cdot \cos\chi_0) + 0(\bar\beta^3) , \tag{55}$$

$$r_0 = 2 a_0 \sin\chi_0 , \quad r_0' = \cos\chi_0 ,$$

---

x/ Eq. (31) is rewritten in this case in the form $[(1-r'^2)\, r]' = = \frac{\kappa}{c^4}\frac{e_0^2}{r^2} r'$ from where $r'^2 = 1 - \frac{2\kappa m_0}{c^2 r} + \frac{\kappa e_0^2}{c^4 r^2}$. Owing to the continuity of r and r' on the matter-vacuum boundary in this domain of r (52) holds.

where

$$0 < \chi_0 < \pi , \quad \chi_0 \equiv \frac{q_0}{2 a_0} .$$

The condition (54) for $\frac{\pi}{2} < \chi_0 < \pi$ and small $\bar\beta$ is written in the form

$$1 + (1 + \frac{3}{4}\bar\beta) \cos\chi_0 = \frac{3}{4}\bar\beta \frac{\chi_0}{\sin\chi_0} . \tag{56}$$

For $\bar\beta = 0$ $\chi_0 = \pi$ i.e. χ_0 reaches its maximum, that is the world becomes a completely closed Friedmann world.

For a small charge $\bar\beta \ll 1$ the desired boundary of the internal (Friedmann) solution must be anywhere near π, i.e. $\chi_0 = \pi - \delta$ where δ is small.

Indeed, as follows from the graph (Fig. 1), eq. (56) has the only solution χ_0.

For $\bar\beta \ll 1$ χ_0 is close to π, for $\bar\beta \to 1$ χ_0' tends to $\frac{\pi}{2}$.

When the world charge (e_0) increases, its external (Schwarzshild) mass and correspondingly the throat radius

$$r_h = \frac{\sqrt{\kappa}\, e_0}{c^2} \tag{57}$$

increase too. It is essential to stress that with increasing e_0 the electric field potential in the throat

$$\phi_h = \frac{e_0}{r_h} \tag{58}$$

does not change and remains constant

$$\phi_h = \frac{c^2}{\sqrt{\kappa}} . \tag{59}$$

The quantity ϕ_h plays the role of a maximum potential; it is composed of world constants and, what is of interest, contains no electric charge.

I. Throat

The condition $\sqrt{\kappa}\ m_0 = e_0$ ensures the throat to be static. An external observer always sees a charged as much as possible continued semi-closed world in the form of a harden sphere x/.

The dynamics of a part of near closed uniformaly charged world remains non-stationary in this case too.

After the moment of maximum expansion the charge cloud described by the internal solution contracts. But the collapse of the system is stopped by electric forces at extremely small radius defined by the sizes of the throat, i.e. by the total electric charge of the system.

It should be stressed that in the throat there is no matter. The fact that the material cloud is non-static does not influence the statical character of the throat. In the throat the field lines are contracted in a maximum possible manner ($\phi_h = \frac{c^2}{\sqrt{\kappa}}$). From the throat the field lines diverge both outside, to the side of the Euclidean infinity and inside the near Friedmann world.

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x/We recall: in the case $\sqrt{\kappa}\ m_0 > e_0$ the throat oscillates between r_1 and r_2. At $e_0 \to \sqrt{\kappa}\ m_0$ $r_1 \to r_2$. In the case $\sqrt{\kappa}\ m_0 = e_0$ at $e_0 \to 0$ the external (Schwarzschild) mass vanishes. The world becomes completely closed, i.e. in the case $\sqrt{\kappa}\ m_0 = e_0$ all the mass is of electric origin. Under these conditions any initial value of the internal mass of non-electromagnetic origin is completely cancelled by the gravitational mass defect.

Thus, the throat imitates the electric field (charges) source though no material source of charges have been localized in the throat.

A more detailed consideration shows that the field in the external space and the field between the matter and the throat have opposite signs.

$$F_{tr} = \frac{e}{r^2}, \qquad \text{(in the external space, domain 0)} \tag{60}$$

$$F_{tr} = -\frac{e}{r^2}. \qquad \text{(between matter and throat, domain I (Fig. II)}$$

Indeed, the connection between $F_{x^0 q}$ and F_{tr} is given by the transformation

$$F_{tr} = \frac{D(t,r)}{D(x^0,q)} F_{x^0 q}. \tag{61}$$

Further it can be shown x/ that sign $\frac{D(t,r)}{D'(x^0,q)}$ = sign r' i.e. the sign of $\frac{D(t,r)}{D(x^0,q)}$ is the same as for r' so we have (60).

---

x/

$$g_{x^0 x^0} = \left(\frac{\partial t}{\partial x^0}\right)^2 g_{tt} + \left(\frac{\partial r}{\partial x^0}\right)^2 g_{rr},$$

$$g_{qq} = \left(\frac{\partial t}{\partial q}\right)^2 g_{tt} + \left(\frac{\partial r}{\partial q}\right)^2 g_{rr},$$

but

$$g_{x^0 x^0} > 0, \quad g_{qq} < 0,$$

$$g_{tt} > 0, \quad g_{rr} < 0.$$

Consequently,

$$\left(\frac{\partial t}{\partial x^0}\right)^2 g_{tt} > \left(\frac{\partial r}{\partial x^0}\right)^2 (-g_{rr}), \quad \left(\frac{\partial r}{\partial q}\right)^2 (-g_{rr}) > \left(\frac{\partial t}{\partial q}\right)^2 g_{tt},$$

from where $\left|\frac{\partial t}{\partial x^0}\frac{\partial r}{\partial q}\right| > \left|\frac{\partial r}{\partial x^0}\frac{\partial t}{\partial q}\right|$ and consequently the sign of $\frac{D(t,r)}{D(x^0,q)} = \frac{\partial t}{\partial x^0}\frac{\partial r}{\partial q} - \frac{\partial r}{\partial x^0}\frac{\partial t}{\partial q}$ is defined by the first term, since always $\frac{\partial t}{\partial x^0} > 0$ – the time always increases ("arrow of time").

In the very throat the test electric charge must be at rest.

In domains I and 0 it is easy to realize the static frame of reference using charged weightless particles of dust.

Thus system coincides with the Nordström-Raissner one. As is known the entire description of the Nordström-Reissner metrics (i.e. including the domain between its two pseudosingularities) is given by the Kruskal type coordinates (non-static frame of reference). In our case the domain (r_2, r_1) reduces to one solution $r_1 = r_2 = r_h$ namely to the throat.

The static frame of reference does not cover only this part near the throat.

Polarized Throats

(necessity of quantum description of the throat)

Basing on eq. (57) $\left(r_h = \frac{\sqrt{\kappa}\, e_0}{c^2}\right)$ we are led to the conclusion that the throat radius increases proportionally to the total electric charge. This is the description of the throat from the point of view of the classical theory. But from the point of view of quantum physics such a state of the throat cannot be stable. Indeed, if at some initial moment there arises a throat with the above properties then in its superintense electric field it occurs inevitably a rapid process of production of any kind electrically charged pairs, proton-antiproton pairs, any kind meson pairs and finally, electron-positron pairs. Opposite charges will tend to decrease the throat effective charge while the charges of another component of pairs will flow together to the Euclidean infinity.

In this process the throat charge gradually decreases together with the throat radius and the internal metrics of the system

becomes more and more close. We consider this effect in more detail not so much to give an exhausting quantitative description as to attract attention to this very interesting, in our opinion situation namely the necessity of application of quantum theory in a ultra-macro-world just for the description of such processes which seems to play essential role only in micro-world. The qilalitative estimates being, as yet, far from the desired ones, are in themselves of some interest.

The production of electrically charged pairs in a strong homogeneous electric field has been considered by Nikishov /6/.

If there is a homogeneous electrostatic field of intensity E filling up the space of a cube of dimensions L then the probability of formation of a pair in it (say, electron pair) with a given momentum $(\vec{p})$ and spin (r) for all the time is given by the expression

$$W_{\vec{p}\,r} = \exp\left(-\pi\,\frac{c^2(p_1^2+p_2^2+m_0^2c^2)}{eEhc}\right), \tag{63}$$

$$(E = (0, 0, E)),$$

where m_0 is the particle mass, $\vec{p}$ is the particle momentum of a produced pair after switching off the field.

In such a problem $\vec{p}$ must be assigned to the discrete spectrum, i.e.

$$L\,p_n^{(i)} = 2\pi h n \qquad (i = 1,2,3).$$

Eq. (63) can be rewritten in the form

$$W_{n_1 n_2 n_3 r} = \exp\left(-\frac{\pi m_0^2 c^4}{eEhc}\right)\exp\left(-\frac{\pi c^2}{eEhc}\left(\frac{2\pi h}{L}\right)^2 n_1^2\right)\exp\left(-\frac{\pi c^2}{eEhc}\left(\frac{2\pi h}{L}\right)^2 n_2^2\right). \tag{64}$$

Here the state of the produced particle is characterized by the numbers (n_1, n_2, n_3, r).

Summing $W_{n_1 n_2 n_3 r}$ over all the quantum numbers and then replacing the sum over n_1, n_2 by the integral we get

$$W = 4N \exp\left(-\frac{\pi m_0^2 c^4}{eEhc}\right) eEhc \left(\frac{L}{2\pi hc}\right)^2 \Phi^2(\xi_0), \tag{65}$$

where

$$\xi_0 = \sqrt{\frac{\pi}{eEhc}}\,\frac{2\pi hc}{L} N, \quad \Phi(\xi_0) \equiv \frac{2}{\sqrt{\pi}} \int_0^{\xi_0} e^{-\xi^2} d\xi,$$

$$N = n_{max}, \quad p_{max} = \frac{2\pi h N}{L} = c\left(m_0 + \frac{e\phi_{max}}{c^2}\right). \tag{66}$$

For a large maximum momentum p_{max}, $\xi_0 \gg 1$ and $\Phi(\xi_0) \approx 1$. Following (65),(66) the probability of production of a pair in the unit of volume

$$w = \frac{4}{(2\pi h)^3}\,\frac{p_{max}}{c}\, eEh \exp\left(-\frac{\pi m_0^2 c^4}{eEhc}\right) \Phi^2(\xi_0). \tag{67}$$

Next, we apply illegally eq. (67) to an inhomogeneous static field $E = \frac{Ze}{r^2}$ $\left(\phi = \frac{Ze}{r}\right)$.

In calculating the total number of pairs N_p over the whole space in (67) we assume p_{max} to be dependent on r

$$p_{max} = \frac{Ze^2}{cr}, \tag{68}$$

where Ze is the total charge of the material system.

For the total number of pairs produced in a given field for the all time we obtain

$$N_p = \int_V w\, d^3x = \frac{16\pi Z^2 e^4 h}{(2\pi h)^3 c^2} \int_{a_0}^{\infty} \frac{1}{r} \exp\left(-\frac{\pi m_0^2 c^4}{Ze^2 hc} r^2\right) dr = \tag{69}$$

$$= \frac{16\pi Z^2 e^4 h}{(2\pi h)^3 c^2} \int_{A_0}^{\infty} \frac{e^{-\xi^2}}{\xi} d\xi,$$

where

$$A_0 = \sqrt{\frac{\pi m_0^2 c^4}{Ze^2 hc}}\, a_0,$$

$$a_0 = \frac{\sqrt{\kappa}\, Ze}{c^2}$$

is the minimum radius.

Since

$$\int_{A_0}^{\infty} \frac{e^{-\xi^2}}{\xi} d\xi = \frac{1}{2} \int_{A_0^2}^{\infty} \frac{e^{-x}}{x} dx = -\frac{1}{2} E_i(-A_0^2), \tag{70}$$

then

$$N_p = -\frac{1}{\pi^2} (Z\alpha)^2 E_i(-A_0^2). \tag{72}$$

For small A_0^2 i.e. for $Z \ll \frac{1}{\pi\alpha}\left(\frac{e}{m_0\sqrt{\kappa}}\right)^2 \approx 10^{45}$,

$$E_i(-A_0^2) \approx c + \ln A_0^2, \tag{72}$$

c - is the Euler constant, and consequently,

$$N_p \approx \frac{1}{\pi^2}(Z\alpha)^2 \left[\ln\left(\frac{e}{m_0\sqrt{\kappa}}\right)^2 = c - \ln(\pi Z\alpha)\right] \approx \tag{73}$$

$$\approx \frac{1}{\pi^2}(Z\alpha)^2 \ln\left(\frac{e}{m_0\sqrt{\kappa}}\right)^2.$$

The condition $Z - N_p = \max = Z_f$ gives the value of the charge Z_f which remains noncancelled the pair production effect

$$Z_f = \max \{ Z [1 - Z \frac{a^2}{\pi^2} \ln (\frac{e}{m_0 \sqrt{\kappa}})^2] \} = \frac{\pi^2}{4 a^2 \ln (\frac{e}{m_0 \sqrt{\kappa}})} , \tag{74}$$

$$\ln (\frac{e}{m_0 \sqrt{\kappa}})^2 \approx \frac{1}{a} , \quad Z_f \approx 137 .$$

In other words, the pair production effect in such a strong electric field decreases the effective charge of the throat down to the finite value $Z_f \approx 137$, independently of the magnitude of the initial charge Z x/.

The independence of the final charge of an arbitrarily value of the bare charge follows also from the well-known Landau formula /9/ connecting the value of the bare charge e_1 localized in a small domain with the value of the physically effective charge down to which the vacuum polarization effect decreases the initial charge

$$e^2 = \frac{e_1^2}{1 + \frac{e_1^2}{3\pi} \ln (\frac{\Lambda}{m_0})^2} . \tag{75}$$

For large e_1 or more exactly, for $\frac{e_1^2}{3\pi} \ln (\frac{\Lambda}{m_0})^2 \gg 1$,

$$e^2 \approx \frac{3\pi}{\ln (\frac{\Lambda}{m_0})^2} . \tag{76}$$

x/ This result is rather natural since for $Z > 137$, as is known, begins the process of the real production of pairs /7,8/.

It is interesting to note that in rough estimates of eq. (73) the same characteristic logarithm as in the Landau formula arises and the argument of the logarithm in eq. (73) gives the following value the introduced Landau expression

$$\Lambda = \frac{e}{\sqrt{\kappa}} \approx 10^{28} \text{ ev}. \tag{77}$$

This is just the value of Λ which is discussed in the paper by Landau in connection with the possible role of the gravitation in elementary particle theory. The image of the object under discussion is very complicated even from the point of view of the Schwarzschild observer. The matter is that at the initial moment of the existence of such a system with large electric charge its external sizes proportional to the charge may be very large $r_h^i = \frac{Z_i e \sqrt{\kappa}}{c^2}$. The pair production decreases the initial charge Z_i down to $Z_f \approx 137$ and, consequently x/

$$r_r^f < \frac{137\, e \sqrt{\kappa}}{c^2} \approx 10^{-30} \text{ cm}.$$

But in this region $Z (Z_f \approx 137)$ the shells (round the field source) with radii $\frac{h}{mc}$ (m being the particle masses of produced pairs) begin to fill up. If we take into account that hadron particles (e.g. protons) have their own sizes then round the system in question there arises an original atmosphere which increases its external sizes by about 20 orders of magnitude. Accidently or not accidently, the object characteristic in its external properties

x/ $r_h^f < 10^{-30}$ cm, since in the previous estimates the possibility of production if any kind pairs has not been taken into account.

of micro-world physics arises from a cosmologic object and its internal content remains the same.

In ref.[10] the author introduces for the objects with above properties a special term "friedmons".

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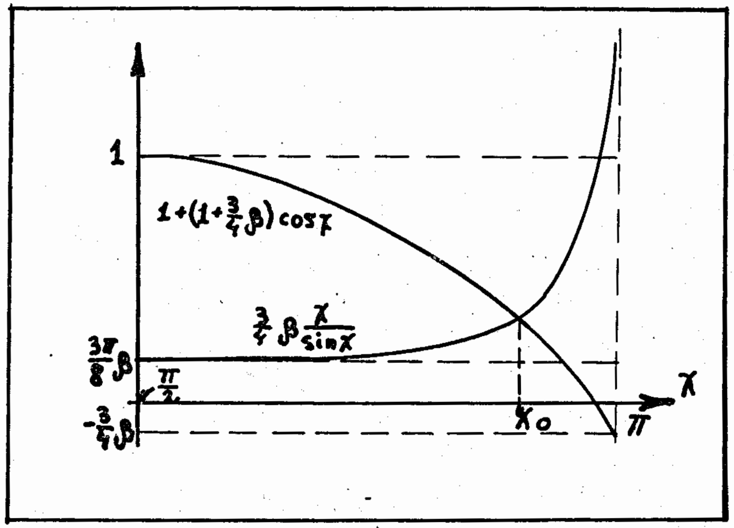

Fig. 1. Graphical finding of the boundary of the extremely continued solution for $\beta \ll 1$.

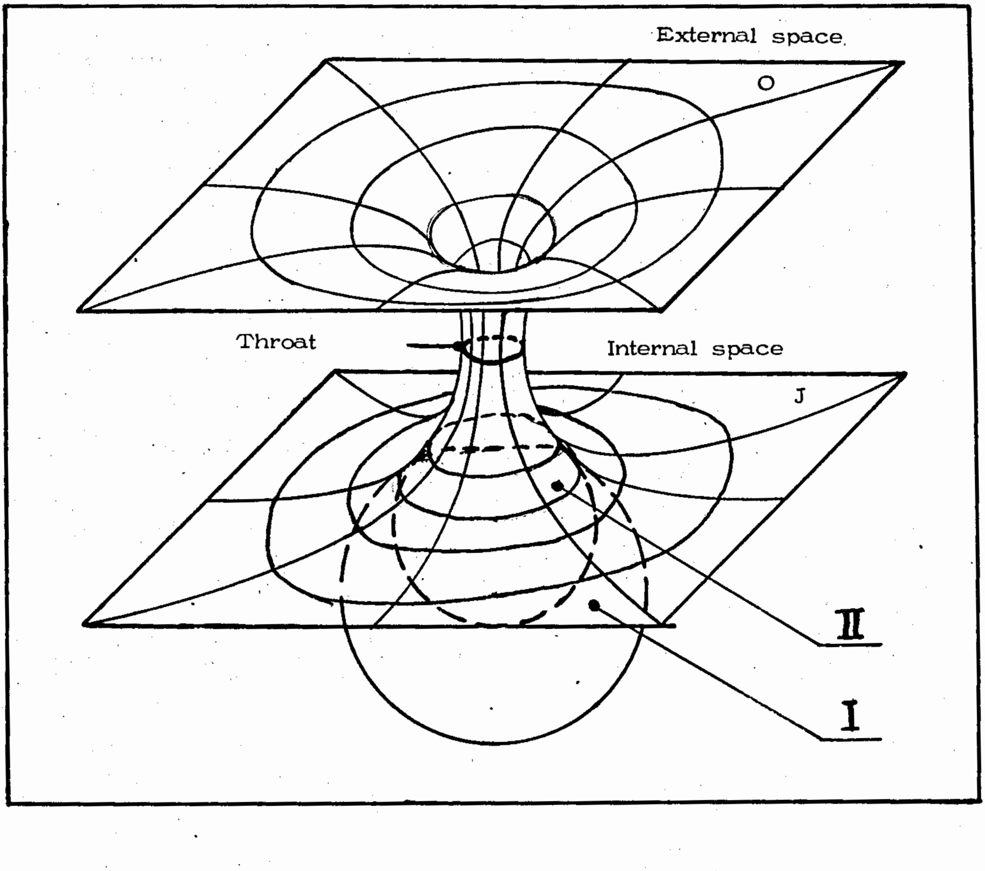

Fig. 2. The section of the space-time for the charged world (τ = const, $\theta = \frac{\pi}{2}$). J - internal space, 0 -external space, I, II - sections of the world ($\epsilon \neq 0$) for different τ .