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ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 4844

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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VENEZIANO'S REPRESENTATION AND PERTURBATION THEORY

1969

E2 - 4844

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8125/2 pr.

VENEZIANO'S REPRESENTATION AND PERTURBATION THEORY

Submitted to Phys. Lett.

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Представление Венециано и теория возмущений

В работе показано, что представления типа Венециано для амплитуды рассеяния может быть получено в рамках квантовой теории поля методами теории возмущений.

**Препринт Объединенного института ядерных исследований.
Дубна, 1969**

Matveev V.A., Tavkhelidze A.N. E2-4844

Veneziano's Representation and Perturbation Theory

The Veneziano type representations for a scattering amplitude can be derived within quantum field theory by the perturbation theory method.

**Preprint. Joint Institute for Nuclear Research.
Dubna, 1969**

A great attention has recently been paid to the study and applications of the Veneziano's representation for scattering amplitudes[1].

The Veneziano's formula incorporates in a simple and economic way many of the features of scattering amplitudes such as, for example, the crossing symmetry, the Regge behaviour in all the channels and the so-called "duality" which is a consequence of the finite-energy sum rules[2].

An interesting problem arises whether it is possible to derive the Veneziano's representation by ordinary quantum field theory methods, in particular by various methods of summing diagrams of perturbation theory. For this reason the above problem is closely connected with that which concerns the derivation of the asymptotic Regge behaviour in quantum field theory[3,4].

In this note we show that the Veneziano type representation for a scattering amplitude can be derived by the perturbation theory methods. We essentially use an auxiliary integral parametric representation for scattering amplitudes which obey unsubtracted dispersion relations.

Consider some invariant amplitude $T(s, t)$

$$T(s,t)=F(s,t)\pm F(u,t), \tag{1}$$

where $F(s,t)$ is analytic in the complex s-plane with the cut at $s \geq s_0$ and obeys unsubtracted dispersion relations

$$F(s,t)=\frac{1}{\pi}\int_{s_0}^{\infty}\frac{\rho(s',t)ds'}{s'-s}. \tag{2}$$

Using the evident identity

$$\frac{1}{s'-s}=\int_0^1 dx \cdot x^{-1-s+s'} \quad ; \quad s<s' \tag{3}$$

we get the following integral parametric representation for the function $F(s,t)$:

$$F(s,t)=\int_0^1 dx \cdot x^{-1-s} \cdot f(x,t) \; ; \qquad s<s_0 \quad , \tag{4}$$

where

$$f(x,t)=\frac{1}{\pi}\int_{s_0}^{\infty} ds' \, \rho(s',t)\, x^{s'}. \tag{5}$$

One can see that eq. (5) defines an analytic function of the variable x inside the circle $|x|<1$ with the cut at $-1<x\leq 0$. For amplitudes $F(s,t)$ which decrease slowly than $1/s$ the function (5) will have a singularity at $x=1$ due to the divergence of the integral in eq. (5).

On the other hand it is easy to see that the asymptotic behaviour of $F(s,t)$ at $s \to -\infty$ and t= fixed is determined by the behaviour of the function $f(x,t)$ near the point $x=1$.

To find the behaviour of the function $f(x, t)$ near the point $x = 1$ we make use of perturbation theory. Suppose that the asymptotic behaviour of $F(s, t)$ is determined by the sum of the ladder type diagrams/3,4/:

$$F(s,t) \underset{s \to -\infty}{\to} \sum_{n=0}^{\infty} F_n(s,t), \tag{6}$$

where

$$F_n(s,t) = -\frac{g^2}{s}\frac{1}{n!}\left[g^2 \Delta(t) \ln(-s)\right]^n \tag{7}$$

and

$$\Delta(t) = \frac{1}{8\pi^2}\int_{4m^2}^{\infty} \frac{1}{\sqrt{t'(t'-4m^2)}} \frac{dt'}{t'-t}. \tag{8}$$

As is well known the sum of the terms (7) gives the asymptotic Regge behaviour

$$F(s,t) \to g^2 (-s)^{\alpha(t)}, \tag{9}$$

where

$$\alpha(t) = -1 + g^2 \Delta(t). \tag{10}$$

However, in doing so we lose the poles of the Regge residue at the positive values of t which should appear in the true Regge-pole term.

For this reason we use the method of summation of perturbation theory expansions for spectral functions rather than for the Green function itself. Such a method was suggested for the investigation of the photon Green function in quantum electrodynamics/5/.

Finding the asymptotic of spectral functions $P_n(s,t)$ from eq. (7) and using formula (5) we get the following behaviour of the functions $f_n(x,t)$ at $x \approx 1$:

$$f_n(x,t) = g^2 \frac{1}{n!}[-g^2\Delta(t)\,\ell n(1-x)]^n. \tag{11}$$

Thus to the asymptotic behaviour of ladder diagrams at $s \to -\infty$ there correspond the logarithmic singularities of the function $f_n(x,t)$ at $x=1$.

Taking the sum of the terms (11) we find that $f(x,t)$ will have a regular singularity at $x=1$ of the form

$$f(x,t) = g^2(1-x)^{-1-\alpha(t)}. \tag{12}$$

Substituting eq. (12) in formula (4) we get just the expression for the Eulerian B -function

$$F(s,t) = g^2\int_0^1 dx\, x^{-1-s}(1-x)^{-1-\alpha(t)} = g^2\,\frac{\Gamma[-\alpha(t)]\Gamma[-s]}{\Gamma[-s-\alpha(t)]}. \tag{13}$$

In a more general case we can write

$$f(x,t) = g^2(1-x)^{-1-\alpha(t)}\,\Phi(x,t), \tag{14}$$

where $\Phi(x,t)$ is a regular at $x=1$ and obeys the condition

$$\Phi(x,t)\big|_{x=1} = 1. \tag{15}$$

Assuming that the function $\Phi(x,t)$ has a regular singularity at $x=0$ we will write

$$\Phi(x,t) = x^{\gamma} \sum_{n=0} C_n x^n , \tag{16}$$

where γ is not integer and

$$\sum_n C_n = 1 . \tag{17}$$

As a result we obtain for the amplitude the Veneziano type representation of the form

$$F(s m, t) = g^2 \sum_{n=0} C_n \frac{\Gamma[-a(t)]\,\Gamma[\gamma + n - s]}{\Gamma[\gamma + n - s - a(t)]} , \tag{18}$$

where the sum of the coefficients is normalized to unity. The asymptotic form of eq. (18) is as follows

$$F(s,t) \underset{s \to -\infty}{\to} g^2\, \Gamma[-a(t)]\,(-s)^{a(t)} . \tag{19}$$

which possesses the poles at the points where $a(t)$ = zero or positive integer. If we now reexpand the expression (19) in powers of g^2 and keep only the principal logarithmic terms we find after summation just the "old" result (9) without any poles.

Thus the method we use here allows one to take into account in some sense the "younger" logarithmic terms as well as the principal ones. Notice that the $s \leftrightarrow t$ crossing symmetry and the linearity of the trajectory $a(t)$ is absent in eq. (18). The latter is probably due to a rather simplified model considered here when amplitudes are dominated at high energies by the two-particle intermediate state in t -channel /6/.

The authors express their deep gratitude to N.N. Bogolubov for stimulating discussion and critical remarks, and also to

D.I. Blokhintsev, A.A. Logunov, V.G. Kadyshevsky, S.P. Kuleshov, R.M. Muradyan, V.P. Shelest, A.N. Šissakian and L. Slepchenko for helpful discussion.

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Received by Publishign Department
on December 8, 1969.