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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ANGLE DISTRIBUTIONS AND POLARIZATIONS IN HIGH ENERGY PION-NUCLEON SCATTERING

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1. Introduction

The high energy scattering differential cross section has two remarkable peculiarities: it decreases rapidly with the momentum transfer increase from zero

$$\frac{d\sigma}{dt} \approx A e^{\gamma t} \quad , \; 0 \leq -t \leq 0.5 \; GeV^2 \tag{1.1}$$

after that the cross section decrease with increasing momentum transfer becomes slower, but as before it remains exponential. It has been shown in paper /1/ that the cross section dependence on the momentum transfer close to the experimentally observed one

$$\frac{d\sigma}{dt} \approx \exp\left[-b\sqrt{-t}\,\right] \tag{1.2}$$

near the diffraction peak can be obtained on the basis of the quasi-optical approach in quantum field theory /2/, supposing that the high energy scattering is described by a smooth potential of the form

$$V(r) = g(r^2)\exp\left[-\phi(r^2)\right] \tag{1.3}$$

in which the functions $g(r^2)$ and $\phi(r^2)$ can contain the energy as a parameter and do not turn into infinity in any finite part of the complex r-plane. In this case the differential cross section dependence on the momentum transfer of Eq. (1.2) is explained simply by the penetration of the scattering particles into the classically forbidden domain. However, it is necessary to add that the existence of the diffraction peak Eq. (1.1) at small momentum transfer is also an essential feature of the scattering on the potentials (1.3).

Thus the introduction of the potentials of Eq. (1.3) even purely phenomenological into field theory is a highly successful step. The fact that the diffraction peak is observed not only in the elastic scattering but in the exchange processes as well is also in favour of the introduction of such potentials. The exponential index γ of the exchange processes such as the charge-exchange scattering is similar to that of the elastic scattering and does not depend very much on the exchanged quantum numbers. It is difficult to submit these facts to the generally accepted views about shadow nature of the diffraction peak and to the scattering picture on the Yukawa potential when the angle distribution depends appreciably on the exchange quantum mass. The description of this scattering by the potentials of Eq. (1.3) allows to explain the regularities of these exchange processes. In paper /3/ where the nonrelativistic particle scattering on the potentials of Eq. (1.3) was studied it was emphasized that these potentials could be described as potentials with a variable interaction radius. In fact if by analogy with the Yukawa potential the interaction radius is defined by the equation

$$\frac{d\phi}{dr} = \text{the inverse interaction radius} \qquad (1.4)$$

then the assumption about the quadratic dependence of the function $\phi(r)$ upon r automatically provides nonconstancy of the interaction radius. We remind that in the case of the Yukawa potential the constancy of the interaction radius led to the poles in the scattering amplitude as a function of the momentum transfer and it caused the strong dependence of the differential cross section on the exchange quantum mass. There are no such poles in the case of the potentials with the variable interaction radius. Now the scattering amplutude is an entire function of t and the effective interaction radius is defined by the momentum transfer. Thus the potentials of Eq. (1.3) lead to new universal scattering characteristics which differ considerably from the more usual Yukawa potential characteristics. In particular, now it is not so difficult to understand the cause of the existence of the exchange scattering diffraction peak which looks like the elastic scattering one. Now the shape of the differential cross section curve is defined mainly by the momentum transfer itself. In recent paper /4/ the high energy scattering amplitude on the complex Gaussian potential was studied with the help of the quasipotential equation for the scattering amplitude in momentum space. The results obtained there are extremely close to the results of the analysis of the fast nonrelativistic particle scattering on the potential of Eq. (1.3) this analysis heaving been made with the help of the Schrödinger equation. It was shown in paper /5/ that this coincidence was not accidental and the phase shifts obtained on the basis of the exact quasipotential equation /2/ coincide in the first approximation with the phase shifts derived from the nonrelativistic Schrödinger equation with the same potential. According to these circumstances the Schrödinger equation is a convenient tool for the investigation of the scattering amplitude even in the high energy region.

In the present paper the method developed in paper /3/ of the evaluation of the scattering amplitude is used for investigation of the spinless particle scattering on 1/2 spin particles. The expressions for the polarizations are obtained both in the small momentum transfer region and in the Orear one where the scattering differential cross section is described by Eq. (1.2). It is shown that at a small scattering angle the method developed in ref. /3/ is equivalent to the widely used eikonal approximation in the scattering theory /6/.

2. Estimation of the Scalar Particle Scattering Amplitude

The possibility of representating the scattering phase shifts on the potential (1.3) in the Born integral form

$$\delta_\ell = -\int_0^\infty x\,dx\, V(x)\, J_\lambda^2 (p x), \quad \lambda = \ell + 1/2 \tag{2.1}$$

and the possibility of estimating further the phase shifts by means of the saddle point method underlie the estimates carried out in ref./3/. Since in this case the phase shifts decrease rapidly (more rapidly than the linear exponential) with increasing ℓ (at $\mathrm{Re}\,\lambda > 0$), it is convenient to expand the exponential in the scattering amplitude partial wave expansion.

$$f(p, \theta) = \frac{1}{2ip} \Sigma\, (2\ell + 1)(e^{2i\delta_\ell} - 1) P_\ell\, (\cos\theta) \tag{2.2}$$

and to estimate the series by means of the saddle point method. Thus the scattering amplitude is represented by the series

$$f(p, \theta) = \frac{1}{2ip} \sum_{n=1}^{\infty} \frac{(2i)^n}{n!} A_n(p, \theta), \tag{2.3}$$

where

$$A_n(p, \theta) = \sum_{\ell=0}^{\infty} (2\ell + 1)\, \delta_\ell^n\, P_\ell\, (\cos\theta). \tag{2.4}$$

To estimate the phase shifts we use the approximation for the Bessel function

$$J_\lambda(p x) = \frac{1}{\sqrt{2\pi\, px\, \mathrm{sh}\,\gamma}} \exp\,[\, px(\mathrm{sh}\,\gamma - \gamma\, \mathrm{ch}\,\gamma)\,], \tag{2.5}$$

where

$$\mathrm{ch}\,\gamma = \frac{\lambda}{p x}. \tag{2.6}$$

In this case the integral in Eq. (1.2) is reduced to the integral

$$\delta_\lambda = -\frac{1}{2\pi p}\int \frac{dx}{\mathrm{sh}\,\gamma} g(x) \exp[F(\dot{x})] \qquad (2.7)$$

in which one can consider $\mathrm{sh}\,\gamma$ and $g(x)$ to be slowly variable functions of x and

$$F(x) = -\phi(x) + 2p\,x(\mathrm{sh}\,\gamma - \gamma\,\mathrm{ch}\,\gamma). \qquad (2.8)$$

Using the definition (2.6) it is easy to show that

$$F'(x) = -\phi'(x) + 2p\,\mathrm{sh}\,\gamma \qquad (2.9)$$

and

$$F''(x) = -\phi''(x) - 2p\,\frac{\mathrm{ch}^2\gamma}{x\,\mathrm{sh}\,\gamma}. \qquad (2.10)$$

The condition $F'(x)=0$ gives the function $\tilde{\tilde{x}}(\lambda)$ which is unobviously defined by the equations.

$$\phi'(\tilde{\tilde{x}}) = 2p\,\mathrm{sh}\,\tilde{\tilde{\gamma}} \qquad (2.11)$$

$$\mathrm{ch}\,\tilde{\tilde{\gamma}} = \frac{\lambda}{p\,\tilde{\tilde{x}}(\lambda)}. \qquad (2.12)$$

The phase shifts themselves are approximated in the following way

$$\delta_\ell = -\frac{g(\tilde{\tilde{x}})}{2\pi p\,\mathrm{sh}\,\tilde{\tilde{\gamma}}} \sqrt{-\frac{2\pi}{F''(\tilde{\tilde{x}})}} \exp[\tilde{\tilde{F}}(\lambda)], \qquad (2.13)$$

where the function $\tilde{\tilde{F}}(\lambda)$ is given by the expression

$$\tilde{\tilde{F}}(\lambda) \equiv F[\tilde{\tilde{x}}(\lambda)] = -\phi(\tilde{\tilde{x}}) + 2p\,\tilde{\tilde{x}}(\mathrm{sh}\,\tilde{\tilde{\gamma}} - \tilde{\tilde{\gamma}}\,\mathrm{ch}\,\tilde{\tilde{\gamma}}). \qquad (2.14)$$

If we substitute the expression for the phase shifts, Eq.(2.13), into the series in Eq. (2.4) and replace the summation by the integration, the series comes to the typical saddle point integral. To estimate it one can use the asymptotic representation for the Legendre polynomial

$$P_\ell(\cos\theta) = \sqrt{\frac{2}{\pi\lambda\sin\theta}}\ \mathrm{Re}\,\exp\left(i\lambda\theta - i\frac{\pi}{4}\right). \tag{2.15}$$

After that $A_n(p,\theta)$ can be represented in the form

$$A_n(p,\theta) = 2\sqrt{\frac{2}{\pi\sin\theta}}\ \{\mathrm{Re}\int d\lambda\,\sqrt{\lambda}\,e^{-i\pi/4}\exp(\Phi_n(\lambda)) \times$$

$$\times\,\mathrm{Re}\left(-\frac{g(\tilde{\tilde{x}})}{2\pi p\,\mathrm{sh}\,\tilde{\tilde{\gamma}}}\sqrt{-\frac{2\pi}{F''(\tilde{\tilde{x}})}}\right) + i\,\mathrm{Re}\int d\lambda\,\sqrt{\lambda}\,e^{-i\pi/4}\exp(\Phi_n(\lambda))\,\mathrm{Im}\left(-\frac{g(\tilde{\tilde{x}})}{2\pi p\,\mathrm{sh}\,\tilde{\tilde{\gamma}}}\sqrt{-\frac{2\pi}{F''(\tilde{\tilde{x}})}}\right)\}. \tag{2.16}$$

The saddle point λ_0 for the integrals in Eq.(2.16) is defined from the equation

$$\phi_n'(s^0) = n\tilde{\tilde{F}}'(s^0) + i\theta = 0 \tag{2.17}$$

It follows from Eq. (2.14) that the point λ_0 is unobviously given by the equation

$$\tilde{\tilde{\gamma}}(\lambda_0) = i\frac{\theta}{2n}. \tag{2.18}$$

However, it is more convenient not to use this equation but to substitute the derived value $\tilde{\tilde{\gamma}}(\lambda_0)$ into Eqs. (2.11) and (2.12) and to express $A_n(p,\theta)$ as a function of $\tilde{\tilde{x}}_0$. In this case the value $\tilde{\tilde{x}}_0$ is directly obtained from the equation

$$\phi'(\tilde{\tilde{x}}) = 2ip\sin\frac{\theta}{2n} \tag{2.19}$$

and Eq. (2.12) serves for λ_0 definition

$$\lambda_0 = p\,\tilde{\tilde{x}}_0 \cos\frac{\theta}{2n}. \qquad (2.20)$$

It is easy to show that at the stationary point the function $\Phi_n(\lambda)$ equals

$$\Phi_n(\lambda_0) = -n\,[\phi(\tilde{\tilde{x}}_0) - \tilde{\tilde{x}}_0\,\phi'(\tilde{\tilde{x}}_0)] \qquad (2.21)$$

and Eq. (2.16) comes to

$$A_n(p,\theta) = 2\left[-i\,\frac{p\tilde{\tilde{x}}_0 \cos\frac{\theta}{2n}}{\sin\theta\,\Phi_n''(\lambda_0)}\right]^{1/2} \exp\left[-n\,\ell n\left(-\frac{p\sin\frac{\theta}{2n}\sqrt{2\pi F''(\tilde{\tilde{x}}_0)}}{g(\tilde{\tilde{x}}_0)}\right) - \right. \qquad (2.22)$$

$$\left. - n\,(\phi(\tilde{\tilde{x}}_0) - \tilde{\tilde{x}}_0\,\phi'(\tilde{\tilde{x}}_0))\right],$$

where

$$\Phi_n''(\lambda_0) = -\frac{4n\,\phi''(\tilde{\tilde{x}}_0)}{\tilde{\tilde{x}}_0\,\phi''(\tilde{\tilde{x}}_0) + 4p^2\,\mathrm{ch}^2\,\tilde{\tilde{\gamma}}(\lambda_0)}. \qquad (2.23)$$

We point out extremely obvious physical meaning of Eq. (2.19) Squaring the both sides of this equation we get

$$[\phi'(\tilde{\tilde{x}}_0)]^2 = -4p^2 \sin^2\frac{\theta}{2n}. \qquad (2.24)$$

There is a momentum transfer of the scattering at the angle θ/n in the right-hand side of Eq. (2.24). Since the derivative of the function $\phi(x)$ is identified with the inverse interaction radius, one may say that the saddle point of the integral in Eq. (2.16) fixes a value of the interaction radius so that the inverse interaction radius squared, equals the momentum transfer of the scattering at the angle θ/n . Eq. (2.3) shows that the wave function which defines the scattering amplitude on the potential of Eq. (1.3) can be represented as a superposition of the divergent spherical waves, each of them appearing due to the interaction at the given distance from the origin, this distance being defined by the scattering angle. So the resulting wave can strongly depend on an angle at which it is observed. Thus sharp change of the angle dependence of the scattering cross section is a characteristic feature of the scattering of the potentials of Eq. (1.3).

Notice that the saddle point $\tilde{\tilde{x}}$ goes to zero with the scattering angle decrease. So one can make the transition to the limit $\theta \to 0$ in Eq. (2.22). After that the forward scattering amplitude is given by the series

$$f(p,0) = -i \frac{p}{a} \sum \frac{(-i\tau)^n}{n!\, n}, \tag{2.25}$$

where

$$a = \phi''(0). \tag{2.26}$$

The value τ equals

$$\tau = \frac{2g(0)}{p\sqrt{2\pi a}} = \tau_0 e^{i\phi}, \tag{2.27}$$

where ϕ is the phase of the function $g(x)$ at $x=0$. The series (2.25) can be written in the integral form

$$f(p,0) = -i \frac{p}{a} \int_0^{\tau_0} \frac{dx}{x} \{ \exp[x e^{i(\phi-\pi/2)}] - 1 \} \equiv -i \frac{p}{a} K(\tau_0) \tag{2.28}$$

and furthermore it is not difficult to get the expression for the total scattering cross section

$$\sigma_{tot} = \frac{4\pi}{a} \int_0^{\tau_0} \frac{dx}{x} \{ 1 - \cos(x\cos\phi) \exp(x\sin\phi) \}, \tag{2.29}$$

It is also simple to evaluate the diffraction peak width

$$\Delta = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right)_{t=0} = \frac{2\,\mathrm{Re}\, f^* \frac{df}{dt}}{|f|^2} \Big|_{t=0}, \tag{2.30}$$

It is easy to show that

$$\frac{df}{dt}\Big|_{t=0} = -i\frac{p}{a^2}\int_0^{\tau_0}\frac{d\xi}{\xi}K(\xi), \qquad (2.31)$$

so

$$\Delta = \frac{1}{a\,|K(\tau_0)|^2}\,\mathrm{Re}\{K^*(\tau_0)\int_0^{\tau_0}\frac{d\xi}{\xi}K(\xi)\}. \qquad (2.32)$$

The expression for the total cross section Eq. (2.29) first has been obtained in paper /6/ in the case of the scattering on the Gaussian potential by means of the eikonal approximation.

We shall show that the method developed here leads to the same results as the eikonal approximation does in the scattering at extremely small angles. Since all time we consider the function $g(x)$ to be a slowly variable one we can suppose it constant deschribing the small angle scattering. If the scattering angle is so small that we can suppose $\sin\frac{\theta}{2n} = \frac{\theta}{2n}$, then we can consider the stationary values $\tilde{\tilde{x}}$ to be small too. In this case the following equations are valid

$$\phi'(\tilde{\tilde{x}}) = a\tilde{\tilde{x}}, \quad \phi(x) = \frac{a\tilde{\tilde{x}}^2}{2} \qquad (2.33)$$

i.e. describing the small angle scattering any smooth potential of Eq. (1.3) can be replaced by Gaussian one. Then with the accepted together with Eqs. (2.33) accuracy the preexponential factors and the logarithm in the exponential in Eq. (2.16) do not depend on the scattering angle the series in Eq. (2.3) comes to the series

$$f(p,\theta) = -\frac{ip}{a}\sum\frac{(-i\tau)^n}{n!\,n}\exp\left[-\frac{(p\theta)^2}{2an}\right]. \qquad (2.34)$$

On using the identity

$$\frac{1}{an}\exp\left[-\frac{(p\theta)^2}{2an}\right] = \int_0^{\infty} db\, b\, J_0(bp\theta)\exp\left(-n\frac{ab^2}{2}\right). \qquad (2.35)$$

one can write the series in Eq. (2.34) in the integral form

$$f(p,\theta)=-ip\int_0^\infty d\,b\,b\,J_0(pb\ \theta)\{\exp(-i\tau e^{-\frac{ab^2}{2}})-1\}. \quad (2.36)$$

It is not difficult to make oneself sure that the exponential in Eq. (2.36) equals

$$i\chi(b)=-\frac{i}{\pi p}\int_{-\infty}^{+\infty}V(\sqrt{b^2+z^2})dz \quad (2.37)$$

and the integral in Eq. (2.36) coincides with the integral which has been obtained as a consequence of solving the Schrödinger equation in the short wave approximation /6/.

Thus the representation of the scattering amplitude in series form (2.4) contains the eikonal approximation as a particular case and besides that it can be used for the estimation of the large angle scattering amplitude. We notice that the transition from the summation to the integration in Eq. (2.4) leads to the sadlle point integral which must approximate the scattering amplitude sufficiently exactly. Supposing $z=\frac{\theta}{n}$ and approximating $n!$ by the Stirling formula we derive the following integral representation for the scattering amplitude

$$f(p,\theta)=\frac{1}{ip}\sqrt{\frac{p\,\theta}{2\pi\sin\theta}}\int_0^\theta dz\left(-\frac{i\tilde{\tilde{x}}(z)\cos z/2}{z^3\,\Phi''(z)}\right)^{1/2}\exp\left[-\frac{\theta}{z}u(z)\right], \quad (2.38)$$

where

$$u(z)=\ell n\frac{\phi'[\tilde{\tilde{x}}(z)]\theta\sqrt{2\pi F''[\tilde{\tilde{x}}(z)]}}{4ezg[\tilde{\tilde{x}}(z)]}+\phi[\tilde{\tilde{x}}(z)]-\tilde{\tilde{x}}(z)\phi'[\tilde{\tilde{x}}(z)]. \quad (2.39)$$

The saddle point position for the integrand is defined by the equation

$$u(z)-zu'(z)=0. \quad (2.40)$$

One can see from the definition of the function $u(z)$ that z_0 (the root of the equation (2.40)) slightly depends on the scattering angle and at small z the equation (2.40) has no solution, $u(z)$ increasing logarithmically with $z \to 0$ and $zu'(z)$ being finite at the point $z = 0$. So at small scattering angle the saddle point lies far from the integration contour and as the first approximation of the scattering amplitude one can take the value of the integrand at the upper integration limit. It means that at a sufficiently small momentum transfer the angle dependence of the scattering amplitude is described satisfactorily by the first term of the series (2.4) i.e. at $\theta < z_0$

$$f(p,\theta) \approxeq \frac{1}{p} A_1(p,\theta). \tag{2.41}$$

In the case of the large angle scattering the evaluation of the integral in Eq. (2.38) by means of the saddle point method leads to the result

$$f(p,\theta) \approxeq \frac{1}{ip}\sqrt{\frac{p}{\sin\theta}}\left(-i\,\frac{\tilde{\tilde{x}}(z_0)\cos z_0/2}{u''(z_0)\, z_0^2\, \Phi''(z_0)}\right)^{1/2} \exp[-\theta u'(z_0)]\,. \tag{2.42}$$

For definiteness we consider the scattering on the Gaussian potential

$$V(x) = g\,\exp\left(-\frac{a x^2}{2}\right). \tag{2.43}$$

The equation defining the stationary point $\tilde{\tilde{x}}$ comes to the following one

$$a\,\tilde{\tilde{x}} = i\,2p\,\sin\frac{\theta}{2n}. \tag{2.44}$$

The expressions for the second derivatives of the functions F and Φ also look simple

$$F''(\tilde{\tilde{x}}) = a\,\frac{\cos\theta/n}{\sin^2\theta/2n} \tag{2.45}$$

$$\Phi''(\tilde{\tilde{x}}) = -\frac{a\,n}{p^2 \cos\theta/n}. \tag{2.46}$$

In this case the general term of the series in Eq. (2.4) equals

$$A_n(p,\theta) = \frac{2p^2}{a}\left(\frac{\sin 2\theta/n}{2n\sin\theta}\right)^{1/2}\left(-\frac{g}{p\sqrt{2\pi a\cos\theta/n}}\right)^n \exp\left(-n\frac{2p^2}{a}\sin^2\frac{\theta}{2n}\right). \tag{2.47}$$

The small angle scattering is defined by the amplitude

$$f(p,\theta) = -g\sqrt{\frac{2}{\pi a^3}}\exp\left(\frac{t}{2a}\right). \tag{2.48}$$

It is not difficult to show that it is necessary to suppose the coupling constant g proportional to the momentum to provide the scattering total cross section constancy. However, even in this case one can still define the scattering phase shifts by the Born integral of Eq. (2.1). For the Gaussian potential the equation (2.40) for the saddle point z_0 comes to the equation

$$\frac{p^2 z_0^2}{a} + \ell n\left(\frac{p^2 z_0^2}{a}\right) = \ell n\left(\frac{\pi p^2 t}{2g^2}\right). \tag{2.49}$$

At a large momentum transfer one can consider the logarithm in the left hand side of Eq. (2.49) to be a slowly variable function of z_0 and take as an approximate solution of Eq. (2.49)

$$z_0^2 = \frac{a}{p^2}\,\ell n\left(\frac{\pi p^2 t}{g^2}\right). \tag{2.50}$$

Notice that we can consider the value standing under the logarithm in Eq. (2.50) to be positive since the phase of the root z_0 due to the logarithm complexity is negligible in that case. The large momentum transfer scattering amplitude equals

$$f(p,\theta) = -ip\sqrt{\frac{1}{-at}}\exp\left[-\sqrt{\frac{t}{a}\,\ell n\left(\frac{\pi p^2 t}{2g^2}\right)}\right]. \tag{2.51}$$

In the general case the scattering amplitude is defined by the following expression

$$f(p,\theta) = -i\, p \sqrt{\frac{1}{-at}} \exp\left[-\sqrt{-t}\,\frac{p\, z_0(t)}{a}\right], \tag{2.52}$$

where $z_0(t) = |z_0(t)| \exp[i\,\psi(t)]$ is the solution of the saddle point Eq. (2.49). We get the formula

$$\frac{d\sigma}{d|t|} = \frac{2|g|^2}{p^2 a^3} \exp\left(\frac{t}{a}\right) \tag{2.53}$$

for the scattering differential cross section at small angles $\theta < z_0$. Here we use Eq. (2.48) for the scattering amplitude. Such behaviour agrees well with the differential cross section behaviour in the diffraction region which is observed in the experiment. Thus our approximation (2.41) works rather well in the diffraction peak region. Now we consider the scattering at angles $\theta > z_0$. On using the estimate in Eq. (2.52) of the series in Eq. (2.3) we get the differential cross section in the form

$$\frac{d\sigma}{d|t|} = \frac{\pi}{-at} \exp\left[-\sqrt{-t}\,\frac{2p\,|z_0(t)|\cos\psi(t)}{a}\right]. \tag{2.54}$$

As one can see from Eq. (2.51) at a large momentum transfer $|z_0(t)|$ is a slowly variable function of t and the saddle point phase $\psi(t) \approx 0$. In this case

$$\frac{d\sigma}{d|t|} \approx \exp\left[-b\sqrt{-t}\right]. \tag{2.55}$$

Such a behaviour agrees well with the differential cross section behaviour in the Orear region 2.0 $GeV^2 \le -t \le$ 5.0 GeV^2. Thus the useful method allows one to describe the experimentally observable change of the regimes in the differential cross section behaviour. We cannot describe the more detailed structure in the intermediate domain 1.0 $GeV^2 \le -t \le$ 2.0 GeV^2, for our evaluations allow to pick out only the main exponential part in the behaviour of the scattering amplitude and cross section. We notice that the experimental precision in the region $1.0\ \mathrm{GeV}^2 \leq -t \leq 2.0\ \mathrm{GeV}^2$ is not enough to determine the analytic form of the differential cross section dependence on the momentum transfer. We point out only the possibility of the oscillations in this region that follows from general Eq.(2.54).

3. 0 Spin Particle Scattering on 1/2 Spin Particle

In this section we take into consideration the spin in the simplest case of the scalar particle scattering on 1/2 spin particle. We assume that the interaction potential has the usual form

$$V(x) = V_1(x) + V_2(x)(\vec{\sigma} \cdot \vec{\ell}), \tag{3.1}$$

where $V_1(x)$ and $V_2(x)$ are the central and the spin-orbital potentials correspondingly. Further we shall assume the form of Eq. (1.3) for them. $\vec{\ell}$ is the orbital moment operator of two particles in the centre of mass system. If the total moment $J(\vec{J} = \vec{\ell} + \frac{\vec{\sigma}}{2})$ of the system is fixed the orbital moment ℓ can take two values $\ell = J \pm \frac{1}{2}$. The potential (3.1) conserves parity, so the transitions between these states are absent. As usually we label by $\ell_\pm$ the values related to the states with $J = \ell \pm \frac{1}{2}$. The scattering in these states is described by the Schrödinger equation with the potentials $V_{\ell+}$ and $V_{\ell-}$ accordingly.

$$V_{\ell+}(x) = V_1(x) + \ell V_2(x) \tag{3.2}$$

$$V_{\ell-}(x) = V_1(x) - (\ell+1) V_2(x). \tag{3.3}$$

We use Eq. (2.1) for the phase shifts $\delta_{\ell\pm}$ which turns out valid because of the assumption about smoothness of the potentials V_1 and V_2:

$$\delta_{\ell\pm} = -\int_0^\infty dx\, x\, V_{\ell\pm}(x) J_\lambda^2(px). \tag{3.4}$$

The scattering amplitude

$$T(p,\theta) = f(p,\theta) + i(\vec{\sigma}\cdot\vec{n})\, g(p,\theta)\sin\theta, \tag{3.5}$$

where $\vec{n} = \frac{\vec{p}\times\vec{p}'}{|\vec{p}\times\vec{p}'|}$ is the unit vector in the direction of the normal to the scattering plane. Here g and f are the spin-flip and spin-non-flip scattering amplitudes for which the following partial wave expansions are valid

$$f(p,\theta) = \frac{1}{2ip}\sum_{\ell=0}^{\infty}\left[(\ell+1)(e^{2i\delta_{\ell+}}-1) + \ell\,(e^{2i\delta_{\ell-}}-1)\right]P_\ell(\cos\theta) \tag{3.6}$$

$$g(p,\theta) = \frac{1}{2ip}\sum_{\ell=1}^{\infty}(e^{2i\delta_{\ell+}} - e^{2i\delta_{\ell-}})P_\ell'(\cos\theta). \tag{3.7}$$

As in the spinless case the amplitudes f and g are represented in the form of the expansions in the double series

$$f(p,\theta) = \frac{1}{2ip}\sum_{n=1}^{\infty}\frac{(2i)^n}{n!}(A_n^+ + A_n^-) \tag{3.8}$$

$$g(p,\theta) = \frac{1}{2ip}\sum_{n=1}^{\infty}\frac{(2i)^n}{n!}(B_n^+ - B_n^-), \tag{3.9}$$

where

$$A_n^+(p,\theta) = \sum_{\ell=0}^{\infty}(\ell+1)\,\delta_{\ell+}^n(p)P_\ell(\cos\theta) \tag{3.10}$$

$$A_n^-(p,\theta) = \sum_{\ell=0}^{\infty}\ell\,\delta_{\ell-}^n(p)P_\ell(\cos\theta) \tag{3.11}$$

$$B_n^{\pm}(p,\theta) = \sum_{\ell=1}^{\infty}\delta_{\ell\pm}^n(p)P_\ell'(\cos\theta). \tag{3.12}$$

We shall estimate these expressions replacing the sum by the integral. The integrals obtained in such a way have the saddle points.

We use the asymptotic formula of Eq. (2.15) for $P_\ell(\cos\theta)$. The similar asymptotic formula for $P_\ell'(\cos\theta)$ has the form

$$P_\ell'(\cos\theta) \cong \frac{1}{\sin\theta}\sqrt{\frac{2\lambda}{\pi\sin\theta}}\ \mathrm{Re}\exp\left(i\lambda\theta - i\frac{3\pi}{4}\right). \tag{3.13}$$

Thus we get the following representation for the expressions (3.10) - (3.12)

$$A_n^{\pm}(p,\theta) = \sqrt{\frac{2}{\pi\sin\theta}}\int d\lambda\sqrt{\lambda}\,\delta^n_{\lambda\pm}(p)\ \mathrm{Re}\exp\left(i\lambda\theta - i\frac{\pi}{4}\right) \tag{3.14}$$

$$B_n^{\pm}(p,\theta) = \frac{1}{\sin\theta}\sqrt{\frac{2}{\pi\sin\theta}}\int d\lambda\sqrt{\lambda}\ \delta^n_{\lambda\pm}(p)\ \mathrm{Re}\exp\left(i\lambda\theta - i\frac{3\pi}{4}\right), \tag{3.15}$$

We write down the phase shifts in the form

$$\delta_{\lambda\pm} = \delta_{1\lambda} \pm \lambda\delta_{2\lambda}, \tag{3.16}$$

where $\delta_{1\lambda}$ and $\delta_{2\lambda}$ are defined by the potentials $V_1(x)$ and $V_2(x)$. The evaluation of the corresponding Born integrals such as in Eq. (2.1) has been considered in detail in the previous section. The following formulas take place

$$\delta_{1,2\lambda}(p) = -\frac{g_{1,2}(x_{1,2})}{2\pi p\,\mathrm{sh}\,\gamma(x_{1,2})}\sqrt{-\frac{2\pi}{F''_{1,2}(x_{1,2})}}\exp\left[F_{1,2}(x_{1,2})\right] \tag{3.17}$$

$$F_{1,2}(x_{1,2}) = -\phi_{1,2}(x_{1,2}) + 2px_{1,2}\left[\mathrm{sh}\,\gamma(x_{1,2}) - \gamma(x_{1,2})\,\mathrm{ch}\,\gamma(x_{1,2})\right], \tag{3.18}$$

where $x_{1,2} \equiv x_{1,2}(\lambda)$ are defined from the equations

$$\phi'_{1,2}(x_{1,2}) = 2p\,\mathrm{sh}\,\gamma(x_{1,2}) \tag{3.19}$$

$$\mathrm{ch}\,\gamma(x_{1,2}) = \frac{\lambda}{p\,x_{1,2}}. \tag{3.20}$$

Further we shall assume for simplicity that the potentials V_1 and V_2 are Gaussian:

$$V_1(x) = g_1 \exp(-\frac{a x^2}{2}) \tag{3.21}$$

$$V_2(x) = g_2 \exp(-\frac{b x^2}{2}). \tag{3.22}$$

No difficulties of principle arise in the consideration of the general form potentials $V(x) = g(x^2)\exp[-\phi(x^2)]$. However the expressions obtained in such a way turn out to be very complex.

Using the binomial expansion for $\delta^n_{\lambda\pm}(p)$ we get

$$A_n^{\pm}(p,\theta) = \sqrt{\frac{2}{\pi \sin\theta}} \sum_{k=0}^{n} \binom{n}{k} g^{n-k} g_2^k \, \mathrm{Re} \int d\lambda \sqrt{\lambda}\, (\pm\lambda)^k e^{-i\pi/4} \times \tag{3.23}$$

$$\times \left[-\frac{1}{2\pi p \,\mathrm{sh}\,\gamma(x_1)} \sqrt{-\frac{2\pi}{F_1''(x_1)}}\right]^{n-k} \left[-\frac{1}{2\pi p\,\mathrm{sh}\,\gamma(x_2)} \sqrt{-\frac{2\pi}{F_2''(x_2)}}\right]^{k} \exp[\Phi_{nk}(\lambda)],$$

where

$$\Phi_{nk}(\lambda) \equiv (n-k) F_1(x_1) + k F_2(x_2) + i\lambda\theta. \tag{3.24}$$

The stationary point λ_0 for the function $\Phi_{nk}(\lambda)$ is defined from the equation $\Phi'_{nk}(\lambda) = 0$. It is not difficult to reduce it to

$$(n-k)\gamma[x_1(\lambda)] + k\gamma[x_2(\lambda)] = i\frac{\theta}{2}. \tag{3.25}$$

The second relation connecting $\gamma(x_1)$ with $\gamma(x_2)$ follows from Eqs. (3.19) and (3.20)

$$\frac{\gamma[x_1(\lambda)]}{\gamma[x_2(\lambda)]} = \frac{a}{b}. \tag{3.26}$$

It follows immediately from Eqs. (3.25) and (3.26) that at the saddle point λ_0 :

$$\gamma(x_1) = i\,\frac{ay}{2b} \tag{3.27}$$

$$\gamma(x_2) = i\,\frac{y}{2}, \tag{3.28}$$

where

$$y = \frac{\theta}{k + (n-k)\dfrac{a}{b}}. \tag{3.29}$$

Returning to Eqs. (3.19) and (3.20) we get

$$\lambda_0 = p\,x_1 \operatorname{ch}\gamma(x_1) = i\,\frac{p^2}{a}\sin\left(\frac{ay}{b}\right) \equiv i\,\tilde{\tilde{\lambda}}. \tag{3.30}$$

It also follows from Eqs. (3.27)-(3.30) that

$$\Phi_{nk}(\lambda_0) = -(n-k)\,\frac{2p^2}{a}\sin^2\left(\frac{ay}{2b}\right) - k\,\frac{2p^2}{b}\sin^2\frac{y}{2} \tag{3.31}$$

$$\Phi''_{nk}(\lambda_0) = -\,\frac{a(n-k)}{p^2\cos\left(\frac{ay}{b}\right)} - \frac{bk}{p^2\cos y} \tag{3.32}$$

$$F''_1[x_1(\lambda_0)] = \frac{a\cos\left(\frac{ay}{b}\right)}{\sin^2\left(\frac{ay}{2b}\right)}\,;\quad F''_2[x_2(\lambda_0)] = \frac{b\cos y}{\sin^2\frac{y}{2}}. \tag{3.33}$$

Taking account of these formulas the expression for $A_n(p,\theta) = A_n^+(p,\theta) + A_n^-(p,\theta)$, obtained by means of the saddle point method has the form

$$A_n(p,\theta) = 2\sum_{k=0}^{n}\binom{n}{k}\cos\frac{\pi k}{2}\left(-\frac{\tilde{\tilde{\lambda}}}{\sin\theta\,\Phi''_{nk}(\lambda_0)}\right)^{1/2} D_{nk}(p,\theta)\exp\left[\Phi_{nk}(\lambda_0)\right], \tag{3.34}$$

where

$$D_{nk}(p,\theta) = g_1^{\,n-k}\,(\tilde{\tilde{\lambda}}\,g_2)^k\left[-p\sqrt{2\pi a\cos\left(\frac{ay}{b}\right)}\,\right]^{-(n-k)}\left[-p\sqrt{2\pi b\cos y}\,\right]^{-k}. \tag{3.35}$$

By repeating the above arguments it is not difficult to receive the following expression for $B_n(p,\theta)=B_n^+(p,\theta)-B_n^-(p,\theta)$:

$$B_n(p,\theta)=2\sum_{k=0}^{n}\binom{n}{k}\sin\frac{\pi k}{2}\cdot\frac{1}{\sin\theta}\left(-\frac{\tilde{\tilde{\lambda}}}{\sin\theta\Phi''_{nk}(\lambda_0)}\right)^{1/2} D_{nk}(p,\theta)\exp[\Phi_{nk}(\lambda_0)]. \quad (3.36)$$

Now we shall write the series (3.8) and (3.9) for the amplitudes $f(p,\theta)$ and $g(p,\theta)$ correspondingly, retaining in $A_n(p,\theta)$ and $B_n(p,\theta)$ only the first-order terms in g_2 . Thus we shall assume, that the spin-orbital interaction is included in the potential $V(x)$ as a correction.

$$f(p,\theta)=\frac{1}{ip}\sum_{n=1}^{\infty}\frac{(2i)^n}{n!}\left(-\frac{\tilde{\tilde{\lambda}}}{\sin\theta\Phi''_{n0}(\lambda_0)}\right)^{1/2} D_{n0}(p,\theta)\exp[\Phi_{n0}(\lambda_0)] \quad (3.37)$$

$$g(p,\theta)=\frac{1}{ip}\sum_{n=1}^{\infty}\frac{(2i)^n}{(n-1)!}\frac{1}{\sin\theta}\left(-\frac{\tilde{\tilde{\lambda}}}{\sin\theta\Phi''_{n1}(\lambda_0)}\right)^{1/2} D_{n1}(p,\theta)\exp[\Phi_{n1}(\lambda_0)]. \quad (3.38)$$

In the previous section it was shown (see (2.36) and (2.37)) that our formulas were turned into the corresponding expressions of the eikonal approximation x) when the small angle $(\sin\frac{\theta}{2n}=\frac{\theta}{2n})$ spinless particle scattering was considered. This is easily verified here also for the scattering amplitudes (3.37) and (3.38). Again we use the identity (2.37) and the following one

$$\frac{p\theta}{a^2(n+\frac{b}{a})^2}\exp\left[-\frac{(p\theta)^2}{2a(n+\frac{b}{a})}\right]=\int_0^{\infty}db_0\, b_0^2\, J_1(b_0 p\theta)\exp\left(-n\frac{ab_0^2}{2}-\frac{bb_0^2}{2}\right). \quad (3.39)$$

The final expressions take the form

$$f(p,\theta)=-ip\int_0^{\infty}db_0 b_0 J_0(p\theta b_0)[\exp(i\chi_0)-1] \quad (3.40)$$

x) Recently, the eikonal approximation was applied /7/ to the explanation of the polarization data in πp small-angle scattering.

$$g(p,\theta)\sin\theta = p^2\int_0^\infty db_0 b_0^2 J_1(p\theta b_0)\exp(i\chi_0)\,\chi_f\,, \tag{3.41}$$

where

$$\chi_0(b_0) = -\frac{1}{\pi p}\int_{-\infty}^{\infty} V_1(\sqrt{b_0^2+z^2})\,dz \tag{3.42}$$

$$\chi_f(b_0) = -\frac{1}{\pi p}\int_{-\infty}^{\infty} V_2(\sqrt{b_0^2+z^2})\,dz\,. \tag{3.43}$$

It should be noted that the assumption about the smallness of $V_2(x)$ is equivalent to the first approximation in the eikonal χ_f: $\sin\chi_f = \chi_f$. In the following we shall use the expressions (3.37) and (3.38) for receiving the information about the large-angle scattering too.

We estimate the amplitudes f and g replacing the sums in n by the integrals which can be evaluated with the help of the saddle point method without any trouble. The final expressions take the form

$$f(p,\theta) = -ip\sqrt{\frac{1}{-at}}\exp\left[-\sqrt{-t}\,\frac{p z_0(t)}{a}\right] \tag{3.44}$$

$$g(p,\theta)\sin\theta = -\frac{|g_2|p^2}{a}\sqrt{\frac{2}{-\pi\, abt}}\left(\frac{\sqrt{-2\pi at}}{|g_1|}\right)^{b/a}\times$$
$$\times|z_0(t)|^{1-b/a}\exp\left\{i\xi(t)-\sqrt{-t}\,\frac{p z_0(t)}{a}\right\}, \tag{3.45}$$

where

$$g_1 = |g_1|e^{i\phi},\quad g_2 = |g_2|e^{i\chi},\quad z_0(t) = |z_0(t)|e^{i\psi(t)} \tag{3.46}$$

$$\xi(t) = \frac{b}{a}\left(\frac{\pi}{2}-\phi\right)+\chi+\left(1-\frac{b}{a}\right)\psi(t) \tag{3.47}$$

and $z_0(t)$ is the solution of the saddle point equation

$$\frac{p^2 z_0^2}{2a} + \ln\left(\frac{p^2 z_0^2}{2a}\right) = \ln\left(\frac{\pi p^2 t}{2g_1^2}\right). \tag{3.48}$$

For small scattering angles $\theta < z_0$ we cannot apply the saddle point method to the calculation of the integrals, representing the amplitudes $f(p,\theta)$ and $g(p,\theta)$ as the saddle point is far from the integration contour in this case. We retain only the first terms in the sums (3.37) and (3.38). It is equivalent to the approximation of the integrals by the values of the integrands at the upper integration limit. We obtain

$$f(p,\theta) = - g_1 \sqrt{\frac{2}{\pi a^3}} \exp\left(\frac{t}{2a}\right) \qquad (3.49)$$

$$g(p,\theta)\sin\theta = - g_2\, p \sqrt{\frac{2}{\pi b^5}}(-t)^{1/2} \exp\left(\frac{t}{2b}\right). \qquad (3.50)$$

Having the expressions for the amplitudes $f(p,\theta)$ and $g(p,\theta)$ it is easy to calculate the polarization parameter. We give the corresponding expressions below without discussion which will be given in the next section in connection with the isotopic structure of πp scattering.

If the initial particles are unpolarized the scattering results in the polarization

$$\vec{P}(\theta) = \vec{n}\, \frac{2\,\mathrm{Im}(fg^*)\sin\theta}{|f(p,\theta)|^2 + |g(p,\theta)|^2\sin^2\theta}. \qquad (3.51)$$

For the small angle scattering when the formulas (3.49) and (3.50) take place, we obtain

$$\vec{P}(t) = \vec{n}\left(\frac{a}{b}\right)^2 \frac{2p}{\sqrt{ab}}\sqrt{-t}\,\exp\left[\frac{t}{2}\left(\frac{1}{b}-\frac{1}{a}\right)\right]\left|\frac{g_2}{g_1}\right| \sin(\phi-\chi). \qquad (3.52)$$

The polarization parameter for the scattering at angles $\theta > z_0$, when the estimates (3.44) and (3.45) are correct takes the form

$$\vec{P}(t) = \vec{n}\, \frac{2|g_2|p}{a}\sqrt{\frac{2}{\pi b}}\left[\frac{\sqrt{-2\pi a t}}{|g_1|}\right]^{b/a} |z_0|^{1-b/a} \cos\xi(t) \times$$

$$\times\left[1 + \frac{2|g_2|^2p^2}{\pi a^2 b}\left(-\frac{2\pi a t}{|g_1|^2}\right)^{b/a} |z_0|^{2(1-b/a)}\right]^{-1}. \qquad (3.53)$$

It is not difficult also to consider the case of the nonvanishing initial polarization $\vec{P}_i \neq 0$. The general expressions for the differential cross section and the polarization are well known

$$\frac{d\sigma(\vec{P}_i)}{d\Omega} = |f(p,\theta)|^2 + |g(p,\theta)|^2 \sin^2\theta + 2(\vec{P}_i\,\vec{n})\sin\theta\,\mathrm{Im}(fg^*) \tag{3.54}$$

$$\vec{P} = \left[\frac{d\sigma(\vec{P}_i)}{d\Omega}\right]^{-1} \{\vec{n}[2\,\mathrm{Im}(fg^*)\sin\theta + 2|g|^2\sin^2\theta(\vec{P}_i\,\vec{n})] + \vec{P}_i(|f|^2 - |g|^2\sin^2\theta) + [\vec{P}_i \times \vec{n}]\, 2\mathrm{Re}(fg^*)\sin\theta\}. \tag{3.55}$$

Using the above formulas for the scattering amplitudes $f(p,\theta)$ and $g(p,\theta)$ it is easy to calculate such values as the asymmetry parameter and the polarization rotation. The former takes place in the scattering of the particles, polarized perpendicularly to the scattering plane, the latter one if the vector $\vec{P}_i$ lies in the scattering plane.

4. Isotopic Structure of the Scattering Amplitude

In this section we shall consider $\pi^{\pm}p$ elestic scattering and the charge exchange reaction $\pi^- p \to \pi^0 n$. Let us recall the basic experimental facts concerning the polarizations and the differential cross sections. The $\pi^- p$ elastic scattering polarization has opposite sign with respect to the $\pi^+ p$ elastic scattering polarization. The polarization of recoil neutron is not zero in the charge exchange reaction $\pi^- p \to \pi^0 n$ and it coincides in sign with the $\pi^+ p$ scattering polarization. The sign of the difference of the $\pi^+ p$ and $\pi^- p$ elastic differential cross sections changes. We shall give a qualitative description of the above mentioned experimental results with the help of the formulas of preceding section.

First of all, it is necessary to take into account the isotopic structure of πp scattering. Let T_1 and T_3 be, the scattering amplitudes in the states with the isotopic spin 1/2 and 3/2 correspondingly. Each of them has the form (3.5):

$$T_{1,3}(p, \theta) = f_{1,3}(p, \theta) + i(\vec{\sigma} \cdot \vec{n}) g_{1,3}(p, \theta) \sin\theta . \tag{4.1}$$

The scattering amplitudes of the three processes mentioned above are expressed in terms of the T_1 and T_3 by the well-known means

$$T(\pi^+ p \to \pi^+ p) = T_3 \tag{4.2}$$

$$T(\pi^- p \to \pi^- p) = \frac{1}{3}(T_3 + 2T_1) \tag{4.3}$$

$$T(\pi^- p \to \pi^0 n) = \frac{\sqrt{2}}{3}(T_3 - T_1). \tag{4.4}$$

In accordance with the preceding considerations let us take the potentials corresponding to the scattering in the states with the definite isotopic spin, in the form

$$V_1(x) = g_{11} \exp\left(-\frac{a_1 x^2}{2}\right) + g_{21} \exp\left(-\frac{b_1 x^2}{2}\right)(\vec{\sigma} \cdot \vec{\ell}) \tag{4.5}$$

$$V_3(x) = g_{13} \exp\left(-\frac{a_3 x^2}{2}\right) + g_{23} \exp\left(-\frac{b_3 x^2}{2}\right)(\vec{\sigma} \cdot \vec{\ell}), \tag{4.6}$$

where

$$g_{11} = |g_{11}| e^{i\phi_{11}}, \quad g_{13} = |g_{13}| e^{i\phi_{13}}$$

$$g_{21} = |g_{21}| e^{i\chi_{21}}, \quad g_{23} = |g_{23}| e^{i\chi_{23}}. \tag{4.7}$$

Now it remains only to use the results of section 3. In the small-angle scattering, when the angle distribution is described by the Born approximation well enough, we must use the expressions (3.49) and (3.50). Let us write the scattering amplitudes in an explicit form

$$T(\pi^+p\to\pi^+p) = -g_{13}\sqrt{\frac{2}{\pi a_3^3}}\exp\left(\frac{t}{2a_3}\right) - i(\vec{\sigma}\vec{n})g_{23}\,p\sqrt{\frac{2}{\pi b_3^5}}\sqrt{-t}\exp\left(\frac{t}{2b_3}\right) \quad (4.8)$$

$$T(\pi^-p\to\pi^-p) = -\frac{1}{3}\left[g_{13}\sqrt{\frac{2}{\pi a_3^3}}\exp\left(\frac{t}{2a_3}\right) + 2g_{11}\sqrt{\frac{2}{\pi a_1^3}}\exp\left(\frac{t}{2a_1}\right)\right] -$$

$$-i(\vec{\sigma}\cdot\vec{n})\frac{p\sqrt{-t}}{3}\left[g_{23}\sqrt{\frac{2}{\pi b_3^5}}\exp\left(\frac{t}{2b_3}\right) + 2g_{21}\sqrt{\frac{2}{\pi b_1^5}}\exp\left(\frac{t}{2b_1}\right)\right] \quad (4.9)$$

$$T(\pi^-p\to\pi^0n) = -\frac{\sqrt{2}}{3}\left[g_{13}\sqrt{\frac{2}{\pi a_3^3}}\exp\left(\frac{t}{2a_3}\right) - g_{11}\sqrt{\frac{2}{\pi a_1^3}}\exp\left(\frac{t}{2a_1}\right)\right] -$$

$$-i(\vec{\sigma}\cdot\vec{n})\frac{\sqrt{2}\,p}{3}\sqrt{-t}\left[g_{23}\sqrt{\frac{2}{\pi b_3^5}}\exp\left(\frac{t}{2b_3}\right) - g_{21}\sqrt{\frac{2}{\pi b_1^5}}\exp\left(\frac{t}{2b_1}\right)\right]. \quad (4.10)$$

The normalization is defined by

$$\frac{d\sigma}{d|t|} = \frac{\pi}{p^2}\left(|f|^2 + |g|^2\sin^2\theta\right). \quad (4.11)$$

Let us give the corresponding formulas for the differential cross sections supposing the scattering angle θ to be small. It permits one to use the expressions (3.49) and (3.50)

$$\frac{d\sigma}{d|t|}(\pi^+p\to\pi^+p) = \frac{2}{p^2}\left[\frac{|g_{13}|^2}{a_3^3}\exp\left(\frac{t}{a_3}\right) + (-t)p^2\frac{|g_{23}|^2}{b_3^5}\exp\left(\frac{t}{b_3}\right)\right] \quad (4.12)$$

$$\frac{d\sigma}{d|t|}(\pi^-p\to\pi^-p) = \frac{2}{9p^2}\left\{\left[\frac{|g_{13}|^2}{a_3^3}\exp\left(\frac{t}{a_3}\right) + \frac{4|g_{11}|^2}{a_1^3}\exp\left(\frac{t}{a_1}\right) + 4C_{11;13}(t)\right] +\right.$$

$$\left.+(-t)p^2\left[\frac{|g_{23}|^2}{b_3^5}\exp\left(\frac{t}{b_3}\right) + \frac{4|g_{21}|^2}{b_1^5}\exp\left(\frac{t}{b_1}\right) + 4C_{21;23}(t)\right]\right\} \quad (4.13)$$

$$\frac{d\sigma}{d|t|}(\pi^-p\to\pi^0n) = \frac{4}{9p^2}\left\{\left[\frac{|g_{13}|^2}{a_3^3}\exp\left(\frac{t}{a_3}\right) + \frac{|g_{11}|^2}{a_1^3}\exp\left(\frac{t}{a_1}\right) - 2C_{11;13}(t)\right] +\right. \quad (4.14)$$

$$\left.+(-t)p^2\left[\frac{|g_{23}|^2}{b_3^5}\exp\left(\frac{t}{b_3}\right) + \frac{|g_{21}|^2}{b_1^5}\exp\left(\frac{t}{b_1}\right) - 2C_{21;23}(t)\right]\right\}.$$

We use the following notations

$$C_{ij,k\ell}(t) = \frac{|g_{ij}||g_{k\ell}|}{(d_i)_j^{i+1/2}(d_k)_\ell^{k+1}} \cos[(\zeta_i)_{ij} - (\zeta_k)_{k\ell}] \exp\{\frac{t}{2}[\frac{1}{(d_i)_j} + \frac{1}{(d_k)_\ell}]\} \quad (4.15)$$

$$S_{ij;k\ell}(t) = \frac{|g_{ij}||g_{k\ell}|}{(d_i)_j^{i+1/2}(d_k)_\ell^{k+1}} \sin[(\zeta_i)_{ij} - (\zeta_k)_{k\ell}] \exp\{\frac{t}{2}[\frac{1}{(d_i)_j} + \frac{1}{(d_k)_\ell}]\}, \quad (4.16)$$

where $i,k = 1,2$; $j,\ell = 1,3$; $d_1 = a$, $d_2 = b$; $\zeta_1 = \phi, \zeta_2 = \chi$. If the initial protons are unpolarized, the expressions for the polarizations $P(t)$, take the form

$$P(\pi^+ p \to \pi^+ p) = [\frac{d\sigma_1}{d|t|}]^{-1} \frac{2\sqrt{-t}}{p} S_{13;23}(t) \quad (4.17)$$

$$P(\pi^- p \to \pi^- p) = [\frac{d\sigma_2}{d|t|}]^{-1} \frac{4\sqrt{-t}}{p} [S_{13;23}(t) + 4S_{11;21}(t) + 2S_{11;23}(t) + 2S_{13;21}(t)] \quad (4.18)$$

$$P(\pi^- p \to \pi^0 n) = [\frac{d\sigma_3}{d|t|}]^{-1} \frac{4\sqrt{-t}}{p} [S_{13;23}(t) + S_{11;21}(t) - S_{11;23}(t) - S_{13;21}(t)]. \quad (4,19)$$

Let us consider the scattering at angles $\theta > z_0$, where z_0 is the solution of the saddle point equation (3.48). In this case the amplitudes $f(p,\theta)$ and $g(p,\theta)$ are given by the expressions (3.44) and (3.45), using this it is not difficult to receive the formulas for the scattering amplitudes, differential cross sections and polarizations. We shall write the corresponding expressions for $\pi^+ p$ elasic scattering only

$$T = \frac{-ip}{\sqrt{-a_3 t}} \exp\{-\sqrt{-t}\,\frac{p\,z_{03}(t)}{a_3}\}[1 + (\vec{\sigma}\cdot\vec{n})|g_{23}|\,p\sqrt{\frac{2}{\pi a_3^2 b_3}}\,(\frac{\sqrt{-2\pi a_3 t}}{|g_{13}|})^{b_3/a_3} \times |z_{03}|^{1-b_3/a_3} \exp(i\xi_3(t))] \quad (4.20)$$

$$\frac{d\sigma}{d|t|} = \frac{\pi}{-a_3 t}[1 + \frac{2p^2|g_{23}|^2}{\pi a_3^2 b_3}(-\frac{2\pi a_3 t}{|g_{13}|^2})^{b_3/a_3} |z_0|^{2(1-b_3/a_3)}]\exp\{-\sqrt{-t}\,\frac{2p|z_{03}(t)|\cos\psi_3(t)}{a_3}\} \quad (4.21)$$

$$P(t) = \frac{2p|g_{23}|}{a_3}\sqrt{\frac{2}{\pi b_3}}\left[\frac{\sqrt{-2\pi a_3 t}}{|g_{13}|}\right]^{b_3/a_3} |z_{03}|^{1-b_3/a_3} \cos\xi_3(t) \times$$

$$\times\left[1 + \frac{2p^2|g_{23}|^2}{\pi a_3^2 b_3}\left(-\frac{2\pi a_3 t}{|g_{13}|^2}\right)^{b_3/a_3} |z_{03}|^{2(1-b_3/a_3)}\right]^{-1}, \tag{4.22}$$

where $\xi_3(t)$ is defined by (3.47).

Also one may consider the case, when the initial protons are polarized $(\vec{P}_i \neq 0)$. Using the general formulas (3.54), (3.55) and also (3.44), (3.45), (3.49), (3.50) it is easy to get the expressions for the cross sections and polarizations. We shall not give the corresponding formulas because of their complication.

5. Discussion of the Results

In this section we are going to accomplish the qualitative agreement of the received formulas with the experimental results/8/ in the elastic $\pi^{\pm}p$ scattering and the charge exchange reaction $\pi^- p \to \pi^0 n$.

First of all, let us consider the behaviour of the coupling constants g_1 and g_2 with the energy. It is known that the interaction is weakly spin-dependent at high energies. Therefore it is natural to suppose that the spin-orbital coupling constant g_2 decreases with increasing energy. Let

$$g_2 \approx p^{-a}, \quad a > 0 \tag{5.1}$$

for example. Neglecting the spin-flip part of the scattering amplitude at high energy and small momentum transfer and using the optical theorem we immediately obtain

$$g_1 \approx p \tag{5.2}$$

if the assumption about the constancy of the total cross section have been made.

Now let us consider the expressions (4.17)-(4.19) for the polarizations $P(t)$. The experiment reveals the sufficiently rapid falling of the polarization with increasing energy. It follows from our formulas that $P(t) \approx p^{-a}$. Thus the behaviour of the polarization $P(t)$ is similar to that of the spin-orbital coupling constant g_2.

The expressions for the differential cross sections and the polarization include sufficiently considerable number of the parameters. We are not going to accomplish the quantitative comparison with the experimental points, but we consider the qualitative picture alone.

First of all, let us simplify our expressions (4.17)-(4.19). Assume $a_1 = a_3 = a$ and $b_1 = b_3 = b$. This leads to that the form of the dependence of the polarizations on t becomes the same for all three processes. Also we simplify the denominators of these formulas, omitting the term $|g|^2 \sin^2\theta$ in the expressions for the differential cross secrions. Finally, we obtain

$$P_1(t) = A(t) \frac{|g_{13}||g_{23}|}{|g_{13}|^2} \sin(\phi_{13} - \chi_{23}) \tag{5.3}$$

$$P_2(t) = \frac{9A(t)}{|g_{13}|^2} \{ |g_{13}||g_{23}| \sin(\phi_{13} - \chi_{23}) + 4|g_{11}||g_{21}| \sin(\phi_{11} - \chi_{21}) + \\ + 2|g_{11}||g_{23}| \sin(\phi_{11} - \chi_{23}) + 2|g_{13}||g_{21}| \sin(\phi_{13} - \chi_{21}) \} \tag{5.4}$$

$$P_3(t) = \frac{9A(t)}{2|g_{13}|^2} \{ |g_{13}||g_{23}| \sin(\phi_{13} - \chi_{23}) + |g_{11}||g_{21}| \sin(\phi_{11} - \chi_{21}) - \\ - |g_{11}||g_{23}| \sin(\phi_{11} - \chi_{23}) - |g_{13}||g_{21}| \sin(\phi_{13} - \chi_{21}) \}, \tag{5.5}$$

where

$$A(t) = 2p\left(\frac{a}{b}\right)^{3/2} \sqrt{-t} \exp\left[\frac{t}{2}\left(\frac{1}{b} - \frac{1}{a}\right)\right]. \tag{5.6}$$

Thus first $P(t)$ grows as $\sqrt{-t}$. When the exponential becomes essential, the curve is bending $(b < a)$ and then it begins to fall. The maximum takes place at $-t_{max} \doteq \frac{ab}{a-b}$. This relation fixes the parameter b if the a is taken from the calculations of the width of the diffraction peak. The expressions (4.17)-(4.19) have been obtained with the help of the Born approximation. We mentioned that this approximation gave good agreement for small momentum transfer $-t \lesssim 0.4$ GeV^2. In this region the existing experimental data are well described by the dependence (5.6).

To demonstrate the sign change of the $\pi^- p$ elastic scattering polarization, let us make the following choice of the phases

$$\phi_{11} = \phi_{13} = -\frac{\pi}{2}, \quad \chi_{21} = 0, \quad \chi_{23} = \pi. \tag{5.7}$$

With this choice of the phases, the nonflip amplitudes, which give the main contributions to the total amplitudes $T(p,\theta)$ are purely imaginary and positive defined. Let us introduce the two parameters

$$\alpha = \left|\frac{g_{11}}{g_{13}}\right|, \quad \beta = \left|\frac{g_{21}}{g_{23}}\right|. \tag{5.8}$$

It is easy to see from (5.3) - (5.5) that the right signs for the polarizations (P_1, $P_3 > 0$, $P_2 < 0$) take place when

$$\begin{aligned} &1 - 4\alpha\beta - 2(\alpha - \beta) < 0 \\ &1 - \alpha\beta + (\alpha - \beta) > 0. \end{aligned} \tag{5.9}$$

Setting the ratios P_2/P_1 and P_3/P_1 we obtain the system of equations in respect of the α and β:

$$\begin{aligned} &4\alpha\beta + 2(\alpha - \beta) = 1 + \frac{1}{9}\left|\frac{P_2}{P_1}\right| \\ &-\alpha\beta + (\alpha - \beta) = -1 + \frac{2}{9}\left|\frac{P_3}{P_1}\right|. \end{aligned} \tag{5.10}$$

This system has always the non-negative solutions. Let $P_2/P_1 = -1$, and $P_3/P_1 = 1/2$, then $\alpha = 0{,}55$, $\beta = 0{,}9$.

For the present there is no any experimental results about the measuring of the large angle scattering polarization. We shall not discuss the expression (4.22). Let us only point to the possible appearance of the oscillations because of the factor $\cos\xi(t)$.

Finally we consider the quantity

$$\Delta \equiv \frac{d\sigma}{d|t|}(\pi^- p \to \pi^- p) - \frac{d\sigma}{d|t|}(\pi^+ p \to \pi^+ p). \qquad (5.11)$$

Neglecting the spin dependence, putting $a = 0.5$ and taking into account the conditions (5.7), we obtain

$$\Delta = \frac{8}{9p^2 a_3^3}\exp\left(\frac{t}{a_3}\right)\left[-2+\frac{1}{4}\left(\frac{a_3}{a_1}\right)^3\exp\left\{t\left(\frac{1}{a_1}-\frac{1}{a_3}\right)\right\}+\frac{1}{\sqrt{2}}\left(\frac{a_3}{a_1}\right)^{3/2}\exp\left\{\frac{t}{2}\left(\frac{1}{a_1}-\frac{1}{a_3}\right)\right\}\right]. \qquad (5.12)$$

The experimental data show /8/ that $\frac{1}{a_3}=7$ in a sufficiently wide energy region. If we put now $\frac{1}{a_1}=14$, then it is found that the crossover effect for the differential cross sections of $\pi^{\pm}p$ elastic scattering takes place at $-t \approx 0.13$ $(\text{GeV/c})^2$. It is in good agreement with the experimental results.

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R e f e r e n c e s

1. S.P. Alliluyev, S.S. Gerstein, A.A. Logunov. Phys. Lett., 18, 195 (1965).
2. A.A. Logunov, A.N. Tavkhelidze. Nuovo Cimento, 29, 380 (1963).
3. V.I. Savrin, O.A. Khrustalev, Preprint IHEP 68-19-K (1968).
4. V.R. Garsevanishvili, V.A. Matveev, L.A. Slepchenko, A.N.Tavkhelidze. Preprint E2-4251, Dubna (1969), Talk given at the Coral Gables Conference, Miami (1969); V.R. Garsevanishvili,

S.V. Goloskokov, V.A. Matveev, L.A. Slepchenko. Preprint E2-4361, Dubna (1969).

5. O.A. Khrustalev, Preprint IHEP 69-24 (1969).
6. R.I. Glauber, Lectures in Theoretical Physics, vol. 1, New York 1959.
7. R.C. Arnold, M.L. Blackmon. Phys. Rev., 176, 2082 (1968).
8. D. Harting et al., Nuovo Cimento, 38, 60 (1965). M. Borghini et. al., Phys. Lett., 24B, 77 (1966). P. Bonamy et al., Phys. Lett., 23, 501 (1966). D. Drobnis et al., Phys. Rev. Lett., 20, 274(1968).

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