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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

PROTON STRUCTURE AND HYPERFINE SPLITTING IN THE HYDROGEN ATOM

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1. Analysis of the Pole Terms

In part I of the paper[1] a correction to the hyperfine splitting in the S-state of the hydrogen atom, proportional to αm; was evaluated

$$\delta = (\delta_N - \delta^0_{stat}) + \Delta_{cut} \qquad (1)$$

$$\delta_N - \delta^0_{stat} = \frac{\alpha m}{\pi} 8 \int_0^\infty \frac{dk}{k^2} [G_N(-k^2)\mathcal{H}(k) - \mathcal{H}(0)] \equiv \delta_{TL} + \delta_{TT} - \delta^0_{stat} \qquad (2)$$

$$\mathcal{H}(k) = \frac{k^3}{\pi} \int_0^{\pi/2} \frac{d\phi \sin^2\phi}{k^2 + 4m^2\cos^2\phi} [f_1(-k^2) \frac{2}{\cos^2\phi + (k^2/4)} + (f_1(-k^2) + 3\mu f_2(-k^2)) \frac{\cos^2\phi}{\cos^2\phi + (k^2/4)}] \equiv \qquad (3)$$

$$\equiv \mathcal{H}_{TL}(k) + \mathcal{H}_{TT}(k)$$

$$G_N(x) = \frac{f_1(x) + \mu f_2(x)}{1+\mu}; \qquad \mathcal{H}(0) = (1+m)^{-1}.$$

(We set the proton mass equal to one and use the same symbols as in part I). In these formulae δ_{TL} is the contribution from the exchange of longitudinal and transversive photons, and δ_{TT} is the contribution from the exchange of two transversive photons (fig. 1b), δ^0_{stat} corresponds to the diagram of fig. 1c (see[1]). (The cross diagrams follow automatically)

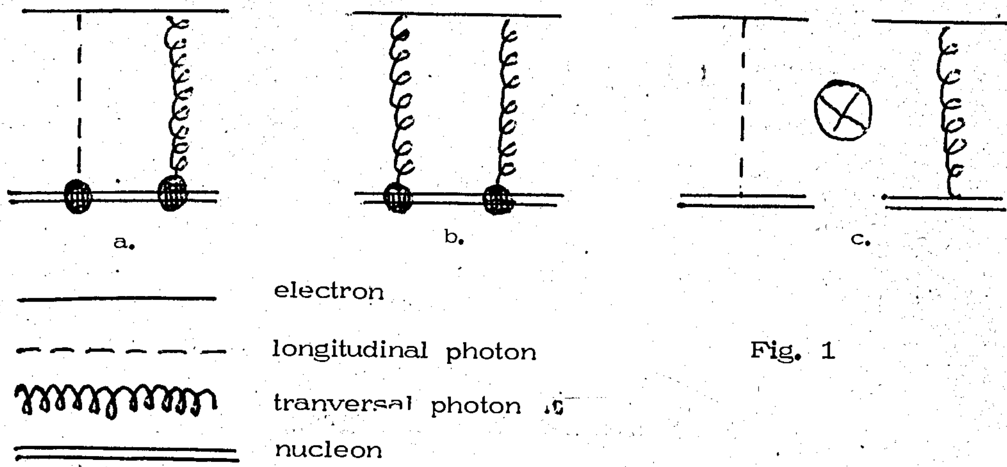

Fig. 1

$$\Delta_{out} = \frac{\alpha m 2}{\pi^3(1+\mu)} \int \frac{d^4k}{k^2(k^4+4m^2k_0^2)} [(2k^2+k_0^2) H_1^{out}(-k^2, ik_0) - 3k^2 k_0^2 H_2^{out}(-k^2, ik_0)] \quad (4)$$

(Note that in formulae (3) and (4) some of the terms proportional to m^2 (from the electron propagator) are retained since in the limit $m \to 0$ we get logarithmic divergences in δ_N and $\Delta_{H_1\,out}$).

To evaluate (1) we choose the nucleon form factor in the form

$$f_1(-k^2) = f_2(-k^2) = f(-k^2) = (1 + \frac{k^2}{m_0^2})^{-2}; \quad m_0^2 = 0{,}71 \text{ BeV}^2 . \quad (5)$$

It is convenient to present the integral (2) in the form

$$G_N(-k^2)\, \mathcal{H}(k) - \mathcal{H}(0) = f^2(-k^2)\,[(\mathcal{H}'_{TL}(k) - \mathcal{H}'(0)) + \mathcal{H}'_{TT}(k)] + \mathcal{H}'(0)[f^2(-k^2) - 1] \quad (6)$$

$$\mathcal{H}'(k) = \mathcal{H}(k)/f^2(-k^2)$$

since in the static limit $M_N \to \infty$, $\delta_{TT} \to 0$ and $\delta_{TL} \to \delta^0_{stat}$ (if $f(-k^2) = 1$). The first and the second summands in (6) can be interpreted as "the effect of the finite nucleon mass" and "the effect of the form factor". The first bracket yealds a value 26,1 ppm (1 ppm $= 10^{-6}$) (the contribution from $\mathcal{H}_{TT}$ being equal to 53,4 ppm), whereas the second gives -43,2 ppm . The total being thus -17,1 ppm .

For the estimation of the contribution from the cut we evaluate that from the N^*_{33} resonance in (4) (see Appendix II).

Substituting

$$\Pi^{N^*_{33}}_{1,2}(-k^2; ik_0) = \frac{2\nu_0(-k^2)}{\nu_0^2(-k^2)+k_0^2} R^{N^*_{33}}_{1,2}(-k^2)$$

and performing numerical integration we obtain the value $\cong -35$ ppm (in fact, the whole contribution comes from $\Pi_1^{N^*_{33}}$, $\Pi_2^{N^*_{33}} \approx 0{,}5$ ppm). Therefore Δ_{cut} is in-general not small as compared to the first summand in (1). Since little is known about the behaviour of the amplitudes $\Pi_{1,2}$ on the cut the application of the expression (4) is rather difficult.

If we turn back to the integral (1) and to the contribution from N^*_{33} in (4) and consider the integrals in k as functions of the upper limit of integration then we can see that, in fact, the whole contribution to the integrals comes from the region of small values of k : $0 \leq k \leq m_\pi$. One of the reasons for this is the smallness of the electron mass and the logarithmic divergence of the integrals at $m \to 0$. The situation remains unaltered even in the case of much slower decrease of the nucleon and N^*_{33} form factors. It could thus be expected, that has we found the contribution from cuts at small k^2 the evaluation of the contribution from the remaining region would not be substantial.

Let us present Δ_{cut} in the form

$$\Delta_{cut} = \frac{\alpha m}{\pi(1+\mu)} \cdot \frac{2}{\pi^2} \int \frac{d^4k}{k^6} [(2k^2+k_0^2)(\Pi_{1\,cut}(-k^2, ik_0) - \Pi_{1\,cut}(0,0)\phi(-k^2)) -$$

$$- 3k^2k_0^2\, \Pi_{2\,cut}(-k^2, ik_0)] + \frac{\alpha m}{\pi(1+\mu)} \frac{2}{\pi^2} \Pi_{1\,cut}(0,0)\,\mu \int \frac{d^4k}{k^4} \frac{k^2}{k^4+4m^2k_0^2}\phi(-k^2) \equiv$$

$$\equiv S_2 + S_3 + S_B \,;\quad \phi(0) = 1. \qquad (7)$$

We have added and subtracted a term proportional to $H_{1\,cut}(0,0) = H_1(0,0)$ then in the first term we have set $m = 0$. $H_1(0,0)$ can be evaluated by the low energy theorem

$$H_1(0,0) = \frac{1}{\pi} \int_{\nu_t(0)}^{\infty} \frac{d\nu^2}{\nu^2} \, \mathrm{Im}\, H_1(0,\nu) = -\frac{\mu^2}{4}. \tag{8}$$

At small k S_2 behaves like $\int dk\, k\, \mathrm{const}$ and such a contribution from the cut of the amplitude H_1 in the region of small k^2 reduces to the term S_B in (7). Choosing $\phi(-k^2) = f^2(-k^2)$ we get $S_B = -22{,}6$ ppm (In this case the sum of the nucleon pole contribution plus S_B is equal to the contribution of the modified Born diagram with such a vertex as if the nucleon were on the mass shell). We note that we have not transformed the term H_2^{cut} in (7), since the region of small k^2 is already demped in S_3.

2. Estimation of S_2 and S_3

Our calculations of S_2 and S_3 are considerably facilitated by the fact that we try to obtain a reasonable upper limit since too little is known about the amplitude $H_{1,2}$. If the estimated upper bound will turn out to be small, this will be sufficient for our program.

Now we assume that in the region of large k^2 the amplitudes $H_{1,2}$ behave not worse than those obtained by Björken[3], i.e.

$$H_1(k^2,\nu)\Big|_{\substack{k^2\to-\infty \\ (\nu/k^2)\to 0}} \lesssim \frac{z_1}{k^2}, \qquad H_2(k^2,\nu)\Big|_{\substack{k^2\to-\infty \\ (\nu/k^2)\to 0}} \lesssim \frac{z_2}{k^4} \tag{9}$$

A rough estimation of S_2 and S_3 can be obtained in the following way. Separating the integrals in (7) into region of "small" $(0 \leq |k^2| \leq 1)$ and "large" $(|k^2| \geq 1$ values of k^2, we can set in the first region (we admit the possibility of the existence of the asymptotic $\approx 0(\frac{1}{k^4})$ instead of $0(\frac{1}{k^5})$ in[4] though in the equal time commutator $[j_\mu(x), j_\nu(0)]_{x_0=0}$ the possible contribution to H_2 vanishes at $\vec{p}=0$, since this difference is inessential in the estimation of the contribution from the region of large k^2 in S_3):

$$[H_1^{out}(-k^2, ik_0) - \phi(k^2) H_1^{out}(0,0)] \approx k^2 |H_1^{out}(0,0)| \tag{10}$$

$$|S_2|_{k^2\leq 1} \approx \frac{\alpha m 4}{\pi^2(1+\mu)} \int_0^1 dk^2 \int_0^\pi d\phi \sin^2\phi (2+\cos^2\phi) |H_1(0,0)| \approx \frac{4\alpha m}{\pi(1+\mu)} |H_1(0,0)| \approx 2 \text{ ppm}.) \tag{11}$$

For H_2 we assume $H_2^{out}(-k^2, ik_0) \approx |H_2^{out}(0,0)| \approx |H_2^{N^*_{33}}(0,0)|$

$$|S_3|_{k^2\leq 1} \approx \frac{4\alpha m}{\pi^2(1+\mu)} \int_0^1 dk^2 \int_0^\pi d\phi \sin^2\phi\, 3\cos^2\phi\, |H_2^{N^*_{33}}(0,0)| \approx \frac{3\alpha m}{2\pi(1+\mu)} |H_2^{N^*_{33}}(0,0)| \approx 4 \text{ ppm} \tag{11'}$$

(We note that the contribution from N^*_{33} in H_2 in fact is $\approx$ 0.5 ppm).
One can put in the second region

$$|S_2 + S_3|_{k^2 \geq 1} \approx \frac{6\alpha m}{\pi^3(1+\mu)} \int_1^\infty \frac{d^4k}{k^4} [\ |H_1^{asympt}(k^2)| + k^2 |H_2^{asymp}(k^2)|] \approx \frac{6\alpha m}{\pi(1+\mu)} \int_1^\infty \frac{dk^2}{k^4} [|z_1| + \tag{12}$$

$$+ |z_2|] \approx \frac{6\alpha m}{\pi(1+\mu)} (|z_1| + |z_2|) \approx 6 \text{ ppm} \qquad |z_1| \approx |z_2| \approx 1$$

One sees that the contributions obtained are really small. Alternatively we can damp in S_2 the region of small and large k^2 and transmitt their contribution to S_B by choosing e.g. the function $\phi(-k^2)$ in (7) in such a way that $\phi(0)=1$ and $\phi(-k^2)|_{-k^2>>1} \to \frac{z_1}{k^2}$. We have numerically $S_B = -25{,}5$ ppm if $\phi(-k^2) = (1+\frac{k^2}{m_0^2})^{-1}$, $m_0^2 = 0.71$ (Bev)^2 and $S_B = -22{,}6$ ppm if $\phi(-k^2) = (1+\frac{k^2}{m_0^2})^{-4}$. For a possibility of more accurate estimate we use dispersion relations for $\Pi^{cut}_{1,2}$

$$\Pi^{cut}_{1,2}(k^2, \nu) = \frac{1}{\pi}\int_{\nu_t(k^2)}^{\infty} \frac{d\nu'^2}{\nu'^2-\nu^2}\, \mathrm{Im}\,\Pi_{1,2}(k^2,\nu) \tag{13}$$

and substitute them into (7). We obtain (see[2])

$$S_2 = \frac{\alpha m}{\pi(1+\mu)}\int_0^{\infty}\frac{dk}{k}\left[\frac{9}{4}\mu^2\phi(-k^2) - \frac{4}{\pi}\int_{\nu_t(-k^2)}^{\infty}\frac{d\nu^2}{\nu^2}\,\mathrm{Im}\,\Pi_1(-k^2,\nu)\,\beta\left(\frac{\nu^2}{k^2}\right)\right] \tag{14}$$

$$S_3 = -\frac{12\alpha m}{\pi^2(1+\mu)}\int_0^{\infty}\frac{dk}{k}\int_{\nu_t(-k^2)}^{\infty} d\nu^2\, \mathrm{Im}\,\Pi_2(-k^2,\nu)\;\delta\left(\frac{\nu^2}{k^2}\right), \tag{15}$$

where

$$\beta(x) = 3x - 2x^2 - 2(2-x)\sqrt{x(x+1)}, \quad \delta(x) = 1+2x-2\sqrt{x(x+1)} \tag{16}$$

$$\nu_t(-k^2) = \frac{2m_\pi + m_\pi^2 + k^2}{2} .$$

We now concentrate our attention on the high energy (Regge) region $(\nu/\nu_t) \gg 1$ (for the region of small ν/ν_t we retain the N^*_{33} resonance only), since the behaviour of $\mathrm{Im}\, H_{1,2}$ in the "low energy region" was studied in detail in[4].

The asymptotic behaviour of the amplitudes $H_{1,2}$ is treated in Appendix I.

The calculations were performed in the following way. We have chosen for H_1^{cut} the representation

$$\frac{1}{\pi}\,\mathrm{Im}\, H_1^{cut}(k^2,\nu) = \delta(\nu-\nu_0(k^2))\, R_1^{N^*_{33}}(k^2) + \theta(\nu-\nu_0(k^2))\,\beta_1^{a_1}(k^2)\left(\frac{\nu}{\nu_t(k^2)}\right)^{a_1} \quad (17)$$

where θ is the step θ function. The Regge-residue was chosen in the form

$$\beta_1^{a_1}(k^2) = \beta_1^{a_1}(0)\,\Gamma_1(k^2), \quad \Gamma_1(0) = 1 \,. \qquad (18)$$

The norm was chosen in accordance with the low energy theorem, by a substitution of (17) into (8). The behaviour of $\Gamma_1(k^2)$ was chosen "maximal" allowed by the condition (8). Also, we have put $\Gamma_1(k^2) = (1+\frac{k^2}{m_0^2})^{-1}$.

The parameters of the N^*_{33} resonance are given in the Appendix II.

The calculations give a small value of S_2 not exceeding $\approx 1{,}5$ppm.

The H_2^{cut} amplitude representation was chosen in analogy with (17), (18) , $a_2 = a_1 = -1$, $\Gamma_2(k^2) = (1+\frac{k^2}{m_0^2})^{-2}$ and $\beta_2(0)$ was chosen in accordance with the condition (9) ,

$$H_2(k^2,\nu)\Big|_{\substack{k^2\to-\infty\\ \nu/k^2\to 0}} \to H_2(k^2,0) = \int_{\nu_t}^{\infty} \frac{d\nu^2}{\nu^2}\beta_2(0)\Gamma_2(-k^3)\left(\frac{\nu}{\nu_t}\right)^{-1} \approx \frac{2m_0^2\beta_2(0)}{k^4} \to \frac{z_2}{k^4}$$

at $|z_2| \approx 1$, $|\beta_2(0)| \approx |z_2| \approx 1$.

The calculations give a value of $S_3 \approx 1-2$ ppm.

3. Summary

Collecting the results obtained above we have for the correction to the Fermi term, proportional to αm the following value

$$\delta = (-40 \pm 6)\,\mathrm{ppm}\ .$$

The error in δ was chosen as a "reasonable upper bound" from the estimates obtained earlier. It is known [8] that the figure presented above agrees with the value $\alpha^{-1} = 137{,}0359$.

The situation in hyperfine splitting can be summarized as follows: according to the calculations performed in [4,3] and in the present paper there is no realistic "candidate" which could contribute in the terms of the type S_2 and S_3. According to our point of view this is not surprising since (by the assumptions used above) the dominance of the contribution from the region of small mass photons $0 \lesssim |k^2| \ll 1$ is quite natural.

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Appendix I

There is some contraversy among the results on the asymptotical behaviour of the virtual compton scattering amplitude[6]. Therefore we devote a special section to this problem.

Let us write the covariant wirtual Compton scattering amplitude in the forward direction in the form[7].

$$\Psi^{ba}_{\lambda',\lambda}(p,k) = \bar{U}(p,\lambda')e^{*b}_{\mu}(k)C^{N}_{\mu\nu}(p,k)e^{a}_{\nu}(k)U(p,\lambda)$$

$$C^{N}_{\mu\nu}(p,k) = (k_{\mu}k_{\nu} - g_{\mu\nu}k^2)t_1(k^2,\nu) + [\nu^2 g_{\mu\nu} + k^2 p_{\mu}p_{\nu} - \nu(p_{\mu}k_{\nu} + p_{\nu}k_{\mu})]t_2(k^2,\nu) + \tag{1.1}$$

$$+ \frac{1}{i}\epsilon_{\mu\nu\sigma\tau} k_{\sigma}\gamma_{\tau}\gamma_5 D(k^2,\nu) - i\epsilon_{\mu\nu\sigma\tau} k_{\sigma}p_{\tau}\hat{k}\gamma_5 G(k^2,\nu).$$

(Here and in what follows we use the c.m.s. with the nucleon momentum along p axis z . $e^{a}_{\nu}(k)$ and $e^{*b}_{\mu}(k)$ are the polarization vectors of the incident and scattered photons with momenta k , $U(p,\lambda)$ and $\bar{U}(p,\lambda')$ are the spinors of the initial and final nucleon with spin projections λ and λ' on the z axis, $\bar{U}U = 1$, $\nu = (pk)$ the nucleon mass is equal to one).

The four independent helicity amplitudes in the S -channel are related to the invariant amplitudes as follows $f^{s}_{0-\frac{1}{2},1\frac{1}{2}} \equiv \langle 0-\frac{1}{2}|1\frac{1}{2}\rangle$

$$f^{s}_{0\frac{1}{2},0\frac{1}{2}} = k^2(t_1 - t_2), \qquad f^{s}_{0-\frac{1}{2},1\frac{1}{2}} = \sqrt{2k^2}\,D \tag{1.2}$$

$$f^{s}_{\frac{1}{2}\pm 1,\frac{1}{2}\pm 1} = (k^2 t_1 - \nu^2 t_2) \mp (\nu D + (\nu^2 - k^2)G)$$

$$t_1(k^2,\nu) = \frac{1}{k^2-\nu^2}\left[\frac{1}{2}\left(f^s_{\frac{1}{2}-1,\frac{1}{2}-1} + f^s_{\frac{1}{2}1,\frac{1}{2}1}\right) - \frac{\nu^2}{k^2} f^s_{\frac{1}{2}0,\frac{1}{2}0}\right]$$

$$t_2(k^2,\nu) = \frac{1}{k^2-\nu^2}\left[\frac{1}{2}\left(f^s_{\frac{1}{2}-1,\frac{1}{2}-1} + f^s_{\frac{1}{2}1,\frac{1}{2}1}\right) - f^s_{\frac{1}{2}0,\frac{1}{2}0}\right]$$

$$D(k^2,\nu) = \frac{1}{\sqrt{2k^2}} f^s_{0-\frac{1}{2},1\frac{1}{2}} \tag{1.3}$$

$$G(k^2,\nu) = \frac{1}{\sqrt{(k^2-\nu^2)^2}}\left[\frac{1}{2}\left(f^s_{\frac{1}{2}1,\frac{1}{2}1} - f^s_{\frac{1}{2}-1,\frac{1}{2}-1}\right) + \frac{\nu}{\sqrt{2k^2}} f^s_{0-\frac{1}{2},1\frac{1}{2}}\right]$$

In t -channel there are six helicity amplitudes different from zero, with crossing symmetry ($f^t_{10,\frac{1}{2}-\frac{1}{2}} \equiv \langle 10 | \frac{1}{2} - \frac{1}{2} \rangle$)

$$f^s_{\frac{1}{2}\pm1,\frac{1}{2}\pm1} = \frac{1}{4}\left[2f^t_{\frac{1}{2},\frac{1}{2},00} \mp 2\sqrt{2}\left(f^t_{\frac{1}{2}-\frac{1}{2},10} - f^t_{-\frac{1}{2}\frac{1}{2},10}\right) + f^t_{\frac{1}{2}\frac{1}{2},11} + f^t_{\frac{1}{2}\frac{1}{2},-1-1} + 2f^t_{\frac{1}{2}\frac{1}{2},1-1}\right]$$

$$f^s_{\frac{1}{2}0,\frac{1}{2}0} = \frac{1}{2}\left[f^t_{\frac{1}{2}\frac{1}{2},11} + f^t_{\frac{1}{2}\frac{1}{2},-1-1} - 2f^t_{\frac{1}{2}\frac{1}{2},1-1}\right]$$

$$f^s_{\pm\frac{1}{2},0\mp\frac{1}{2}1} = \frac{1}{2\sqrt{2}}\left[\mp\sqrt{2}\left(f^t_{\frac{1}{2}-\frac{1}{2},10} - f^t_{-\frac{1}{2}\frac{1}{2},10}\right) + f^t_{\frac{1}{2}\frac{1}{2},11} - f^t_{\frac{1}{2}\frac{1}{2},-1-1}\right] \tag{1.4}$$

$$f^s_{\frac{1}{2}-1,\frac{1}{2}1} = \frac{1}{4}\left[2\left(f^t_{\frac{1}{2}\frac{1}{2},1-1} - f^t_{\frac{1}{2}\frac{1}{2},00}\right) + f^t_{\frac{1}{2}\frac{1}{2},11} + f^t_{\frac{1}{2}\frac{1}{2},-1-1}\right].$$

Since at $t=0$ $f^s_{\frac{1}{2}0,-\frac{1}{2}1} = f^s_{\frac{1}{2}-1,\frac{1}{2}1} = 0$ we get the following two constraints

$$\sqrt{2}\left(f^t_{\frac{1}{2}-\frac{1}{2},10} - f^t_{-\frac{1}{2}\frac{1}{2},10}\right) = f^t_{\frac{1}{2}\frac{1}{2},11} - f^t_{\frac{1}{2}\frac{1}{2},-1-1} \tag{1.5}$$

$$2f^t_{\frac{1}{2}\frac{1}{2},00} = f^t_{\frac{1}{2}\frac{1}{2},11} + f^t_{\frac{1}{2}\frac{1}{2},-1-1} + 2f^t_{\frac{1}{2}\frac{1}{2},1-1} \qquad (1.6)$$

Now, taking into account (1.5) and (1.6) the crossing can be written in the form

$$t_0(k^2,\nu) = k^2 t_1(k^2,\nu) - \nu^2 t_2(k^2,\nu) = \tilde{\tilde{f}}^t_{\frac{1}{2}\frac{1}{2},00}$$

$$t_2(k^2,\nu) = \frac{2}{k^2}\tilde{\tilde{f}}^t_{\frac{1}{2}\frac{1}{2},1-1}$$

$$H_1(k^2,\nu) = \frac{1}{2\sqrt{2k^2}}\left(\tilde{\tilde{f}}^t_{\frac{1}{2}-\frac{1}{2},10} - \tilde{\tilde{f}}^t_{-\frac{1}{2}\frac{1}{2},10}\right) \qquad (1.7)$$

$$\nu H_2(k^2,\nu) = \frac{1}{2k^2\sqrt{2}}\left(\tilde{\tilde{f}}^t_{\frac{1}{2}-\frac{1}{2},10} + \tilde{\tilde{f}}^t_{-\frac{1}{2}\frac{1}{2},10}\right),$$

where $\tilde{\tilde{f}}^t_{\lambda\mu}$ are the reduced helicity amplitudes

$$\tilde{\tilde{f}}^t_{\lambda\mu} = \left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{-|\lambda-\mu|}\left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} f^t_{\lambda\mu}$$

$$H_1 = \frac{1}{2}(D+\nu G), \qquad H_2 = -\frac{1}{2\nu}G$$

the scattering angle in the t channel is $\cos\theta_t = z_t = \frac{\nu}{\sqrt{k^2}}$.

It can be seen from (1.7) that the invariant amplitudes have no kinematical singularities. For our considerations the amplitudes H_1 and H_2 will be necessary.

The decomposition on the states with the given value of the total momentum has the form[8]

$$\tilde{\tilde{f}}^{t}_{\frac{1}{2}-\frac{1}{2},10} \pm \tilde{\tilde{f}}^{t}_{-\frac{1}{2}\frac{1}{2},10} = \tilde{\tilde{f}}^{t\pm}_{\frac{1}{2}-\frac{1}{2},10} = \sum_{J\,even} F^{J+}_{\frac{1}{2}-\frac{1}{2},10}\, e^{J\pm}_{11}(z_t) + \sum_{J\,odd} F^{J-}_{\frac{1}{2}-\frac{1}{2},10}\, e^{J\mp}_{11}(z_t). \quad (1.8)$$

States with $P=C=\sigma=1, I=0,1$ will contribute to the first sum in (1.8), those with $P=C=-\sigma=1, I=0,1$ to the second one (σ is the signature)

$$\tilde{\tilde{f}}^{t}_{\frac{1}{2}\frac{1}{2},11} - \tilde{\tilde{f}}^{t}_{\frac{1}{2}\frac{1}{2},-1-1} = \tilde{\tilde{f}}^{t-}_{\frac{1}{2}\frac{1}{2},11} = \sum_{J\,even} F^{J-}_{\frac{1}{2}\frac{1}{2},11}\, e^{J+}_{00}(z_t). \quad (1.9)$$

Here the contribution comes from the states with $C=-P=\sigma=1, I=0,1$.

In the pure Regge pole model (and in the absence of the fixed singularities) we have

$$\tilde{\tilde{f}}^{t\pm}_{\frac{1}{2}-\frac{1}{2},10} \underset{(\nu/\nu_t)\to\infty}{\to} \sum \alpha^{\pm}\beta^{\pm}\left(\frac{\nu}{\nu_t}\right)^{\alpha^{\pm}(0)-1} + \sum(\alpha^{\mp}-1)\beta^{\mp}\left(\frac{\nu}{\nu}\right)^{\alpha^{\mp}(0)-2} \quad (1.10)$$

$$\tilde{\tilde{f}}^{t-}_{\frac{1}{2}\frac{1}{2},11} \underset{(\nu/\nu_t)\to\infty}{\to} \sum \beta^{t}\left(\frac{\nu}{\nu_t}\right)^{\alpha^{t}(0)}, \quad (1.11)$$

where $\alpha^{\pm}$ is the trajectory with the positive (negative) signature.

The leading terms will be in $\tilde{\tilde{f}}^{t+}_{\frac{1}{2}-\frac{1}{2},10}$ the Pomeranchuk trajectory , in $\tilde{\tilde{f}}^{t-}_{\frac{1}{2}-\frac{1}{2},10}$ the A_1-trajectory, in $\tilde{\tilde{f}}^{t-}_{\frac{1}{2}\frac{1}{2},11}$ – the π-trajectory ($\alpha_{A_1}(0) \approx \alpha_{\pi}(0) \approx -0{,}02$) . One sees that in this case the asymptotic of the right-and left-hand sides of the equality (1.5) differ essentially ($\approx \nu^{a_P(0)}$ in the r.h.s. and $\approx \nu^{a_\pi(0)}$ in the l.h.s.). In the framework of the Regge pole model it is necessary thus to set at $t=0$ the residua of every trajectory in the r.h.s. with $a_i(0) > a_\pi(0)$ equal to zero. (The π' trajectory, e.g. with $a_{\pi'}(0) = a_\pi(0)$ at $t=0$ will be a leading one in (1.5)). The amplitudes H_1 and H_2 have asymptotics

$$H_1 \approx \nu^{a_{A_1}(0)-1}, \quad H_2 \approx \nu^{a_\pi(0)-2} \tag{1.12}$$

H_2 satisfying the superconvergent sum rule

$$\frac{\mu(\mu+1)}{4} f^2(k^2) = \frac{1}{\pi} \int_{\nu_t(k^2)}^{\infty} d\nu^2 \, \mathrm{Im}\, H_2(k^2, \nu). \tag{1.13}$$

Let us consider now the possible changes of asymptotics (1.12) taking tinto account the contribution of the J-plane cuts.

Following the fact that the cut asymptotically has definite signature[9] from (1.18) the important conclusions can be made for the amplitude H_1 : a) the leading cut with positive signature (P−P) gives a contribution $\approx \frac{1}{\nu \ln \nu}$; b) the leading cut with negative signature (P−A_1) gives a contribution $\approx \frac{\nu^{a_{A_1}(0)-1}}{\ln \nu}$. Thus, the asymptotics of H_1 when no other singularities are present is $\lesssim \frac{1}{\nu}$. The behaviour $H_1 \to \mathrm{const}$ and hence the subtraction in the dispersion relation for the amplitude H_1 can result only from the δ-Kronecker type of fixed singularity at $J=1$. We assume that such a singularity is absent. (Besides that an estimate of the type (11),(12)

(assuming that condition (9) is satisfied) does not depend on the form of the dispersion relations) .
For the amplitude H_2 the asymptotics in any case is $\lesssim \frac{1}{\nu}$.

Appendix II

We present now an example how to calculate the contribution from the N^*_{33} resonance. Contributions from the other resonances can be calculated in a very similar way.

For convenience we choose c.m.s. photon and nucleon (in the rest system of the resonance). The nucleon momentum goes in the z direction.

The imaginary part of the v.c.s. amplitude

$$\mathrm{Im}\,\Psi^{\mu\nu}_{\lambda'\lambda}(p,k) = \frac{E_p}{2} \int dx\, e^{ikx} \langle p,\lambda' |[\, j_\mu(x), j_\nu(0)\,]|\, p,\lambda\rangle \qquad (11.1)$$

(we use the norm $\langle p',\lambda' | p,\lambda\rangle = (2\pi)^3 \delta_{\lambda'\lambda}\, \delta(\vec{p}-\vec{p}\,')$, $\bar{U}U=1$ the nucleon mass equal unity) in the resonance approximation (vanishing widths) has the form

$$\frac{1}{\pi}\,\mathrm{Im}\,\Psi^{\mu\nu}_{\lambda'\lambda}(p,k) = \sum_{J,\lambda''} \delta(\nu-\nu_J) M_J\, \Phi^{*\mu}_{\lambda''\lambda'}(p,k,J)\, \Phi^{\nu}_{\lambda''\lambda}(p,k,J) + \text{cross}, \qquad (11.2)$$

where

$$\Phi^{\mu}_{\lambda\lambda}(p,k,J) = \sqrt{E_p}\, \langle \lambda', J \,|\, j_\mu(0) \,|\, p,\lambda\rangle \qquad (11.3)$$

is the matrix element of the electromagnetic current between the nucleon and a resonance of spin I, $\nu_J = \frac{M_J^2-1-k^2}{2}$, M_J is the resonance mass, λ' and λ are the projections of the nucleon and resonance angular momenta on the Z axis.

For a photon with given helicity $\Phi^{a}_{\lambda'\lambda}(p,k,J) = e^{a}_{\mu}(k)\Phi^{\mu}_{\lambda'\lambda}(p,k,J)$ $a = \pm 1, 0$, $e^{a}_{\mu}(k)$ is the polarization vector of the photon. P and T invariance give

$$\Phi^{-a}_{-\lambda'-\lambda} = \eta_{JN}(-1)^{J-1/2}\Phi^{a}_{\lambda'\lambda}, \quad \Phi^{*a}_{\lambda'\lambda} = \Phi^{a}_{\lambda'\lambda} \tag{11.4}$$

η_{JN} –being nucleons and resonances relative parity.

Now we write the γNN^{*}_{33} vertex in the form [12]

$$\Phi^{\mu}_{\lambda'\lambda}(p,k,J3/2^{+}) = \sqrt{E} < J = \frac{3}{2}^{+}, \lambda' | j_{\mu}(0) | p, \lambda > =$$

$$= \bar{U}_{\nu}(p', \lambda')\gamma_5 [-(\hat{k} g_{\mu\nu} - k_{\nu}\gamma_{\mu}) C_3 + (kp' g_{\mu\nu} - k_{\nu}p'_{\mu}) C_4 + \tag{11.5}$$

$$+ (kp g_{\mu\nu} - k_{\nu} p_{\mu}) C_5] U(p, \lambda).$$

where $U_{\nu}(p', \lambda')$ is the N^{*}_{33} spinor $\bar{U}_{\nu}U_{\nu} = 1$ $p' = (p+k) = (M, 0, 0, 0)$, M is the N^{*}_{33} mass, E –the nucleon energy in the c.m.s., $C_{3,4,5}$ are the three independent form factors depending on k^2 .

The contribution of N^{*}_{33} to each of the invariant amplitudes can be found from the relations (1.3), (1.4), (11.2), (11.4) and from

$$\Phi^{0}_{\frac{1}{2}\frac{1}{2}} = -\sqrt{2k^2}\frac{1}{\sqrt{3}}\sqrt{\frac{E-1}{2}}(C_3 + MC_4 + EC_5) \equiv -\sqrt{2k^2}\frac{1}{\sqrt{3}}\sqrt{\frac{E-1}{2}}\zeta_0$$

$$\Phi^{1}_{\frac{3}{2}\frac{1}{2}} = -\sqrt{\frac{E-1}{2}}[(M+1)C_3 + (kp')C_4 + (kp)C_5] = -\sqrt{\frac{E-1}{2}}\zeta_{3/2}$$

$$\Phi^{1}_{\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{3}}\sqrt{\frac{E-1}{2}}[(M-1-2E)C_3 + (kp')C_4 + (kp)C_5] \equiv \frac{1}{\sqrt{3}}\sqrt{\frac{E-1}{2}}\zeta_{1/2}$$

$$(kp) = \nu_0, \; (kp') = k_0 M = k^2 + \nu_0, \; E = \frac{\nu_0 + 1}{M}, \; \nu_0 = \frac{M^2 - 1 - k^2}{2}. \tag{11.6}$$

We get for H_1 and H_2

$$\frac{1}{\pi}\,\mathrm{Jm}\,H_{1,2}(k^2,\nu) = [\delta(\nu-\nu_0)-\delta(\nu+\nu_0)]\,R_{1,2}^{N^*_{33}}(k^2)$$

$$R_1^{N^*_{33}} = \frac{M(E-1)}{8(k^2-\nu_0^2)}\left[\frac{2}{3}k^2\zeta_0\cdot\zeta_{1/2}+\nu_0\left(\zeta_{1/2}^2-\frac{1}{3}\zeta_{1/2}^2\right)\right]$$

$$R_2^{N^*_{33}} = -\frac{M(E-1)}{8(k^2-\nu_0^2)}\left[\frac{2}{3}\zeta_0\zeta_{1/2}+\frac{1}{\nu_0}\left(\zeta_{3/2}^2-\frac{1}{3}\zeta_{1/2}^2\right)\right]. \qquad (11.7)$$

The multipole decomposition of the vertex (11.5) is of the form[13]

$$(M1) = e\,\frac{3}{2}\sqrt{\frac{E-1}{2}}\left(\zeta_{3/2}-\frac{1}{3}\zeta_{1/2}\right)$$

$$(E2) = e\frac{\sqrt{5}}{2}\sqrt{\frac{E-1}{2}}\left(\zeta_{3/2}+\zeta_{1/2}\right) \qquad \frac{e^2}{4\pi}=\alpha,\quad |\vec{k}|^2 = k_0^2-k^2 \qquad (11.8)$$

$$(\tilde{\tilde{Q}}2) = -e\sqrt{\frac{20}{3}}\sqrt{\frac{E-1}{2}}\,|\vec{k}|\,\zeta_0$$

At $k^2=0$ only two independent constants remain, namely $C_3^0 = C_3(k^2=0)$ and $\lambda^+ = M(C_4^0+C_5^0)/C_3^0$, they can be determined, e.g. from the data on pion photoproduction in the region of the resonance N^*_{33}. One of the conditions reads

$$\lambda^+ = -1+2\,\frac{M+1}{M-1}\,\frac{\delta}{1-\delta},\quad \delta = \frac{1}{\sqrt{5}}\,\frac{(E2)}{(M1)}. \qquad (11.9)$$

The purely magnetic transition approximation corresponds to the values of $\delta=0$ and $\lambda^+=-1$. We note that even small values of δ lead to considerable derivatives of λ^+ from (–1) since the nucleon mass is very close to that of the N^*_{33} resonance. For $\delta \approx 2,5\%$, $\lambda^+ \approx -0.63$. C^0_3 can be determined if we know the cross section of the photoproduction of the pion in the $N^*_{33}(\sigma_{max})$ peak[14]. From the Breit-Wigner formula $\Gamma_\gamma = \frac{1}{16\pi} \frac{(M^2-1)^2}{M^2} \Gamma_{tot} \cdot \sigma_{max}$ Γ_γ being the $N^*_{33} \to N\gamma$ decay width, Γ_{tot} the total $N^*_{33} \to N\pi$ width. Taking into account the smallness of δ, we write

$$\Gamma_\gamma = \alpha \frac{(M^2-1)^3}{16M^5} \frac{(3M+1)^2}{6} (C^0_3)^2 (1 + 2\frac{M-1}{3M+1}\lambda^+ .$$

Combining these formulae and using the values $\Gamma_{tot} \approx 0,128$, $\sigma_{max} \approx 3,9 \cdot 10^{-28} cm^2$, $\lambda^+ \approx -0,63$ we obtain $C^0_3 = 2.0$.

Unfortunately, the available experimental data on electroproduction of the N^*_{33} resonance does not allow us to determine the behaviour of each form factor $C_{3,4,5}(k^2)$ separately. In view of the fast decrease in k^2 [15] we are really interested only in the behaviour in the region of small k^2. Therefore the following recipe was used in our calculations: after we have normalized the form factors at $k^2=0$, $C_{3,4,5}(k^2) = C^0_{3,4,5} \cdot \phi_{3,4,5}(k^2)$, $\phi_{3,4,5}(0) = 1$ we put $\phi_3(k^2) = \phi_4(k^2) = \phi_5(k^2) = \phi(k^2)$ and use for $\phi(k^2)$ data from[15]. The constant $\lambda^- = \frac{M(C^0_4 - C^0_5)}{C^0_3}$ which remains unknown is variated in the region $|\lambda^-| \leq 3$.

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