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QUANTUM FIELD THEORY AND REGGE POLES

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1. Introduction

The Regge behaviour of the scattering amplitudes was discovered first in quantum mechanics but was applied just after for the description of high energy scattering. The terminology and ideas of reggism greatly influence the theory and the description of experiment of the elementary particle physics. But the bases of the transplantation of the Regge ideas into quantum field theory were only a certain analogy and models not so convincing, from our point of view. High energy experiment does not allow us also to make an unambiguous conclusion that the situation is described only by the Regge poles. The cuts of the Mandelstam type due to double exchange of Regge poles do not clear up the picture too. Nevertheless, all this does not stop the flow of papers on the modernization of details of the classical Regge pole scheme.

In the situation of the kind it is especially important, to understand what quantum field theory points to. However we do not see any other possibility than a direct summation of perturbation theory in the asymptotical region. The first work along these lines dealt with summation of senior orders of leading poles of the ladder graphs which resulted in the moving pole /1,2/ for the φ^3- ladder and in the standing out /3,4/ for the φ^4 one. But the question is how this result will be affected by other graphs and the junior orders of the leading pole (at $j=-1$ in φ^3- theory and $j=0$ in φ^4- theory) in perturbation theory, which may be also important for not too small coupling constant. We have succeeded in solving this problem for the classical mesodynamics and for φ^3- and φ^4- theories.

The final result of the summation is especially simple and beautiful for the negative signature scattering amplitude of nonidentical particles in φ^3- theory and for amplitudes in φ^4- theory. It has the form

$$F(j,t)=C(t)\left[w(j)^{-1}-B(t)\right]^{-1}C(t)+R(t) \qquad (1)$$

where all orders of leading singularity are collected only in the t-independent function $w(j)$ and the functions B, C and R are series in the coupling constant each term of which being regular in the neighbourhood of this singularity. This expression shows for both theories a moving Regge-poles when $1/w(j)-B(t)=0$. The main difference between the theories is the form of $w(j)$. For φ^3- theory $w(j)=\Gamma(j+1)$ so the amplitude $F(j,t)$ can have only a moving pole when $Re\,j>-1$ (For the ladder graph this is just the result of Polkinghorn/2/). The situation is more complicated for the case of φ^4- theory . We have succeeded in finding $w(j)$

only assuming a finite value of the bare charge. The function $w(j)$ has in this case a square-root branchpoint of the type revealed by Sawyer/3/. So, the amplitude in addition to the moving pole possesses a standing branchpoint. (By the way, this is not astonishing because of an analogy of φ^4 theory with the potential $1/r^2$ for small r).

The case of mesodynamics

$$\mathcal{L}_{int}=g\bar{\psi}\gamma^5\psi\varphi+h\varphi^4 \qquad (2)$$

differs from the simple φ^4 theory only by the matrix character of the functions entering (1) (of the third or the second rank depending on G-parity) and transponation of the right matrix C(t) but the character of the answer is the same: the moving pole and the standing branchpoint. For the moment we can only guess which of the singularities is a leading one for $t\approx 0$ because the functions entering the general expression are found as a series. It seems to us that the standing branch point, instead of the "vacuum pole" P and the moving pole" as P' have to give better discription of experiment than the usual pure pole model. In any way, each of these singularities have the properties which determine the success of the Regge-description: it is universal, i.e. the same for the elastic ($\pi\pi$, πn and nn) and quasielastic amplitudes, and the factorisation theorem holds for the residues.

To obtain these results we give the necessary information about the leading singularity of the Feynman graphs (Sec. 2). Then we deduce formula (1) and give the scheme for calculating $\beta(t)$, $C(t)$ and $w(j)$ for φ^4- theory (Sec. 3) and φ^3- theory (Sec. 4). Section 5 is devoted to the asymptotic of the amplitude in an external particle mass off the mass shell. We find out there that for φ^4- theory the amplitude is not rapidly vanishing when the mass goes to infinity. Some results for the mesodynamics in the approximation of week coupling are given in Sec. 6. Finally in Section 7 we discuss the results obtained.

2. Leading Singularities of Graphs

For the consideration of this problem the Mellin transform of the amplitude with positive and negative signature is especially suitable:

$$f^{\pm}(s,t) = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty} dj\, \frac{(s)^j \pm (-s)^j}{\sin \pi j}\, \pi\, \frac{F^{\pm}(j,t)}{\Gamma(j+1)} \qquad (3)$$

($F^{\pm}(j,t)/\Gamma(j+1)$ has the same leading singularities as the partial wave amplitude).

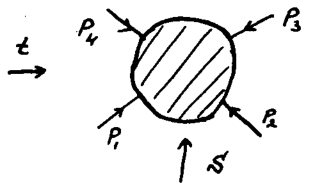

Fig.1

Earlier we have shown [5] that the most right singularity of any graph in φ^4- theory is the set of poles of different orders at the point j=0. The maximal order of the poles is determined by the number of two-particle divisions in t - channel which break the graph into kernels, i.e. structures without two-particle divisions. This connection with topology is more distinct for Feynman parametrization of propagators in the exponential form.

The singularities at j=0 are generated by integration over the region where α - parameters corresponding to the kernels and the connected union of kernels are simultaneously small. (We will say that such a kernel or union of kernels are „in asymptotical regime" or simply „in asymptotic"). They are the only source of singularities at j=0. Just this fact reduces the problem of summation to the ladder-type graphs.

The more kernels and their unions are in asymptotical regime the higher the order of the pole. But only those unions are to be taken into account which either have no common lines or are entirely contained inside one another. Divergent parts (which are not the kernels or unions) increase the order of the pole only when they are into an asymptotical object. The asymptotical regime, i.e. the simultaneous vanishing of some parameters α means graphically the contraction of the corresponding lines.

For the kernel or union of kernels it means the contraction of this object into a point . So, the coefficient for the pole which corresponds to the given set of the objects in asymptotics appears to be the product of factors determined by these objects (independent of t) and t - dependent contribution of a weakly connected graph which is obtained after contraction of these objects into the points. Such a weakly connected graph consists of the components of the type of Fig. 2 and is a product of the contribution denoted by $C_n(t)$(Fig.2a) and $B_n(t)$ (Fig.2b). Each of them is regularized (subtracted) so that it has no singularity at j=0. The prescription for calculation of the functions B_n and C_n can be obtained without difficulty from general expressions of paper[5]. They are given in the work[15]. When j=0 they coincide with the usual Feynman rules.

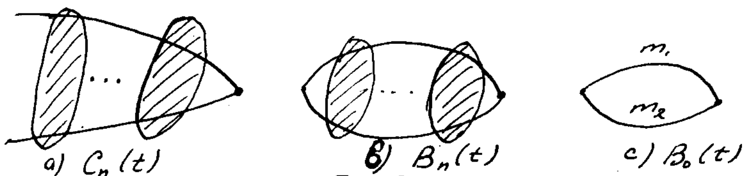

Fig.2

Here we write down only the simplest ones:

$$C_0(t)=1;\quad B_0(t)=\Gamma(j)\int_0^1 dx\left\{\left[xm_1+(1-x)m_2-x(1-x)t\right]^{-j}-1\right\}, \quad (3)$$

where m_1 , m_2 are the masses of virtual particles (Fig. 2c).

3. Summation of Leading Singularities in φ^4-Theory

Account of all possibilities of the asymptotical regime of the kernels and unions of them gives us all singularities at j=0. This oan be done for any n-kernel graph as follows. Let the most left of the contracted kernels have a number $k+1$ and the most right one the number $n-\ell$. Then, the noncontracted "ends" of the graph form $C_k(t)$ and $C_\ell(t)$ and the middle part we denote by $X_r(j,t)$

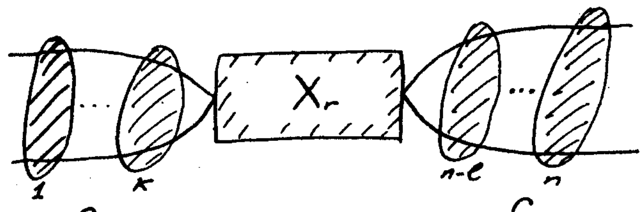

Fig.3

The contribution of the graph F_n is of the form

$$F_n(j,t)=\sum_{k+r+\ell=n} C_k(t)\,X_r(j,t)\,C_\ell(t)+R_n(t),. \quad (4)$$

where R_n is a nonsingular at j=0 residue of F_n

and $C_0(t)=1$ by definition. The part corresponding to $X_n(j,t)$ can be in asymptotical regime as a whole. The corresponding contribution is denoted as $w_n(j)$ which is a t-independent polynomial of $1/j$ because different kernels and unions inside of w_n can be in asymptotics too. In addition, the contribution of X_n has some other possibilities, when the union of the first k kernels is in asymptotics (the contribution $w_k(j)$) the next r kernels are noncontracted $(B_r(t))$ and the residual $n-k-r$ kernels give again contribution $X_{n-k-r}(j,t)$ (fig. 4).

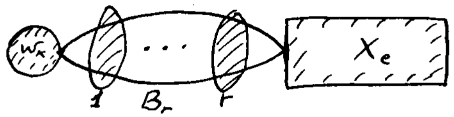

Fig. 4.

So

$$X_n(j,t)=w_n(j)+\sum w_k(j)B_r(t)X_e(j,t). \tag{5}$$

The formulas (4) (5) are valid for any graph with a given number of kernels what permits us to sum it over all possible sorts of kernels and to obtain as a result the recurrent relations. The following summation of it over the number n up to infinity results in the equations

$$F(j,t)=C^2(t)X(j,t)+R(t)$$

$$X(j,t)=w(j)\left(1+B(t)\right)X(j,t) \tag{6}$$

which immediately give the expression (1).

The functions B,C and R in this equation are the sums of quantities $B_n(t)$, $C_n(t)$, $R_n(t)$ over all sorts of kernels and over the number of them. Each term of this sum is regular for $Re\,j>-1$ but as for the whole sum we discuss it in Sec.7.

All singularities at j=0 are collected in the function $w(j)$. But to find this function we need one more assumption about the results of summation of inner divergences, i.e. in fact about the form of the bare charge. There are many people who believe that zero bare charge $h\to h(1-ah\ln S)^{-1}$ is a characteristic feature of field theory. The idea about $w(j)$ in this oase can give the first term, which possesses the essential singularity at j=0 of the type $1/w(j)\sim\exp(-ah/j)$.

But this belief is based only on the properties of the first few terms of divergent asymptotical series. Obviously, it is not enough. The rough estimation of n-th term of the series allows us to sum it in a certain sense The result gives the bases to believe that the renormalization of the charge and the wave function is finite (" the finite bare charge"). This reduces the problem of finding $w(j)$ to the sceleton kernels only (i.e. to the kernels without internal divergent parts like "fish", "open" and "closed" envelope and so on). The leading singularity of any of such kernels is a simple pole at j=0 what reduces the whole problem to the simple ladder[6].

The expression for $w_n(j)$ differs from that for F_n by independence of t and masses. So, the expression of the type (4) (5) can also be written, the independent of t numbers r_κ, c_κ and b_κ entering there instead of R_κ, C_κ and B_κ ($b_0 = c_0 = 1$). Besides, the quantity $\Gamma^{-1}(j)\, w_n(j)$ enters the left-hand side of the expression of the type (4), i.e. the singularity due to the asymptotical regime of the graph as a whole is picked out. This possibility to be excluded from the right hand side of the expression. It gives

$$\frac{1}{\Gamma(j)} w_n(j) = r_n + \sum_{k+r+\ell=n} c_k x_k(j) c_\ell - w_n(j)$$
$$x_n(j) = w_n(j) + \sum_{k+r+\ell=n} w_k(j)\, b_r\, x_\ell(j) , \tag{7}$$

The summation of these relations over all sorts of kernels and the number of it leads to the system

$$\frac{1}{\Gamma(j)} w(j) = r + c^2 x(j) - w(j) \tag{8}$$
$$x(j) = w(j) + w(j)\, b\, x(j)$$

with the solution

$$\frac{1}{w(j)} = \frac{1}{2r}\left(br - c^2 + 1 + 1/\Gamma(j) + K\right) \tag{9}$$
$$K = \sqrt{\left(br + c^2 - 1 - \frac{1}{\Gamma(j)}\right)^2 - 4c^2 br} .$$

(The sign of the square-root is choosen from the correspondence with perturbation theory).

In the limit of weak coupling $r = r_1 \sim h^2$ and $b = c = 1$

$$\frac{1}{w(j)} = \frac{1}{2}\left(1 + \frac{1}{\Gamma(j) r} + \sqrt{\left(1 - \frac{1}{r\Gamma(j)}\right)^2 - \frac{4}{r}}\right)$$

which gives the branchpoints of

$$j \approx \frac{1}{\Gamma(j)} = \pm 2\sqrt{r} + r$$

4. Summation of Leading Singularities in φ^3 Theory

φ^3- theory has no graphs vanishing not slower than $S^{-1+\varepsilon}$, i.e. any graph is regular when $Re\, j > -1$

Let us consider the negative signature amplitude of scattering of nonidentical particles. For this amplitude the only singular at j=-1 graphs will be those which contain , at least, one "step" i.e. the kernel with one line (Fig. 5)

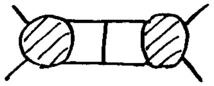

Fig.5

The singularities at j=-1 are due to asymptotical regime of some of those steps. There are no other sources of singularities at this point. All the reasonings which have led to Eq. (6) and expression (1) are the same. However, unlike φ^4- theory, here the unions of kernels do not work and $w(j)$ is a contribution of one step only, so

$$w(j) = w_1(j) = h^2\,\Gamma(j+1). \tag{11}$$

The expression for the terms of series for $B(t)$ and $C(t)$ are also little different, eaoh of them being regular now when $Re\,j > -2$, for example

$$B_0(t) = \Gamma(j+2)\int_0^1 dx\,\left[m_1^2 x + m_2^2(1-x) - x(1-x)t\right]^{-j-2}. \tag{12}$$

So, negative signature amplitude in this theory can have only moving poles. (For the positive signature a set of condensing to j=-1 poles is added).

5. Asymptotics in the External Mass

This scholastic, at first sight, problem becames recently of great importance, in connection with the cuts due to exchange by two virtual reggions [7-9]. The modern method of finding such cuts assumes the reggion contribution to be rapidly vanishing with growth of the masses of external

particles. Otherwise the existence of cuts is problematic.

Our method [5] allows us, without any difficulty, to obtain some indication to whether this assumption is probable in field theory. For this purpose we consider the region where $p_1^2 = M^2 >> t, m_i^2, p_i^2$ and $s = a_1 M^2$, $u = a_2 M^2$ ($a_1 + a_2 \sim 1$) (Fig. I). In the φ^3- theory only the Born term does not depend on M, the others graphs vanishing at least not slower than M^{-2}

The situation in φ^4- theory is quite different. the behaviour of scattering amplitude for large M^2 is here determined by the asymptotical regime of the kernel and the unions of kernels in t - channel adjacent to the external line p_1 . To sum up all this contribution it is necessary to repeat all the reasonings of Sec. 3 starting from the Mellin transformation of M^2 variable to ν variable (instead of j).

The leading singularity of any graph will be in this case the poles at ν =0 due to asymptotical regime of the abovementioned kernels and unions of kernels. The account of all such possibilities gives us all singularities at ν =0. Consider first the graph with n kernels in t-channel. All these possibilities are taken into account by the following procedure. Let the union of first k kernels be in asymptotics (i.e. be contracted into the point). The noncontracted right end of the graph forms the function

$\tilde{C}_{n-k}(t)$ which is semilinear our functions $C_{n-k}(t)$.
The contribution of the contracted union is denote by
$v_k(\nu)$ (Fig. 6)

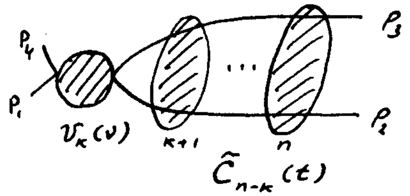

Fig.6

In other words the contribution of the graph F_n has the form

$$F_n(\nu,t) = Q_n(t) + \sum_{k=1}^{n} v_k(\nu)\,\tilde{C}_{n-k}(t)\,, \tag{13}$$

where $\tilde{C}_k(t)$ and $Q_n(t)$ are regular when $Re\,\nu > -1$
The summation of this relation over the all sorts of kernels and over n gives us by analogy with (6)

$$F(\nu,t) = Q(t) + v(\nu)\,\tilde{C}(t)\,.$$

For finding $v(\nu)$ we need again the hypothesis about the form of the bare charge. The finite bare charge reduces again the problem to the sceleton graphs only and allows us to write down equations of type (7). Let the union of first l kernels inside v_k be contracted. They give us v_ℓ again. The remaining non-contracted $k-\ell$ kernels form $\tilde{C}_{k-\ell}$

so that

$$\frac{1}{\Gamma(\nu)}\, v_k(\nu) = \sum_{\ell<k} v_\ell(\nu)\,\tilde{C}_{k-\ell} + q_k \tag{14}$$

($\tilde{C}_k$ and q_k are regular when $Re\,\nu > -1$). The summation of (14) over all sorts of kernels and over the number of them gives a simple equation with the solution

$$F(\nu,t) = Q(t) + \frac{\tilde{C}(t)\,q}{\frac{1}{\Gamma(\nu)} - \tilde{C}}\,. \tag{15}$$

This means

$$F \sim M^{2\tilde{C}} \quad \text{when} \quad M^2 \to \infty\,, \tag{16}$$

In the approximation of weak coupling $0<\tilde{C}\sim h^2$ and the amplitude increases in contrast to φ^3 – theory.(By the way, we note that the zero bare charge could give a decrease of the type $\tilde{C}(t)/\ln M^2$)
This result is valid also for $M^2 >> S$ with an unessential modification. The consideration of (1) shows us that for $S >> M^2 >> m_i^2 t,\ p_i^2$ the M- dependence disappears in any theory. This is in accordance with the result of paper /14/.

So, the hypotheses about the rapid vanishing of the scattering amplitude with the external mass increase is justified for φ^3 - theory in accordance with the result of the paper /8/. As for φ^4 - theory, which is more close to the mesodynamics by its character, the conclusion of the kind is at least doubtfull. (The simple ladder, in particular, give the growth M^{2h^2}).

5. Some Features of Mesodynamics

In mesodynamics the asymptotic of graphs for the positive signature amplitudes with integer angular momentum in t-channel is determined by the same topological elements as in φ^4-theory i.e. by the kernels and unions of kernels. After the normalization to the firstBorn term the leading singularity of any such graph appears to be at j=0 (as in the case of φ^4-theory). This allows to repeat the reasoning of Sec.3, but now it is more complicated because of interlacing of the amplitudes for $\pi\pi$-, πn- and nn-scatterings. So, the algebraic equations (6),(8) turns into two systems of equation which corresponds to positive and negative G-parity in t-channel. The solution and investigation of these systems will be the subject of separate paper/10/. Here we restrict ourself only by some general features of the result.

The formula (1), as it was metioned already, has a matrix character entering there functions are of the form

$$W^{(+)}(j)=\begin{pmatrix} W_{\pi\pi}, & jW_{\pi V}, & 0 \\ jW_{\pi V}, & W_{VV}, & 0 \\ 0 & 0 & W^{(+)}_{TT} \end{pmatrix}, \quad B^{(+)}=\begin{pmatrix} B_{\pi\pi}, & jB_{\pi V}, & jB_{\pi T} \\ jB_{\pi V}, & B_{VV}, & B_{VT} \\ jB_{\pi T}, & B_{VT}, & B^{(+)}_{TT} \end{pmatrix}, \tag{17}$$

$$W^{(-)}(j)=\begin{pmatrix} W_{AA} & 0 \\ 0 & W^{(-)}_{TT} \end{pmatrix}, \quad B^{(-)}=\begin{pmatrix} B_{AA} & B_{AT} \\ B_{TA} & B^{(-)}_{TT} \end{pmatrix}, \tag{18}$$

where (+) and (-) means G-parity, indecies V,T,A, and π denotes the standart structures of two-nucleon state and two-pion state. The matrix C has more complicated nonquadratic form. It does not influences the character of singularities of the amplitudes, so we does not write it down. In the approximation of week coupling C=1, $B_{\pi V}=B_{\pi T}=0$, $B_{\pi\pi}$ is the former expression (3) and, for instance,

$$B_{VV}=\Gamma(j)\int_0^1 dx\left\{\left[m^2-x(1-x)t\right]^{-j}-1+ j\left[m^2-x(1-x)\tfrac{t}{2}\right]\left[m^2-x(1-x)t\right]^{-j-1}\right\}. \tag{19}$$

The functions $W(j)$ can be also found here in the assumption of the finite bare charge. In the limit of week coupling each of them contains the same standing branch points at $j\approx\pm g,\pm h$ This corresponds to our earlier result/11/ from the summation of senior orders of singularities at J=0. It is not dificult to check the validity of the factorization theorem for the residues at the pole and for the branch points: $F_{\pi\pi}F_{nn}=F^2_{\pi n}$.

7. Discussion

We have to discuss now the meaning of the results obtained. The main result is the representation for the amplitude (1), the first term of which absorbs all the singularities of perturbation theory at J=0 for φ^4-theory and mesodynamics and at j=-1 for φ^3-theory (all logarithmic terms). We have there some singular function $W(j)$ and for B,C, and R there are perturbation expansions the singularity of each term in j-plane being shifted by unity to the left

of the leading one. It is natural to ask whether after summation these singularities can turn into the leading ones or be located near the leading one. As we already saw in the weak coupling approximation the singularity goes away from zero at a distance of order of h. There is no reason to think that the singularity resulting from j=-1. will move much more rapidly. That is why we believe that the expression (1), picks out the leading singularity of the amplitude at least for not too large coupling constant. This shows what the quantum mechanical Regge poles transform to in field theory. When the coupling constant increases the singularities shift to the right and the singularity due to j=-1 can reach the singularity due to j=0.
The results of refs.[7-9] mean in this language that when the latter reaches the point j_0 the former comes to the point $2j_0 - 1$, i.e. when $j_0 = 1$ they overlap. The paper[9] is devoted to the modernization of the Regge-pole picture for this case. But the investigation of the mass asymptotics (Sec. 5) shows that the assumption made in these papers is essentially based on the properties of φ^3– theory which can not be transferred to φ^4– theory and mesodynamics. In this sense the mentioned results seem to be unconvincing.

Interlacing of channels in mesodynamics brings some complication in the representation of the type (1) without any changes of principle (see (17)). We have

succeeded so far in obtaining this representation only for the amplitudes with integer spin in t – channel and positive signature. One of the interesting properties of mesodynamics in this case is the factorization of amplitudes $f_{\pi\pi}(s,t)\, f_{nn}(s,t) = f^2_{\pi n}(s,t)$. The investigation of graphs shows[12] that this property has more general character and takes place for quasielastic scattering as well.

The processes with isospin $I=2$ in t– channel have no nucleon–antinucleon intermediate states and for this case the simple representation (1) is valid.
The suppresion of these processes in comparison with vacuum one (I= 0) could be explained by the smallness of the coupling constant h. The singularities of $w(j)$ can be correctly reproduced by the summation of senior logarithm[11] but as we have seen the amplitude $F(j,t)$ possesses in addition the moving pole. In spite of the fact that the form of $w(j)$ is determined by the hypothesis on the bare charge, none of them (zero bare charge or finite bare charge) can lead to a polynomial form of $1/w(j)$. As a result the amplitudes (1) and (17) have both the moving pole and standing singularities carried by $w(j)$. The hypothesis about the finite bare charge, which seems to us more likely, leads to the square root branch point.

In the approximation of weak coupling and $t \sim 0$ the leading singularity is the branch point. It is difficult to say now what happens when the coupling constant grows. We believe that the standing branchpoint is a more suitable candidate for the Pomeranchuk's singularity P and the moving pole for P'.

The amplitudes with negative signature are complicated by the accumulation of poles of the type of ref. [13]. (The spinor structure shifts the point of accumulation from j=-1 to j=0). However, the results of investigation of the asymptotics in this case [5] allows us to hope to consider such amplitudes as well. Unfortunately, we can not say the same about the amplitudes with fermion trajectories, where the three--particle divisions are also important.

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