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1. Introduction

Calculation of the hyperfine splitting of the S-level of the hydrogen atom ΔE_{hfs} has been carried out in many papers. An up-to-date survey of the problem with complete reference can be found in the excellent review article by S.Drell[1] as well as in the paper by S.Drell and I.Sullivan[2]; we shall therefore only briefly mention some of the results on calculations of the corrections to the Fermi formula due to the structure of the proton.

A set of diagrams contributing in the appropriate order is depicted on fig.1. Diagram 1d contains the amplitudes of the virtual Compton scattering (v.C.s) on the electron and proton.

In the papers by R.Arnowitt[3] and W.Newcomb and E.Salpeter[4] by means of the perturbation theory for the Bethe- Salpeter equation ΔE_{hfs} was evaluated neglecting the electromagnetic form factors of the proton (i.e. assuming the proton to be point like with anomalous magnetic moment) and using the Born approximation for the diagram (fig.1d). The result is

$$\frac{\Delta E^{0}_{hfs}}{\Delta_F} = \Delta_0 + \ln\frac{\Lambda}{M_N} \qquad (1)$$

(Δ_F is the Fermi splitting described by the diagram (fig.1a) without proton form factors, M_N is the proton mass and Λ is a cut-off parameter). A logarithmic divergence appears in the term with the double Pauli interaction in the diagram (fig.1d). Δ_0 is finite and depends on the way in which the cut-off is introduced (see[5]). $\Delta_0 \approx -(2\div4)\,ppm$. The important role played by the form factors in the electromagnetic vertex of the proton for the first time was stressed by Z.Zemach[6].

In the papers by C.Iddings and P.Platzman[7] and by C.Iddinds[8] the corrections to the formula (1) arising as an effect of the proton form factors were studied in detail. In [8] the application of the dispersion relations for the amplitude v.C.s. on the proton was also suggested. The final result may be presented in the form

$$\Delta E_{hfs} = \Delta_F + \Delta E^{0}_{hfs} + \Delta E^{1}_{hfs} \qquad (2)$$

$$\frac{\Delta E^{1}_{hfs}}{\Delta_F} = \Delta_1 - \ln\frac{\Lambda}{M_N} + S_2 + S_3 \qquad \Delta_1 \approx -32{,}5\,ppm \quad (1\,ppm \equiv 10^{-6})$$

S_2 and S_3 being the contributions from the cut of the amplitude v.C.s. on the proton modified by the inclusion of a term proportional to μ^2 - the square of the proton anomalous magnetic moment. (See[1], formulae (1.83), (1.84)). The contribution coming from the region of strongly virtual photons in the diagram (fig.1d) was estimated by J.Bjorken[9], who used a method developed by himself.

Finally, in the paper by I.Drell and I.Sullivan[2] the relativistic and non-relativistic models for the behaviour of the imaginary part of the v.C.s. amplitude in the low energy region of the cut were studied in detail.

The relatively high polarizability of the nucleon indicates that the contributions to the hyperfine splitting from higher inetrmediate states in the amplitude of v.C.s. cannot be, in general, small as compared with the nucleon contribution. This fact always stimulated various attempts to take into account also other (besides the one-nucleon) intermediate states. The contributions from these states to the quantities of the type S_2 and S_3 proved to be surprisingly small as compared with expected effect.

From our point of view this result is not accidental neither it is caused by the shortcomings of the models. It nearly reflects the characteristic features of the problem related, in our opinion, to the fact that the main contribution to the two-photon contributions under consideration comes from photons of relatively small values of $-k^2 \ll M_N^2$, $k^2 = \nu^2 - \vec{k}^2$ is the photon mass).

Roughly speaking: the presence of two-photon and one-electron propagators in the corresponding integrals (note the smallness of the electron mass $m_e / M_N \approx 10^{-3}$) sharply increases the contribution from the region of small k^2 contributing at the same time with a factor k^{-6} at $|k^2| \to \infty$, thus even for relatively weak restrictions on the rate of decrease of v.C.s. amplitude (while $-k^2$ increases) the region of strongly virtual photons gives only a small contribution (10%) as compared with the region of small values of k^2 x/. Taking into account the fact that the region of small va-

x/ This is an essential difference between the problem of hyperfine splitting and that of the mass differences of the proton and neutron since , in latter case, the "absence" of the electron propagator and of the second photon propagator results in the fact that the main contribution comes from the region of "intermediate" and large values of k^2 (if only the integral converges).

lues of k^2 in the integral $\dot{S}_3$ is suppressed and that the modification of the cut (see[1], formulae (1.84), (1.85)) results also in suppression of the region of small k^2 - in the integral S_2 we conclude in accordance with the above-mentioned arguments - that S_2 and S_3 should be small.

We stress that the divergences appearing in the expressions (1') and (2') do not allow us, in fact, to ascribe any physical sense to Δ_0 and Δ_1. Furthermore, such a method of calculations does not permit to determine the relative values of the different contributions - the one nucleon contribution, the cuts etc., thus handicaping one to get a clear picture of the characteristic features of the problem.

Our first goal therefore is the derivation of the appropriate formulae free from the above-mentioned disadvantages and admitting a simple physical interpretation. This subject is tackled in part I of our paper.

In part II the formulae obtained in part I are analyzed and the terms S_2 and S_3 are estimated. The contributions from the N^*_{33} resonance and the behaviour of the imaginary part of v.C.s. amplitude in the high energetic region of the cut are studied in detail.

As a general scheme for the study of the bound states energy levels the quasipotential method of A.Logunov and A.Tavkhelidze[10] is adopted. For the first time the quasipotential equation for the system of two particles of spin 1/2 was applied to hyperfine splitting by R.Faustov[11]. This method (because of the three-dimensional character of the equation) has some advantages as compared with the Bethe-Salpeter equation and considerably facilitates all the calculations for the hydrogen-like systems[12].

2. Quasipotential Equation for the Bound State of Two Particles

Consider the scattering amplitude for two particles with masses m and M off the mass shell in the center of mass system (c.m.s) (see fig.2).

As it was shown in paper /11/, the quasipotential equation for the bound state of two spin 1/2 particles has the form

$$(E-\sqrt{\vec{p}^{\,2}+m^2}-\sqrt{\vec{p}^{\,2}+M^2})\Psi(\vec{p}) = \frac{1}{(2\pi)^3}\int d\vec{q}\, V(\vec{p},\vec{q};E)\,\Psi(\vec{q})\,, \qquad (1)$$

where the wave function $\Psi(\vec{p})$ has two spinor indices.

The quasipotential operator V is determined by a Lippman-Schwinger type equation for the scattering amplitude

$$T_+ = V + VG_0T_+ \qquad \text{or} \qquad V = T_+(1+G_0T_+)^{-1}\,, \qquad (2)$$

where

$$T_+(\vec{p},\vec{q};E) = \bar{u}_1(\vec{p})\,\bar{u}_2(-\vec{p})\,T(\vec{p},\vec{q};E,\epsilon=\epsilon'=0)\,u_1(\vec{q})\,u_2(-\vec{q}) \qquad (3)$$

($u_{1,2}$ are Dirac spinors for states with positive energy and norm $u^*u=1$)

$$G_0(\vec{p},\vec{q};E) = \frac{(2\pi)^3\,\delta(\vec{p}-\vec{q})}{E-\sqrt{\vec{p}^{\,2}+m^2}-\sqrt{\vec{p}^{\,2}+M^2}} \equiv (2\pi)^3\delta(\vec{p}-\vec{q})\,F(\vec{p})\,. \qquad (4)$$

Multiplication in the equation (2) is to be understood in the operator sense, namely as integration over the 3-dimensional momentum volume $\int d\vec{p}\,/(2\pi)^3$.

In the nonrelativistic limit $\vec{p}^{\,2} \ll m^2, M^2$ equation (1) reduces to the usual Schrödinger equation

$$\left(W - \frac{\vec{p}^{\,2}}{2m^*}\right)\Psi(\vec{p}) = \frac{1}{(2\pi)^3}\int d\vec{q}\;V(\vec{p},\vec{q};E)\,\Psi(\vec{q})\,, \qquad (5)$$

where $W = E - m - M$; $m^* = mM/m + M$.

Expanding (2) into a perturbation series we obtain

$$V^{(2)} = T_+^{(2)},\; V^{(4)} = T_+^{(4)} - T_+^{(2)}\,G_0\,T_+^{(2)}\,, \qquad (6)$$

etc.

Extracting from $V^{(2)}$ the Coulomb potential (without form factor)

$$V^{(2)} = v_c + \Delta V^{(2)}, \quad v_c(\vec{p}, \vec{q}) = - \frac{e^2}{(\vec{p} - \vec{q})^2} \tag{7}$$

and considering $\Delta V^{(2)}$ and $V^{(4)}$ as perturbations, we obtain a correction to the Coulomb energy levels

$$\left(W_n = E_n - m - M = - \frac{m^* \alpha^2}{2n^2}, \quad n = 1, 2, \ldots \right)$$

in the form

$$\Delta E_n = \langle n | \Delta V^{(2)} | n \rangle + \langle n | V^{(4)} | n \rangle + \sum_{m \neq n} \langle n | \Delta V^{(2)} | m \rangle \frac{1}{E_n - E_m} \langle m | \Delta V^{(2)} | n \rangle , \tag{8}$$

where the brackets $\langle \ldots | \ldots | \ldots \rangle$ denote matrix elements with the wave functions of the equation (1) and with the Coulomb potential. Since the relativistic corrections are of the order $\vec{p}^{\,2}/m^2 \approx \alpha^2$ (in the hydrogen atom $\vec{p}^{\,2} \approx \alpha^2 m^2$) the known solutions of the equation (5) can serve as a sufficiently good for our purpose approximation for the wave functions. For the ground state the wave functions are of the form

$$\Psi_0(\vec{p}) = \Psi_c(\vec{p}) w_1 w_2 , \quad \Psi_c(\vec{p}) = \phi_c(0) \frac{8\pi m^* \alpha}{(\vec{p}^{\,2} + m_*^2 \alpha^2)^2} , \quad \phi_c(0) = \frac{1}{(2\pi)^3} \int d\vec{p} \;\; \Psi_c(\vec{p}) , \tag{9}$$

where $w_{1;2}$ are the two component Pauli spinors

$$\Psi(\vec{p}) \cong \frac{E - m - M - \vec{p}^{\,2}/2m^*}{E - \sqrt{\vec{p}^{\,2} + m^2} - \sqrt{\vec{p}^{\,2} + M^2}} \Psi_0(\vec{p}) = \Psi_0(\vec{p}) + O(\alpha^2). \qquad (10)$$

The next term in the expansion $\Psi(\vec{p})$ is of the order $O(\alpha^2)$ and is unessential for our considerations.

In the last summand of the formula (8) the sum over the intermediate states of the bound state can be replaced by a sum over the states of free particles since such a substitution does not worsen the accuracy of the whole approximation. After this operation formula (8) can be written in the form

$$\Delta E_n = <\Delta V^{(2)}> + <V^{(4)}> + <\Delta V^{(2)} G_0 \Delta V^{(2)}>. \qquad (8a)$$

Now, substituting the expression $V^{(4)}$ from (6) into (8a) we obtain

$$\Delta E = <\Delta V^{(2)}> + <T^{(4)}> - <v_c G_0 \Delta V^{(2)}> - <\Delta V^{(2)} G_0 v_c> - <v_c G_0 v_c>. \quad (11)$$

Let us now evaluate the quasipotential (it is covenient to use the photon propagator in Coulomb gauge)

$$D_{00}(k) = -\frac{1}{\vec{k}^2}; \; D_{ij}(k) = -\frac{1}{k^2 + i\epsilon}\left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\right)$$

$$D_{0i} = D_{i0} = 0 \quad (i, j = 1,2,3),$$

where a partition into the longitudinal (Coulomb) and transversal parts is carried out.

Diagram "a" of fig.1 yields (spinors polarization indices are dropped)

$$T_{+}^{(2)}(\vec{p},\vec{q}) = e^2\ \bar{u}_1(\vec{p})\bar{u}_2(-\vec{p})[\gamma_0^{(1)}\Gamma_0^{(2)}(k)D_{00}(k)+\gamma_i^{(1)}\Gamma_j^{(2)}(k)D_{ij}(k)]u_1(\vec{q})u_2(-\vec{q}),$$

$$\vec{k} = \vec{p} - \vec{q},\ k_0 = 0$$

where the electromagnetic vertex of the proton is

$$\Gamma_\mu(k) = \gamma_\mu f_1(k^2) - \frac{\mu}{4M}[\gamma_\mu, \gamma_\nu] k_\nu f_2(k^2) \tag{13}$$

$f_{1,2}(k^2)$ being the usual Dirac and Pauli form factors with the normalization $f_1(0) = f_2(0) = 1$, μ – the anomalous magnetic moment of the proton $\mu = 1,79$.

Performing in (12) expansion in $\vec{p}^2/m^2$, $\vec{p}^2/M^2$ and m/M and retaining only that part of the quasipotential which gives the hyperfine splitting of the levels we get

$$\Delta V_{hfs}^{(2)}(\vec{p},\vec{q}) = \frac{e^2}{\vec{k}^2}[f_1(k^2)+\mu f_2(k^2)]\frac{1}{4mM}\{[\vec{k}^2(\vec{\sigma}_1\vec{\sigma}_2)-(\vec{k}\vec{\sigma}_1)(\vec{k}\vec{\sigma}_2)] - 2i(\vec{p}\times\vec{q})\vec{\sigma}_2\}. \tag{14}$$

Thus, in the lowest order the splitting of the S -level of the hydrogen equals

$$\Delta E^{(2)}_{hfs} = \langle \Delta V^{(2)}_{hfs} \rangle = \frac{2\pi\alpha}{3mM} \langle \vec{\sigma}_1 \vec{\sigma}_2 \rangle \int \frac{d\vec{p}}{(2\pi)^3} \frac{d\vec{q}}{(2\pi)^3} \Psi_c(\vec{p}) [f_1(k^2) + \mu f_2(k^2)] \Psi_c(\vec{q}) =$$

$$= \frac{2\pi\alpha}{3mM} (1+\mu) \langle \vec{\sigma}_1 \vec{\sigma}_2 \rangle \{ |\phi_c(0)|^2 + \tag{15}$$

$$+ \int \frac{d\vec{p}}{(2\pi)^3} \frac{d\vec{q}}{(2\pi)^3} \Psi_c(\vec{p}) [\frac{f_1(k^2) + \mu f_2(k^2)}{1+\mu} - 1] \Psi_c(\vec{q}) \},$$

where

$$\phi_c(\vec{x}) = \int \frac{d\vec{p}}{(2\pi)^3} e^{i\vec{p}\vec{x}} \Psi_c(\vec{p}), \quad |\phi_c(0)|^2 = \frac{(m^*\alpha)^3}{\pi}. \tag{16}$$

The first summand in (15) gives the well known Fermi formula, whereas the second one determines the corrections due to the electromagnetic structure of the proton. Since k^2 in (15) is small, the second summand in fact is of the same order as the two photon diagrams. To make the relation with the two photon diagrams more explicit we utilize the approximate expression for the wave function $\Psi_c(\vec{p})$ from formula (15)

$$\Psi_c(\vec{p}) = \phi_c(0) [(2\pi)^3 \delta(\vec{p}) + F(\vec{p}) v_c(\vec{p}) + 0(\alpha^2)] w_1 w_2 \tag{17}$$

which can be obtained by the expansion of the Coulomb wave function of the S state of $\Psi_c(\vec{p})$ [see (9)] in powers of a and by the utilization of the equation (1) with the Coulomb potential. Formula (17) (holding when the appropriate operator acting on $\Psi(\vec{p})$ has no singularities at $\vec{p}=0$) is an expansion of the Coulomb ladder, where every subsequent term includes an additional exchange of one longitudinal photon and the "static propagator" $F(\vec{p})$. Substituting (17) equation (15) takes the form

$$\Delta E^{(2)}_{hfs} = \frac{2\pi a}{3mM}(1+\mu)\langle\vec{\sigma}_1\vec{\sigma}_2\rangle|\phi_c(0)|^2\{1+2\int\frac{d\vec{k}}{(2\pi)^3}v_c(\vec{k})F(\vec{k})[\frac{f_1(k^2)+\mu f_2(k^2)}{1+\mu}-1]\}\quad(18)$$

The comparison of the second summand in (18) with the diagram of fig.3 clarifies physical sense. Here (fig.3) $\otimes$ means that $F(\vec{p})$ plays the role of the propagator, - - - - is a longitudinal photon, and  is a tranversal one.

Let us consider now the corrections to the Fermi formula related to the two photon diagrams (fig.1b, c,d). By the evaluation of these corrections, in accordance with the remarks preceeding formula (9) one can put $\vec{p}=\vec{q}=0$ and $E=m+M$ in the corresponding expressions for the quasipotential.

We shall not be dealing with diagrams 1b and 1c. Just note that fig.1c in the appropriate order does not contribute and the role of fig. 1b reduces just to the description of the anomalous moment of the electron /11/, i.e. it gives a common factor $(1+a/2\pi)$ in the formula (18).

The corrections in energy from the diagram 1d together with a term of the second order of the perturbation theory (see formula (11)) has the form (the last summand in (11) does not contribute)

$$\Delta E^{(4)}_{hfs} = |\phi_c(0)|^2 \{<T^{(4)}_{2\gamma}> - 2 < v_c F \Delta V^{(2)}_{hfs}>\} , \tag{19}$$

where the matrix elements are now taken over spinor indices only. Here $T^{(4)}_{2\gamma}$ corresponds to the diagram 2d at $\vec{p} = \vec{q} = 0$ and $E = m + M$ and the second summand can be written as

$$< v_c F \Delta V^{(2)}_{hfs} > = \frac{2\pi\alpha}{3mM} < \vec{\sigma}_1 \vec{\sigma}_2 > \int \frac{d\vec{k}}{(2\pi)^3} v_c(\vec{k}) F(\vec{k}) [f_1(k^2) + \mu f_2(k^2)] \tag{20}$$

and corresponds to the diagram 3a.

Adding eqs. (18) and (20) we obtain

$$\Delta E^{(2)}_{hfs} + \Delta E^{(4)}_{hfs} = |\phi_c(0)|^2 \{ \frac{2\pi\alpha}{3mM} (1+\mu)(1 + \frac{\alpha}{2\pi}) < \vec{\sigma}_1 \vec{\sigma}_2 > + < T^{(4)}_{2\gamma} > - \tag{21}$$

$$- \frac{4\pi\alpha}{3mM} (1+\mu) < \vec{\sigma}_1 \vec{\sigma}_2 > \int \frac{d\vec{k}}{(2\pi)^3} v_c(\vec{k}) F(\vec{k}) \} \equiv \Delta E_F + \Delta E^{2\gamma}_{hfs} .$$

Now the magnitude of the hyperfine splitting of the triplet and singlet S levels is

$$\Delta E_{hfs} = E(^3S_1) - E(^1S_0) = \Delta_F [1 + \delta] , \tag{22}$$

where

$$\Delta_F = \frac{8\pi\alpha}{3mM}(1+\mu)(1+\frac{\alpha}{2\pi})|\phi_c(0)|^2 \qquad (23)$$

is the Fermi splitting

$$\delta \equiv \delta_{2\gamma} - \delta^0_{stat} = \frac{\alpha m}{\pi(\mu+1)}[\frac{3M}{i\pi^2}\int\frac{d^4k}{k^4}N^{(p)}_{\mu\nu}(k)N^{(e)}_{\mu\nu}(k) - 8(1+\mu)\int_0^\infty\frac{dk}{k^2-2mW_1}] \qquad (24)$$

$$N^{(p,e)}_{\mu\nu} = \frac{1}{4}Sp\,(C^{(p,e)}_{\mu\nu}\frac{1+\gamma_0}{2}\gamma_z\gamma_5),\; W_1 = -\frac{m^*\alpha^2}{2} \qquad (25)$$

$C^{(p)}_{\mu\nu}(k)$ and $C^{(e)}_{\mu\nu}(k)$ are the amplitudes of the virtual scattering on the proton and electron (see[8]) which now will be the subject of our study. Since the effects of the bound state in the two photon diagrams can be neglected we can carry out a fully covariant description of the Compton scattering.

3. Amplitude of Virtual Forward Compton Scattering

The amplitudes $C^{(e)}_{\mu\nu}$ and $C^{(p)}_{\mu\nu}$ correspond to the following diagrams (figs. 4a and 4b)

$$C_{\mu\nu} = \gamma_\mu\frac{1}{\hat{p}+\hat{k}-m}\gamma_\nu + \gamma_\nu\frac{1}{\hat{p}-\hat{k}-m}\gamma_\mu \,. \qquad (26)$$

From (25) and (26) we obtain

$$N^{e}_{\mu\nu} = \frac{k^2}{k^4 - 4m^2\nu^2} \epsilon_{\mu\nu\sigma z} k_\sigma + O(\alpha m^2), \quad \nu = k_0 \tag{27}$$

then $C^{(p)}_{\mu\nu}$ can be presented in the form (see[8,1])

$$C^{(p)}_{\mu\nu} = \frac{1}{M}(k_\mu k_\nu - k^2 g_{\mu\nu})A + \frac{1}{M}[\nu^2 g_{\mu\nu} + k^2 p_\mu p_\nu - \nu(p_\mu k_\nu + p_\nu k_\mu)]\, B +$$

$$+ \frac{1}{M^3}\{M\nu[\gamma_\mu, \gamma_\nu] - p_\nu[\gamma_\mu, \hat{k}] + p_\mu[\gamma_\nu, \hat{k}]\}H_1 + \frac{\nu}{M^3}\{k^2[\gamma_\mu, \gamma_\nu] -$$

$$- k_\nu[\gamma_\mu, \hat{k}] + k_\mu[\gamma_\nu, \hat{k}]\}H_2 .$$

The invariant amplitudes A, B, H_1, H_2 are functions of k^2 and ν with crossing symmetry in ν . We have from (25) and (26)

$$3N^{(e)}_{\mu\nu}(k) N^{(p)}_{\mu\nu}(k) = -\frac{2(k^2/M^2)}{k^4 - 4m^2\nu^2}[(2k^2 + \nu^2)H_1(k^2, \nu) + 3k^2\nu^2 H_2(k^2, \nu)] . \tag{29}$$

Carrying out now in the integral (24) the Wick rotation (see[8]) $k_0 \to ik_0$ and substituting (29) we obtain for $\delta_{2\gamma}$

$$\delta_{2\gamma} = \frac{\alpha m}{\pi(1+\mu)M} \frac{2}{\pi^2} \int \frac{d^4k}{k^4} \frac{k^2}{k^4 + 4m^2 k_0^2} [(2k^2 + k_0^2)H_1(-k^2, ik_0) - \tag{30}$$

$$- 3k^2 k_0^2 H_2(-k^2, ik_0)]$$

(in (30) we integrate over the 4-dimensional Euclidean space). In spherical coordinates

$$\int d^4k = 4\pi \int_0^\infty k^2\, dk \int_0^\pi d\phi \sin^2\phi, \quad k_0 = k\cos\phi).$$

Using a dispersion relation for $H_{1,2}$ in ν for $k^2 < 4m_\pi^2$ and separating explicitly the one nucleon contribution

$$H_{1,2}(k^2,\nu) \equiv H^N_{1,2} + H^{cut}_{1,2} = -\frac{4k^2M^2}{k^4 - 4M^2\nu^2} R^N_{1,2}(k^2) + \frac{1}{\pi}\int_{\nu_t(k^2)}^\infty \frac{d\nu'^2}{\nu'^2 - \nu^2} \operatorname{Im} H_{1,2}(k^2,\nu'), \quad (31)$$

where

$$R^N_1(k^2) = \frac{1}{4} f_1(k^2)\,[f_1(k^2) + \mu f_2(k^2)]$$

$$\nu_t(k^2) = \frac{1}{2M}(2Mm_\pi - m_\pi^2 - k^2), \quad (32)$$

$$R^N_2(k^2) = \frac{1}{4k^2} f_2(k^2)\,[f_1(k^2) + \mu f_2(k^2)]$$

we write formula (30) in the form (see[1])

$$\delta_{2\gamma} = \delta_N + \Delta_{cut}, \quad (33)$$

where δ_N is the one nucleon pole contribution, Δ_{cut} is a contribution from the cuts of the amplitudes H_1 and H_2

$$\delta_N = \frac{\alpha m}{\pi M} 8 \int_0^\infty \frac{dk}{k^2} G_N(k^2)\, \mathcal{K}(k), \quad G_N(x) = \frac{f_1(x) + \mu f_2(x)}{1+\mu}$$

$$\mathcal{H}(k) = \frac{k^3}{2\pi}\int_0^{\pi/2} \frac{d\phi \sin^2\phi}{k^2+4m^2\cos^2\phi}\left[f_1(-k^2)\frac{2+\cos^2\phi}{\cos^2\phi+(k^2/4M^2)} + \mu f_2(-k^2)\frac{\cos^2\phi}{\cos^2\phi+(k^2/4M^2)}\right]. \tag{35}$$

Our final expression of the correction to the Fermi splitting has thus the form[x/]

$$\delta = \delta_{2\gamma} - \delta^0_{stat} = (\delta_N - \delta^0_{stat}) + \Delta_{cut} \tag{36}$$

$$\delta_N - \delta^0_{stat} = \frac{\alpha m}{\pi M}\, 8 \int_0^\infty \frac{dk}{k^2}\left[G_N(-k^2)\,\mathcal{H}(k) - \mathcal{H}(0)\right] \tag{37}$$

$$\Delta_{cut} = \frac{\alpha m}{\pi(1+\mu)M}\,\frac{2}{\pi^2}\int \frac{d^4k}{k^4}\,\frac{k^2}{k^4+4m^2k_0^2}\left\{(2k^2+k_0^2)H_1^{cut}(-k^2, ik_0) - 3k^2k_0^2 H_2^{cut}(-k^2, ik_0)\right\} \tag{38}$$

The derivation of formulae (36)–(38) is the main purpose of this paper. An analysis of these expressions and the results of numerical calculations are presented in the other paper [13].

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[x6/] The linear divergence in δ_N arises because we have neglected the effects of bound states. The latter would result in an effective cut off at $|k^2| \lesssim \alpha^2 m^2$. In the expression (24) one can, in general (up to the terms of higher orders), tend $W_1 \to 0$ and consider integration in k^2 to zero obtaining thus the finite expression(37).

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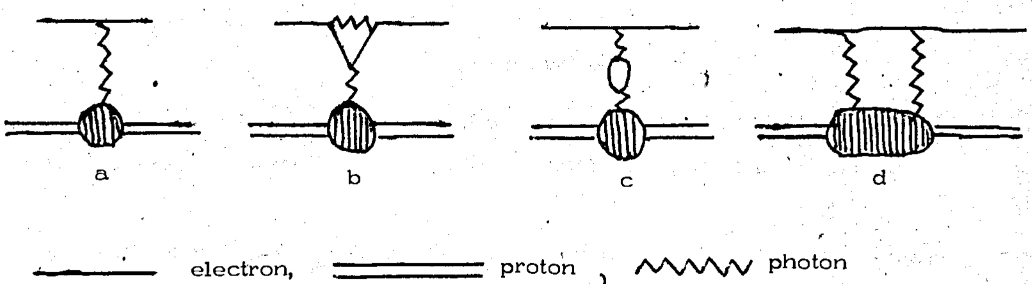

Fig.1

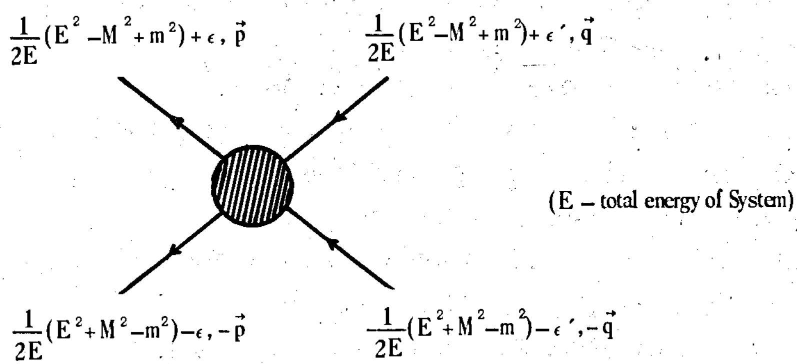

Fig.2

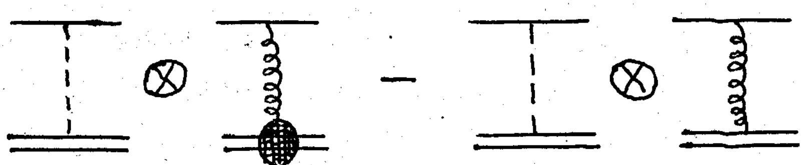

Fig.3

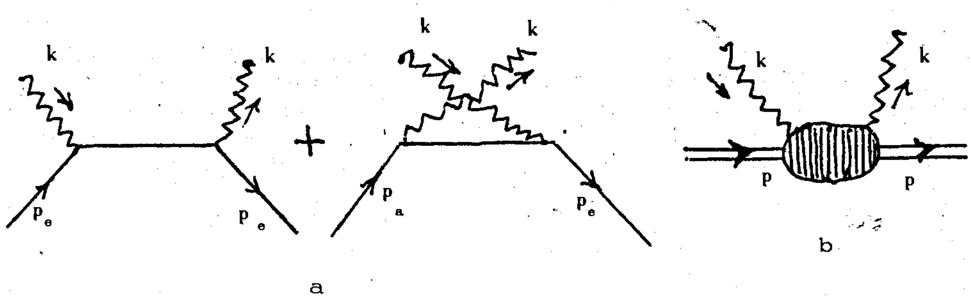

Fig.4