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It is well known, that the widely used now eikonal or impact parameter method [1] rests on the physical assumptions, concerning both the magnitude and slope of the scattering potential. Due to its simplicity and the fact, that the amplitude obtained satisfies the unitarity condition at high energies this approach is often used in the interpretation of the experiment and in theoretical investigations [1]. In some cases there is a fairly good agreement between experiment and high energy approximation calculations even though the small angle applicability condition is broken [2].

In the present paper, the method recently used by one of the authors [3] is developed further to investigate the potential scattering of Dirac particles at high energies. The problem of finding the potential scattering amplitude is set directly on the basis of the integral equation for the amplitude. At high energy and small angles the problem is reduced to the solution of the one-dimensional integral equation for the "local" amplitude. The final result we obtain is close to the Blankenbekler and Goldberger's approximation [1]. Keeping the same accuracy a certain modification appears however, which reflects the fact that the approximate function $\Psi(\bar{r}) = [1 + \frac{i}{4p}\int_{-\infty}^{z} V(\sqrt{b^2 + z'^2})\,dz']^{-2} \cdot e^{i\bar{p}_0\bar{r}}$ used in [1] has no necessary asymptotic form. The new representation of the amplitude satisfies the unitarity condition in the high energy limit as well.

We start from the usual equation for the amplitude

$$\hat{f}(\theta) = -\frac{1}{4\pi}((\bar{a}\,\bar{p}) + \beta m + E)\int e^{-i\bar{p}\bar{r}}\, V(r)\Psi(\bar{r})\, d^3 r,$$

where

$$\Psi(\bar{r}) = u_0 e^{i\bar{p}_0\bar{r}} - \frac{1}{4\pi}\int \frac{e^{ip|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}((\bar{a}\,\hat{p}') + \beta m + E) V(r')\Psi(\bar{r}')\, d^3 r'. \quad (2)$$

Then we introduce the J_0 - representation of the Born-amplitude

$$f_B(\theta) \approx \int e^{i\bar{q}\bar{r}}\, V(r)\, d^3 r = -4\pi \int_0^\infty J_0(qb)\,\chi(b)\, db,$$

$$\chi(b) = -\frac{1}{2}\int_{-\infty}^{\infty} V(\sqrt{b^2+z^2})\, dz; \quad \bar{q} = \bar{p}_0 - \bar{p} \quad (2)$$

and the function

$$\phi(\bar{r}) = e^{-i\bar{p}_0\bar{r}}\,\Psi(\bar{r}).$$

At high energy and small angles ($\theta < \frac{1}{pR}$, R - characteristic linear size of the potential) the problem is reduced to the following equations

$$\hat{f}(\theta) = ((\bar{a}\,\bar{p}) + \beta m + E)\int_0^\infty J_0(qb)\,\hat{\Gamma}(b)\chi(b)\, db, \quad (3)$$

$$\hat{\Gamma}(b) = u_0 + \frac{1}{2\pi^2}\int_0^\infty b'\,db'\,\chi(b')\int \frac{(\bar{a}\bar{\tau}) + \beta m + E}{\tau^2 - p^2 - i\epsilon} J_0(b|\bar{p}_0 - \bar{\tau}|)\, J_0(b'|\bar{p}_0 - \bar{\tau}|)\,\hat{\Gamma}(b')\, d^3\tau, \quad (4)$$

where use has been made of eq. (2) and the integral equation for the function $\phi(\bar{\tau}) = \frac{1}{(2\pi)^3}\int e^{i\bar{\tau}\bar{r}}\,\phi(\bar{r})\, d^3 r$. Keeping only the "singular" part of the integral entering the karnel of eq. (4) we obtain the equation

$$\hat{\Gamma}(b) = \bar{u}_0 + \frac{i}{2p}\chi(b)((\bar{\alpha}\,\bar{p}_0) + \beta m + E)\hat{\Gamma}(b)$$

and then,

$$\hat{\Gamma}(b) = \frac{u_0}{1 - i\frac{E}{p}\chi(b)} \qquad (5)$$

By substituting (5) in eq. (3) it follows the result

$$f(\theta) = 2E(u_f^* u_0)\int_0^\infty J_\sigma(qb)\frac{\chi(b)}{1 - \frac{iE}{p}\chi(b)}\, b\, db \qquad (6)$$

Obviously, it is a generalization of the Blankenbekler and Goldberger's formula to the potential scattering of Dirac particles. A new result is obtained here when taking into account approximately /3/ the "principial" part of the integral in eq. (4). This is the corresponding cross section

$$\frac{\sigma(\theta)}{4E} = \frac{p^2}{E^2}\left(\frac{m^2}{p^2} + \cos^2\frac{\theta}{2}\right)|f_0(\theta)|^2 + \frac{p^2}{E^2}\left(\frac{m^2}{p^2} + \sin^2\frac{\theta}{2}\right)|f_1(\theta)|^2, \qquad (7)$$

where

$$f_0(\theta) = \int_0^\infty J_0(qb)\frac{1-\beta E}{\Delta(b)}\chi(b)b\, db,$$

$$f_1(\theta) = \int_0^\infty J_0(qb)\frac{m\beta}{\Delta(b)}\chi(b)b\, db,$$

$$\Delta(b) = 1 - 2(\alpha+\beta)E + \beta(2\alpha+\beta)\cdot p^2,$$

$$\alpha = \frac{1}{2\pi^2}\int_0^\infty b'db'\chi(b')[I_1(b,b') - I_2(b,b')] + \frac{i}{2p}\chi(b),$$

$$\beta = \frac{1}{2\pi^2}\int_0^\infty b'db'\chi(b')I_2(b,b'),$$

$$I_1(b,b') = 4\pi \int_0^1 dy \, \fint_0^\infty \frac{J_0(br) J_0(b'r)}{r^2 - (2py)^2} r^2 dr \; ; \quad I_2(b,b') = 8\pi \int_0^1 y^2 dy \, \fint_0^\infty \frac{J_0(br) J_0(b'r)}{r^2 - (2py)^2} r^2 dr .$$

For the electron scattering, neglecting the electron mass, we have simpler result

$$\frac{\sigma(\theta)}{4E^2} = \cos^2 \frac{\theta}{2} \left| \int_0^\infty J_0(qb) \frac{\chi(b) b \, db}{1 - \Delta' - i\chi(b)} \right|^2 . \qquad (8)$$

In this case an additional approximation was accepted: $\frac{d\chi}{db} R \ll 1$. As a crude estimation the value $\Delta' \approx \pm \frac{\pi}{4} Z e^2$ for the pure Coulomb interaction can be used. The correction factor Δ' will cause a change of the diffractional peak position and its width depending on the sign and magnitude of the potential.

References

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