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n-POINT LORENTZ INVARIANT DISTRIBUTIONS. I

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1. Introduction

The aim of this paper is to represent the space of the distributions invariant to the orthochronous proper Lorentz group $L^{\uparrow}_{+}$, defined on the topological product M_n of n Minkowski spaces, by the distribution space defined on the manifold of orbits of Lorentz group in M_n. The orbit manifold is concretely realized by the matrix manifold, with the Lorentz invariant matrix elements. For $n=1$ the problem was solved in [1,2,3] and for $n=2,3$ in [4] and [5].

We denote by $M \equiv R^4$ the Minkowski space of real points $x = (x^0, x^1, x^2, x^3)$ with the scalar product $\langle x,y \rangle = x^0 y^0 - \sum_{\ell=1}^{3} x^\ell y^\ell$ for any $x, y \in M$ (R^m is the m-dimensional Euclidean real space).

Let M_n be the topological product of n Minkowski spaces formed with the points $\hat{x} = (x_1, \dots, x_n)$ for $x_i \in M$, $i = 1, \dots, n$. $D(M_n) \equiv D(R^{4n})$ is the Schwartz's space of the complex-valued C^∞- test functions with compact support in M_n and its dual $D'(M_n)$ is the space of the distributions in M_n [6].

The distribution $f \in D'(M_n)$ is said to be Lorentz invariant if $f = f_\Lambda$, where

$$f_\Lambda(\phi) \equiv f(\phi_\Lambda) \;;\; \phi_\Lambda(\hat{x}) \equiv \phi(\Lambda x_1, \dots, \Lambda x_n), \qquad (1)$$

$$\phi \in D(M_n), \quad \hat{x} \in M_n, \quad \Lambda \in L^{\uparrow}_{+}.$$

For $\Lambda_0 \in L^{\downarrow}_{+}$ (where $L^{\downarrow}_{+}$ is antichronous proper Lorentz group) every Lorentz invariant distribution f can be decomposed into the even part $f_+ = \frac{1}{2}(f + f_{\Lambda_0})$ and the odd part $f_- = \frac{1}{2}(f - f_{\Lambda_0})$.

We denote by $D'_+(M_n)$ and $D'_-(M_n)$ the even and odd Lorentz invariant distribution spaces, respectively.

2. Representation of Lorentz Invariant Distributions on Lorentz Orbit Manifold

Let S_n be the manifold of the $n \times n$ real symmetrical matrices. S_n is a $\frac{1}{2}n(n+1)$ -dimensional analytic real manifold.

We define the following determinants

$$G^{j_1 \dots j_p}_{i_1 \dots i_p}(u) \equiv \det(u_{ij}) \quad (i = i_1, \dots, i_p \,;\, j = j_1, \dots, j_p),$$
$$G_{i_1 \dots i_p}(u) \equiv G^{i_1 \dots i_p}_{i_1 \dots i_p}(u), \tag{2}$$

where $u \in S_n$; $p, i_1, \dots, i_p, j_1, \dots, j_p = 1, \dots, n$.

Let us introduce the map $\pi : M_n \longrightarrow S_n$ such that $\pi(\hat{x}) = u$ and $u_{ij} = \langle x_i, x_j \rangle$ $(\hat{x} \in M_n)$, $i, j = 1, \dots, n$.

We showed in/7/ that the image U_n of M_n by mapping π is a semialgebraic manifold formed with the matrices $u \in S_n$ which satisfy the following conditions:

1. $u_{ii} \geq 0$ implies $G_{ij}(u) \leq 0$ and $G_{ijk}(u) \geq 0$, (3)
2. $u_{ii} < 0$ and $G_{ij}(u) \leq 0$ implies $G_{ijk}(u) \geq 0$,
3. $G_{ijk\ell}(u) \leq 0$,
4. $\operatorname{rank} u \leq 4$,

where $i, j, k, \ell = 1, \dots, n$.

We can consider U_n as the orbit manifold of the full Lorentz group L in M_n. Indeed, two points in M_n are equivalent if they belong to the same orbit of L and we devide M_n with respect to this equivalence relation. Then there exists a bijective mapping, induced by projection π, which carries any matrix $u \in U_n \setminus \{ \bar{0} \}$ to the orbit $\pi^{-1}(\bar{0})$ of L in $M_n \setminus \{ \hat{0} \}$ ($\hat{0}$ is the zero point of M_n and $\bar{0}$ is the zero matrix of U_n; $\pi^{-1}(\bar{0})$ is the union of the 2^n orbits of L containing the vectors $x \in M_n$ with $x_1, \ldots, x_n$ isotrope and collinear)/7/. The extension of the projection π for the topological product of n complex Minkowski spaces was studied thoroughly in /8/.

Let us now decompose U_n into analytic submanifolds. To avoid certain complications we introduce the following notations:

$$
\begin{aligned}
&U_{nh} \equiv \{ u \mid u \in U_n ; \ \mathrm{rank}\, u = h \} , \\
&M_{nh} \equiv \pi^{-1}(U_{nh}) , \\
&M_{n-} \equiv \{ \hat{x} \mid \hat{x} \in M_n ; \ < x_i , x_i > < 0, \ i = 1, \ldots, n \}, \\
&M_{nh+} \equiv M_{nh} \setminus M_{n-} , \quad U_{nh+} \equiv \pi (M_{nh+}) ,
\end{aligned} \tag{4}
$$

where $h = 1,2,3,4$. We shall denote by $\bar{A}$ the closure of the set A in M_n or S_n. We consider the closed sets X, $Y \subset R^m$ with $X \subset Y$. $D(Y)$ is the Schwartz space of the restrictions to Y of the functions belonging to $D(R^m)$ and $D(Y \setminus X)$ is the subspace of $D(Y)$ consisting of the functions which vanish over X with all their derivatives.

With the above statements we shall prove that the Lorentz invariant distribution spaces on the submanifolds of M_n are isomorphic to the distribution spaces on the images of the considered submanifolds in U_n.

<u>Theorem 1.</u> a) U_{nh} ($h = 1,2,3,4$) is a real analytic submanifold of S_n of dimension $hn - \frac{1}{2} h(h-1)$, with a unique analytic structure.

b) $D'_+(M_{nh})$ ($h = 2,3,4$) is isomorphic to $D'(U_{nh})$ and $D'_+(\bar{M}_{n1} \setminus \{ \hat{0} \})$ is isomorphic to $D'(\bar{U}_{n1})$.

c) $\bar{D}'_-(M_{nh})$ ($h = 2,3,4$) is isomorphic to $D'(U_{nh+})$ and $D'_-(\bar{M}_{n1})$ is isomorphic to $D'(\bar{U}_{n1+})$.

Proof. a) We begin by introducing the index sets $I_h = \{(i_1, \ldots, i_h)\}$, where $i_1, \ldots, i_h = 1, \ldots, n$; $i_1 < \ldots < i_h$ and we define the algebraic manifolds

$$V_{i_1 \ldots i_h} \equiv \{ u \mid u \in U_{nh}, \; G_{i_1 \ldots i_h}(u) \neq 0 \},$$
$$N_{i_1 \ldots i_h} \equiv \pi^{-1}(V_{i_1 \ldots i_h}). \tag{5}$$

Consider now the following $hn - \frac{1}{2} h(h-1)$ local coordinates in $V_{i_1 \ldots i_h}$:

$$\{u_{i_\ell j_\ell}\} \;\; (j_\ell = 1, \ldots, n; \; j_\ell \neq i_{\ell'}; \; \ell' < \ell; \; \ell, \ell' = 1, \ldots, h). \tag{6}$$

The relations (3) and (4) give the equations

$$G^{i_1 \ldots i_h k}_{i_1 \ldots i_h \ell}(u) = 0 \tag{7}$$

in $V_{i_1 \ldots i_h}$ for $k, \ell = 1, \ldots, n$. From (7) it follows that every $u_{k\ell}$ is a rational function of local coordinates with the nonzero denominator $G_{i_1 \ldots i_h}(u)$. We remark that $\{V_{i_1 \ldots i_h}\}$ for $\{i_1, \ldots, i_h\} \in I_h$ is an open covering of U_{nh} in the topology induced by S_n . Then it follows that the local coordinate system (6) determines on U_{nh} an analytic structure/9/. We shall prove that this structure is unique; we prove this only for $h = 4$ (the proof for $h < 4$ is similar). We consider the following transformation of variables

$$\hat{x} \longrightarrow (\langle x_{i_\ell}, x_{i_\ell} \rangle, x_i^{a_1}, x_i^{a_2}, x_i^{a_3}, x_j^{a_2}, x_j^{a_3}, x_k^{a_3}), \tag{8}$$

where $j_\ell = 1, \ldots, n$; $j_\ell \neq i_{\ell'}$ for $\ell, \ell' = 1$; $\ell' < \ell$ and $i, j, k \in \{i_1, i_2, i_3, i_4\}$; $a_1, a_2, a_3 \in \{0, 1, 2, 3\}$ are fixed with $i \neq j \neq k$, $a_1 \neq a_2 \neq a_3$. For any $\hat{x} \in V_{i_1 i_2 i_3 i_4}$

there exist i, j, k, a_1, a_2, a_3 with nonzero Jacobian of the transformation (8):

$$J = 16 \, | x_i^{a_0} | \cdot \begin{vmatrix} x_i^{a_0} & x_i^{a_1} \\ x_j^{a_0} & x_j^{a_1} \end{vmatrix} \cdot \begin{vmatrix} x_i^{a_0} & x_i^{a_1} & x_i^{a_2} \\ x_j^{a_0} & x_j^{a_1} & x_j^{a_2} \\ x_k^{a_0} & x_k^{a_1} & x_k^{a_2} \end{vmatrix} \cdot \begin{vmatrix} x_i^{a_0} & x_i^{a_1} & x_i^{a_2} & x_i^{a_3} \\ x_j^{a_0} & x_j^{a_1} & x_j^{a_2} & x_j^{a_3} \\ x_k^{a_0} & x_k^{a_1} & x_k^{a_2} & x_k^{a_3} \\ x_\ell^{a_0} & x_\ell^{a_1} & x_\ell^{a_2} & x_\ell^{a_3} \end{vmatrix} \tag{9}$$

where $\ell \in \{ i_1, i_2, i_3, i_4 \}$, $a_0 \in \{ 0, 1, 2, 3 \}$ with $\ell \neq i, j, k$ and $a_0 \neq a_1, a_2, a_3$ (indeed because of the nonvanishing of the Gramm determinant $G_{i_1 i_2 i_3 i_4}(\pi(\hat{x}))$ the vectors $x_{i_1}, x_{i_2}, x_{i_3}, x_{i_4}$ are linearly independent),. The relations (8) and (9) show that the restriction of π to M_{n4} is coregular[9]. But if a factor manifold is an analytic manifold (U_{nh}) and the respective projection (π) is coregular then its analytic structure is unique[9]. A consequence of (3) is that U_{nh} are connected excepting U_{11}, U_{22}, U_{33} and U_{44} which have 2,2,8 and 64 components, respectively.

b) Let $h = 1,2,3,4$.

For any $\phi \in D(M_{nh})$ we define the transformations

$$F_{h+}(\phi)(u) = \int_{\bar{M}_{nh}} \delta_{h+}(\hat{x}, u)\, \phi(\hat{x})\, d\mu_h(\hat{x}) \quad , (u \in U_{nh}), \tag{10}$$

where μ_h is the measure on M_{nh} induced by M_n and

$$\delta_{h+}(\hat{x}, u) = \sum_{(i_1, \dots, i_h) \in I_h} \prod_{\ell=1}^{h} \prod_{\substack{j_\ell = 1 \\ j_\ell \neq i_\ell, \ell' < \ell}}^{n} \delta(\langle x_{i_\ell}, x_{j_\ell} \rangle - u_{i_\ell j_\ell}), \tag{11}$$

where δ is the Dirac distribution.

To make more accurate the meaning of the above transformations, for instance for $h = 4$, we consider in (10) and (11) the local analytic transformations (8) with the Jacobian (9) and the partition of the unity belonging to $\{ N_{i_1 i_2 i_3 i_4} \}$ for $(i_1, i_2, i_3, i_4) \in I_4$ and the Dirac distributions have sense. Using the analytic structure of U_{nh} it follows immediately that any $F_{h+}(\phi)$ is a C^{∞}-function of compact support and $F_{h+} : D(M_{nh}) \longrightarrow D(U_{nh})$ is a linear continuous and surjective mapping.

The following proof generalizes the canonical construction given in/1,2,3/. For any $f \in D'_{+}(M_{nh})$ one defines $F'_{h+}(f) \in D'(U_{nh})$ by

$$f(\bar{\psi}) \equiv F'_{h+}(f), \tag{12}$$

where $\psi \in D'(U_{nh})$, $\bar{\psi} = \Phi(\psi_0 \pi)$ with $\Phi \in D(M_{nh})$ and $F_{h+}(\bar{\Phi}) = 1$. Conversely, for any $F'_{h+}(f)$ one defines the distribution

$$\bar{f}(\phi) \equiv F'_{h+}(f)(F_{h+}(\phi)), \quad \phi \in D(M_{n4}). \tag{13}$$

To show that (13) is correct and that $\bar{f} \in D'_{+}(M_{n4})$ we shall prove that

$$(\bar{f} - f)(\phi) = f(\overline{F_{h+}(\phi)} - \phi), \tag{14}$$

where $\omega = \overline{F_{h+}(\phi)} - \phi$ satisfies the equation $F_{h+}(\omega) = 0$. If we apply now the Gauss-Ostogradski formula to $F_{h+}(\omega) = 0$, using also the partition of the unity and by passing to the variables x (for instance for $h=4$ reversing the transformations (8)), it follows

$$\omega = \sum_{\substack{\alpha,\beta=0 \\ \alpha<\beta}}^{3} A_{\alpha\beta} \, \xi_{\alpha\beta}. \tag{15}$$

where $\xi_{\alpha\beta} \in D(M_{nh})$ and

$$A_{\alpha\beta} = \sum_{i=1}^{n} \left(g_{\alpha\alpha} x_i^{\alpha} \frac{\partial}{\partial x_i^{\beta}} - g_{\beta\beta} x_i^{\beta} \frac{\partial}{\partial x_i^{\alpha}} \right) \tag{16}$$

are the infinitesimal generators of the group $L_+^{\uparrow}$ in M_n and $g_{\alpha\beta} = 2\delta_{\alpha 0}\delta_{\beta 0} - \delta_{\alpha\beta}$ is the Minkowski metric. $f \in D'(M_n)$ is a Lorentz invariant distribution if and only if $A_{\alpha\beta} f = 0$ for any α and β. Then from (14) and (15) it follows $f = \bar{f}$. Therefore the mapping $F'_{h+} : D'_+(M_{nh}) \longrightarrow D'(U_{nh})$ is bijective one.

Taking into account the above results, F'_{h+} is a bijective linear bicontinuous mapping, hence it is a isomorphism.

c) Let us take $f \in D'_-(M_{nh})$. We define $g \in D'(U_{nh})$ so that $f(\phi) = g(F_{h+}(\phi))$, where $\phi \in D(M_{nh})$ with its support in M_{n-}. Consider that the support of ϕ is in a bounded open set invariant to the $\Lambda_0 \in L_+^{\uparrow}$ with $(\Lambda_0) = -g_{\alpha\beta}$ $(\alpha, \beta = 0,1,2,3)$. It follows $f(\phi_{\Lambda_0}) = g(F_{h+}(\phi))$. Then $f(\phi) = f(\phi_{\Lambda_0})$. Since f is odd: $f(\phi_{\Lambda_0}) = -f(\phi)$. Hence $f = 0$ if f has the support in $M_{n-} \cap M_{nh}$. Taking into account this remark we define the transformations

$$F_{h-}(\phi) \equiv \int_{\bar{M}_{nh}} \delta_{h-}(\hat{x}, u)\, \phi(\hat{x})\, d\mu_h(\hat{x}) \equiv$$
$$\equiv \int_{\bar{M}_{nh}} \delta_{h+}(\hat{x}, u)\, \bar{\phi}(\hat{x})\, d\mu_h(\hat{x}), \tag{17}$$

where $\phi \in D(M_{nh})$ and $\bar{\phi}(\hat{x}) = \sum_{i=1}^{n} \operatorname{sgn} x_i^0\, \phi(\hat{x})$ for $\hat{x} \in M_n$. One can show now as in the proof of b) that any F_{h-} : $D(M_{nh}) \longrightarrow D(U_{nh})$ is a linear, continuous and surjective mapping and that there exists respectively the isomorphism F'_{h-} : $D'_-(M_{nh}) \longrightarrow D'(U_{nh+})$ with

$$f(\phi) = F'_{h-}(f)(F_{h-}(\phi)), \tag{18}$$

where $f \in D'_-(M_{nh})$, $\phi \in D(M_{nh})$.

Similarly to[1] and[2] there exist the linear and continuous extensions $F_{1+} : D(\bar{M}_{n1} \setminus \{\hat{0}\}) \longrightarrow D(\bar{U}_{n1})$,

$F_{1-} : D(\bar{M}_{n1}) \longrightarrow D(\bar{U}_{n1})$, $F'_{1+} : D'_{+}(\bar{M}_{n1} \setminus \{\hat{0}\}) \to D'(\bar{U}_{n1})$,

$F'_{1-} : D'_{-}(\bar{M}_{n1}) \longrightarrow D'(\bar{U}_{n1+})$ of F_{1+}, F_{1-}, F'_{1+}, F'_{1-}

respectively ($F'_{1\pm}$ are isomorphisms).

It should be noted that Theorem 1 for $n = 2,3$ was proved by Hepp[5]. The transformations $F_{h\pm}$ are obtained generalizing the Methée and Radon transformations[1],[4].

2. Spectral Representation of Lorentz Invariant Distributions

Now in what follows we extend the isomorphisms given in Theorem 1 to the whole $D'_{\pm}(M_n)$. We begin by introducing the Lorentz invariant distributions with support in $\hat{0}$. Any distribution $f \in D'(\hat{0})$ has the form[6]:

$$f = P\left(\frac{\partial}{\partial \hat{x}}\right) \delta(\hat{x}), \tag{19}$$

where $\delta(\hat{x}) = \prod_{i=1}^{n} \prod_{\mu=0}^{3} \delta(x_i^{\mu})$ and $P\left(\frac{\partial}{\partial \hat{x}}\right)$ is a differential polynomial with the complex coefficients in the $\frac{\partial}{\partial x_i^{\mu}}$ variables. If $f \in D'_{+}(\hat{0})$, then according to the Weyl's theorem[10] (with respect to the theory of invariants) there exists the differential polynomial $P(\hat{\Box})$ in the variables $\Box_{ij} = \frac{\partial^2}{\partial x_i^0 \partial x_j^0} - \sum_{r=1}^{3} \frac{\partial^2}{\partial x_i^r \partial x_j^r}$ $(i,j = 1,\dots,n)$ in such a wav that

$$f = P(\hat{\Box})\, \delta(\hat{x}). \tag{20}$$

Any $g \in D'(\bar{0}) (\subset D'(S_{ln}))$ has the form

$$g = Q\left(\frac{\partial}{\partial u}\right) \delta(u), \tag{21}$$

where $\delta(u)=\prod_{\substack{i,j=1\\ i\le j}}^{n}\delta(u_{ij})$ and $Q(\frac{\partial}{\partial u})$ is a differential polynomial with the complex coefficients in the variables $\frac{\partial}{\partial u_{ij}}$ $(i,j=1,\dots,n,\ i\le j)$.

For any $f\in D'_+(\hat{0})$ we define $F'_{0+}(f)\in D'(\bar{0})$ by

$$f(\psi o\pi)=F'_{0+}(f)(\psi),\quad \psi\in D(U_n). \tag{22}$$

If f has the concrete form (20), then $F'_{0+}(f)$ has the concrete form (21) with $Q(\frac{\partial}{\partial u})=P(\hat{\Box}_u)$, where $P(\hat{\Box}_u)$ is obtained from $P(\Box)$ by the adjoint of the substitution

$$\hat{\Box}_{ij}\to 4(1+\delta_{ij})\frac{\partial}{\partial u_{ij}}+\sum_{k,\ell=1}^{n}(1+\delta_{ik})(1+\delta_{j\ell})u_{k\ell}\frac{\partial^2}{\partial u_{ik}\,\partial u_{j\ell}}. \tag{23}$$

We define $H'_+(0)=\{F'_{0+}(f)\}$ for all $f\in D'_+(\hat{0})$; F'_{0+} : $D'_+(\hat{0})\longrightarrow H'_+(\bar{0})$ is an isomorphism. Let $H_+(\bar{0})$ be a locally convex space with the dual space $H'_+(\bar{0})$.

We define now the following direct sums of locally convex spaces

$$\begin{aligned}H_+(U_n)&\equiv D(U_{n4})\oplus D(U_{n3})\oplus D(U_{n2})\oplus D(U_{n1})\oplus H(0)\\ H_-(U_{n+})&\equiv D(U_{n4+})\oplus D(U_{n3+})\oplus D(U_{n2+})\oplus D(\bar{U}_{n1+}).\end{aligned} \tag{24}$$

Using these notations we shall prove the following theorem:

<u>Theorem 2.</u> $D'_+(M_n)$ and $D'_-(M_n)$ are isomorphic to $H'_+(U_n)$ and $H'_-(U_{n+})$.

<u>Proof.</u> According to the Whitney theorem/11/ we write the direct sums

$$D(M_n)=D(M_{n4})\oplus D(M_{n3})\oplus D(M_{n2})\oplus D(\bar{M}_{n1}\setminus\{\dot{0}\})\oplus D(\dot{0}), \tag{25}$$

$$D(\bar{M}_{n1})=D(\bar{M}_{n1}\setminus\{\hat{0}\})\oplus D(\hat{0}).$$

We obtain the dual sums/11/ of (25)

$$D'_+(M_n) = D'_+(M_{n4}) \oplus D'_+(M_{n3}) \oplus D'_+(M_{n2}) \oplus D'_+(\bar{M}_{n1} \setminus \{\hat{0}\}) \oplus D'_+(\hat{0}),$$
$$D'_-(M_n) = D'_-(M_{n4}) \oplus D'_-(M_{n3}) \oplus D'_-(M_{n2}) \oplus D'_-(\bar{M}_{n1}). \quad (26)$$

Finally, we consider now the isomorphisms given in Theorem 1 and we define the following direct sums of isomorphism

$$F'_+ \equiv \overset{4}{\underset{h=0}{+}} F'_{h+}, \quad F'_- \equiv \overset{3}{\underset{h=1}{+}} F'_{h-}. \quad (27)$$

Hence we obtain the isomorphisms $F'_+ : D'_+(M_n) \to H'_+(U_n)$ and $F'_- : D'_-(M_n) \longrightarrow H'_-(U_n)$. It should be noted that $H_-(U_{n+}) = D(U_{n+})$. Theorem 2 for $n=1$ was obtained by Methée/1/.

It follows from Theorems 1 and 2 that any Lorentz invariant distribution has the following formal spectral representation in the sense used by Rieckers and Güttinger/3/:

$$f(\hat{x}) = \sum_{h=1}^{4} \Big[\int_{\bar{U}_{nh}} g_{h+}(u)\, \delta_{h+}(\hat{x}, u)\, d\bar{\mu}_h(u) + \quad (28)$$
$$+ \int_{\bar{U}_{nh+}} g_{h-}(u)\, \delta_{h-}(\hat{x}, u)\, d\bar{\mu}_h(u) \Big] + P(\hat{\square})\, \delta(\hat{x}),$$

where $g_{h+} \in H'_+(\bar{U}_{nh})$, $g_{h-} \in H'_-(\bar{U}_{nh+})$, and $H'_+(\bar{U}_{nh})$ and $H'_-(\bar{U}_{nh+})$ are the restrictions of $H'_+(U_n)$ and $H'_-(U_{n+})$ to $\bar{U}_{nh}$; $P(\hat{\square})\delta(\hat{x})$ is given in (20). $\bar{\mu}_h$ are the measures on the manifolds U_{nh}; this representation is unique modulo direct sums.

The spectral representation (28) for $n=1$ is just that established by Rieckers and Güttinger/3/.

<u>Remarks</u>: 1. Theorems 1 and 2 can be proved for Lorentz invariant tempered distributions using the proofs given above with unessential modifications. Then the Wightman distributions for $n+1$ points admit the spectral representation (28). The Fourier transform (28) has the structure of Lehmann representation/3/,/12/.

2. Theorems 1 and 2 can be extended also for Lorentz covariant distributions. To express the Lorentz covariant distributions as a finite sum of distributions belonging to $H_{\pm}(U_n)$ multiplicated by Lorentz covariant differential polynomials, it must be used the results established by Hepp[13] extended by using the theory of ideals of differentiable functions[11].

3. Theorem 1. a) on the mass shell was proved by Jacobson[14] and it follows that the restrictions of the Theorems 1 and 2 on the mass shell are true.

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