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CONSISTENCY OF BROKEN CHIRAL SYMMETRY AND VENEZIANO MODELS

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Recently, there has been some interest in the question of implications of the Veneziano model[1] to chiral symmetry breaking. On the basis of the Veneziano model Fayyazuddin and Riazuddin[2] assert that current divergences belong to the $(3,\bar{3}) + (\bar{3},3)$ representation of $SU(3) \times SU(3)$ [3]. On the other hand, Cronin and Kang[4] show that Veneziano's model is incompatible with models for symmetry breaking[3] where the symmetry breaking Hamiltonian belongs to a single $(3,\bar{3}) + (\bar{3},3)$ representation of $SU(3) \times SU(3)$. The statement of ref.[4] is that the σ terms

$$\int d^3x \, [A_0^{\pi^-}(\vec{x},0) \, , \, \partial^\mu A_\mu^{K^+}(0)] \quad \text{and} \quad \int d^3x \, [A_0^{K^+}(\vec{x},0) \, \partial^\mu A_\mu^{\pi^-}(0)]$$

taken between π^+ and K^+ are different functions of the momentum transfer, while according to ref.[3] they are proportional. However, neither[2] nor[4] use a correct off-shell amplitude.

The purpose of this note is to show that using a correct off-mass-shell continuation[5,6], the Veneziano amplitude is not inconsistent with the chiral symmetry breaking model[3]. As an example we treat πK scattering (where the contradiction of ref.[4] is revealed), however, $\pi\pi$ and KK scattering can be treated similarly. $\pi\pi$ scattering has already been considered in refs.[5,6].

The off-mass-shell amplitude is defined by

$$T\left(\begin{matrix} j,\ell,j,n \\ q,p,q',k \end{matrix}\right) = \int d^4x\, e^{-ipx} \langle P^j q' | T(\partial^\lambda A^\ell_\lambda |x| \partial^\mu A^n_\mu(0)) | P^i q\rangle, \qquad (1)$$

where $|P^j q'\rangle$, $|P^i q\rangle$ are pseudoscalar meson states, i, ℓ, j, n are indices of the mesons; q, p are incoming, q', k outgoing momenta. The σ terms are related to the off-shell amplitude by

$$T\left(\begin{matrix} i,\ell,j,n \\ q,p,q',0 \end{matrix}\right) = -\int d^3x \langle P^j q' | [A^n_0(\vec{x},0), \partial^\mu A^\ell_\mu(0)] | P^i q\rangle = \qquad (2)$$

$$= -2i\,\sigma^{n\ell}_{jq',iq}$$

and

$$T\left(\begin{matrix} i,\ell,j,n \\ q,0,q',k \end{matrix}\right) = -\int d^3x \langle P^j q' | [A^\ell_0(\vec{x},0), \partial^\mu A^n_\mu(0)] | P^i q\rangle =$$

$$= -2i\,\sigma^{\ell n}_{jq',iq} \qquad (3)$$

Following ref.[6] we assume for the off-mass-shell continuation of the Veneziano amplitude:

$$T\left(\begin{matrix} i,\ell,j,n \\ q,p,q',k \end{matrix}\right) = \frac{i\,2 f_\ell f_n\, 2(2\pi)^3\, m_\ell^2\, m_n^2}{(k^2-m_n^2)(p^2-m_\ell^2)\,\Phi(p^2,k^2)}\, B(s,t), \qquad (4)$$

where $B(s,t)$ is the relevant on-shell Veneziano amplitude. The factor $[(k^2-m_n^2)(p^2-m_\ell^2)\Phi(p^2,k^2)]^{-1}$ provides for the mass singularities of the amplitude T x/. We also have $\Phi(m_\ell^2, m_n^2) = 1$. From factorization along the leading trajectory it can be shown that $\Phi(p^2,k^2)$ factorizes.

Specialize now eq. (4) to $K\pi$ scattering. In this case the external mass singularities are poles at the $1^-(0^-)$ daughters of the $\pi - A_1$ trajectory as well as at the $\frac{1}{2}(0^-)$ daughters of the K trajectory. For these trajectories we accept[4]

$$\alpha_\pi(s) = \alpha_\rho(s) - \frac{1}{2},$$

$$\alpha_K(s) = \alpha_{K^*}(s) - \frac{1}{2},$$

where $\alpha_\rho(s)$, $\alpha_{K^*}(s)$ are the ρ and K^* trajectories[1,8] respectively, with a universal slope b.

Thus we get for the relevant amplitudes

$$T\left(\begin{matrix} \pi^- K^+ K^+ \pi^- \\ q,p,q',k \end{matrix}\right) = \frac{i\,2 f_K f_\pi\, m_K^2\, m_\pi^2\, 2(2\pi)^3}{f(p^2) f(k^2)}\, b^2\, \Gamma\left(\frac{1}{2} - \alpha_{K^*}(p^2)\right) \times$$

$$\times\, \Gamma\left(\frac{1}{2} - \alpha_\rho(k^2)\right) \beta_0\, \frac{\Gamma(1-\alpha_{K^*}(s))\,\Gamma(1-\alpha_\rho(u))}{\Gamma(1-\alpha_{K^*}(s)-\alpha_\rho(u))}, \qquad (5)$$

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x/ In ref.[2,4] $\Phi(p^2,k^2)$ is omitted, which is wrong if we allow for large variations of p^2 and k^2 as in[4].

$$T\left(\begin{matrix}\pi^{+} K^{+} K^{+} \pi^{+} \\ q, p, q', k\end{matrix}\right) = \frac{i\, 2 f_K f_\pi m_K^2 m_\pi^2 2(2\pi)^3}{f(p^2) f(k^2)} b^2 \Gamma\left(\frac{1}{2} - \alpha_{K^*}(p^2)\right) \times$$

$$\times \Gamma\left(\frac{1}{2} - \alpha_\rho(k^2)\right) \beta_0 \frac{\Gamma(1-\alpha_{K^*}(t))\Gamma(1-\alpha_\rho(u))}{\Gamma(1-\alpha_{K^*}(t)-\alpha_\rho(u))}, \qquad (6)$$

where $(k^2 - m_\pi^2)(p^2 - m_K^2)\Phi(k^2, p^2)$ has been replaced by

$$b^{-2} f(p^2)\, \bar{f}(k^2) [\Gamma(\frac{1}{2} - \alpha_{K^*}(p^2))\, \Gamma(\frac{1}{2} - \alpha_\rho(k^2))]^{-1}.$$

$f(p^2)$, $\bar{f}(k^2)$ are (probably smooth) functions, with the normalization $f(m_K)^2 = 1$, $\bar{f}(m_\pi^2) = 1$.

For the amplitude of eq. (5) the σ term is

$$\int d^3x\, [A_0^{\pi^+}(x,0),\, \partial^\mu A_\mu^{K^+}(0)] = 0.$$

Using eqs. (2), (3) it is easy to see that this is compatible with eq. (5) (as observed in ref./4/), being $1-\alpha_{K^*}(m_\pi^2) - \alpha_\rho(m_K^2) = 0.$

For the amplitude of eq. (6) the σ terms are connected by/3/

$$\int d^3x\, [A_0^{K^+}(\vec{x},0), \partial^\mu A_\mu^{\pi^-}(0)] = \frac{\sqrt{2}+c}{\sqrt{2}-\frac{c}{2}} \int d^3x [A_0^{\pi^-}(\vec{x},0), \partial^\mu A_\mu^{K^+}(0)], \qquad (7)$$

where $c \approx -1{,}25$, as estimated in ref./3/. Using eqs.(2), (3) we get from eq. (6)

$$\sigma^{\pi^- K^+}_{K^+_{q'}, \pi^+_q} = -\frac{f_K f_\pi m_K^2 m_\pi^2 2(2\pi)^3 \Gamma(\frac{1}{2}-\alpha_\rho(0))}{f(t)\bar{f}(0)} b^2 \beta_0 \Gamma(1-\alpha_{K^*}(t))\Gamma(\frac{1}{2}) \qquad (8)$$

$$\sigma^{K^+, \pi^-}_{K^+_{q'}, \pi^+_q} = -\frac{f_K f_\pi m_K^2 m_\pi^2 2(2\pi) \Gamma(\frac{1}{2}-\alpha_{K^*}(0))}{f(0)\bar{f}(t)} b^2 \beta_0 \Gamma(1-\alpha_{K^*}(t))\Gamma(1-\alpha_\rho(m_K^2)). \qquad (9)$$

If we choose $f(k^2) = d\bar{f}(k^2)$ the t dependence of eq. (8) and (9) is the same, so the contradiction of ref./4/ is removed. In addition, from eqs. (7),(8),(9) we get a value $c \approx -1{,}17$, which is reasonable.

In conclusion we may say that the correctly continued off-shell Veneziano amplitude is not incompatible with the chiral symmetry breaking model of Gell-Mann, Oakes and Renner. It is clear, however, that a detailed comparison of the two models is only possible, if we write down a Veneziano amplitude which is off the mass-shell in all the four masses.

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