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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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The recent e^+e^- colliding beam experiments and the accumulation of a large amount of data on vector meson photoproduction have attracted particular interest to different tests of the models based on the vector-dominance hypothesis (VDM). This hypothesis seems to be in reasonable agreement with the majority of experimental results [1,2]. However, the model of Gell-Mann, Sharp and Wagner (GSW) [3], based on VDM, was found to be in drastic disagreement with experimental data [2,4]. In what follows we consider the off-mass-shell corrections to the GSW-model which essentially remove this disagreement. With this aim we introduce the form-factor for the $\omega\rho\pi$ vertex $F\left(\frac{p_\omega^2}{m_\omega^2}, \frac{p_\rho^2}{m_\rho^2}\right)$, $F(1,1)\equiv 1$. We do not consider the form-factors for $\rho\to\gamma$ and $\omega\to\gamma$ transitions because the $\rho\to\gamma$ form-factor seems to be constant [1] in the interval $0\leqslant p_\rho^2 \leqslant m_\rho^2$ and we assume that the same is true for the $\omega\to\gamma$ form-factor. The role of the ρ form-factor may be estimated by the use of the Breit-Wigner approximation, for in the decay $\omega\to\rho\pi\to 3\pi$ the ρ meson is essentially on the mass shell (we shall prove that $p_\rho^2 \simeq m_\rho^2$). The corresponding corrections are rather small and can be neglected. Under the assumptions stated above we first estimate phenomenologically the dependence of $F(x,y)$ on $x=\frac{p_\omega^2}{m_\omega^2}$, $y=\frac{p_\rho^2}{m_\omega^2}$ employing the improved GSW-model. Then we develop a simple dynamical model in which F may be calculated for arbitrary $x=y$.

Following the notations of GSW [3], we find

$$\Gamma(\omega\to 3\pi) = \frac{m_\omega}{3}\,\frac{\gamma_{\rho\pi\pi}^2}{4\pi}\left(\frac{f_{\omega\rho\pi}^2}{4\pi}m_\pi^2\right)\frac{(m_\omega-3m_\pi)^4}{(m_\rho^2-4m_\pi^2)^2}\,\frac{3.56}{\sqrt{3}}\,\overline{F^2(1,y)} \tag{1}$$

$$\Gamma(\omega\to\pi\gamma) = \frac{m_\omega}{3}\,\alpha\left(\frac{f_{\omega\rho\pi}^2}{4\pi}m_\pi^2\right)\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\frac{(m_\omega^2-m_\pi^2)^3}{32 m_\omega^4 m_\pi^2}\,F^2(1,0) \tag{2}$$

$$\Gamma(\pi^0\to 2\gamma) = m_\pi\,\frac{\alpha^2}{64}\left(\frac{f_{\omega\rho\pi}^2}{4\pi}m_\pi^2\right)\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\left(\frac{\gamma_\omega^2}{4\pi}\right)^{-1}F^2(0,0) \tag{3}$$

Here $\overline{F^2(1,y)}$ is used for a certain mean value of the square of the form-factor $F(1,y)$. We now assume for $F(x,y)$ the simplest linear symmetric dependence

$$F(x,y) = F(0,0)\left[1+\lambda(x+y)\right] \quad , 0 \leqslant x \leqslant 1, \ 0 \leqslant y \leqslant 1. \tag{4}$$

To estimate λ we use two sets of experimental data:

1) $\gamma_\rho^2/4\pi = 0.53 \pm 0.04$; $\gamma_\omega^2/4\pi = 4.69 \begin{smallmatrix}+1.24\\-0.81\end{smallmatrix}$; $\gamma_{\rho\pi\pi}^2/4\pi = \frac{1}{4}(2.12 \pm 0.16)$ [1]

2) $\gamma_\rho^2/4\pi = 0.51 \pm 0.03$; $\gamma_\omega^2/4\pi = 3.7 \pm 0.7$; $\gamma_{\rho\pi\pi}^2/4\pi = \frac{1}{4}(2.10 \pm 0.11)$ [2].

The value of γ_ρ is consistent with the vector dominance in photoproduction processes and in the ρ form-factor (It should be stressed, however, that from the world average[4] value for $\Gamma(\rho \to 2\pi)$) we would find somewhat different numbers for quantities considered in what follows).

Comparing $\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\pi\to 2\gamma)}$, $\frac{\Gamma(\omega\to 3\pi)}{\Gamma(\pi\to 2\gamma)}$ predicted by Eqs. (1)-(3) with the experimental values, we obtain for data 1):

$$\frac{F^2(1,0)}{F^2(0,0)} = 2.05 \pm 0.60 , \quad (5a); \quad \frac{\overline{F^2(1,y)}}{F^2(0,0)} = 3.08 \pm 0.80 , \quad (5b).$$

and for data 2):

$$\frac{F^2(1,0)}{F^2(0,0)} = 2.60 \pm 0.60, \quad (6a); \quad \frac{\overline{F^2(1,y)}}{F^2(0,0)} = 4.23 \pm 0.80 , \quad (6b).$$

where we used experimental results[1,2,4]

$$\Gamma(\omega\to 3\pi) = (11 \pm 1.2)\,\mathrm{MeV}; \ \Gamma(\omega\to\pi\gamma) = (1.15 \pm 0.15)\,\mathrm{MeV}; \ \Gamma(\pi^0\to 2\gamma) = (7.4 \pm 1.5)\,\mathrm{eV}.$$

From Eq. (5a) we extract $\lambda \equiv \lambda_a^{(1)} = (0.43 \pm 0.21)$,

and from Eq. (6a) we find $\lambda \equiv \lambda_a^{(2)} = (0.61 \pm 0.20)$. To use Eqs. (5b), (6b) we should find $\overline{F^2(1,y)}$. It can be shown that

$$\overline{F^2(1,y)} = F^2(0,0)(1+2\lambda)^2\left[1 - \frac{4\lambda}{1+2\lambda}\varepsilon\right], \tag{7}$$

where $\mathcal{E} \simeq 0.1$. So we see that $\overline{F^2(1,y)} \simeq F^2(1,1)$ and our assumption $p_\rho^2 \simeq m_\rho^2$ is verified. Substituting Eq. (7) into Eqs. (5), (6) we obtain $\lambda_b^{(1)} = (0.41 \pm 0.13)$, $\lambda_b^{(2)} = (0.58 \pm 0.12)$. All $\lambda_{a,b}^{(i)}$ are consistent with each other (within experimental errors) . Considering that one parameter λ was determined from two independent quantities $\Gamma(\omega \to \pi\gamma)/\Gamma(\pi \to 2\gamma)$ and $\Gamma(\omega \to 3\pi)/\Gamma(\pi \to 2\gamma)$, we may infer that our simple phenomenological model works surprisingly well and gives $\lambda \sim 0.5$.

The dependence of $F(x,y)$ on x and y may be significant in other processes. In this regard the decay $\pi^\circ \to e^+e^-$ is rather interesting. Its partial width may be estimated in the simple model (Fig.1a). For $F(x,y) \simeq F(0,0)$ the branching ratio $\frac{\Gamma(\pi^\circ \to e^+e^-)}{\Gamma(\pi^\circ \to 2\gamma)}$ was found [6] to be of the order $(5\div6).10^{-8}$. By use of the form-factor F(x,y) from (4) this prediction may become $(3\div5)$ times bigger. Similarly, F(x,y) would diminish the prediction of the simple vector-dominated model for $\rho \to 4\pi$ (See Fig. 1b). These effects are worth of particular attention and will be considered in another paper.

Consider now the simple dynamical model, in which F(x,y) may be calculated. Let $\omega\rho\pi$ interaction be described by the Lagrangian $\mathcal{L}_{\omega\rho\pi} = f_{\omega\rho\pi}\, \varepsilon_{\mu\nu\lambda\sigma}\, \partial^\mu \omega^\nu \partial^\lambda \vec{\rho}^{\,\sigma} \cdot \vec{\pi}$. The simplest way to find F(x,y) is the calculation of the third order (in $f_{\omega\rho\pi}$) perturbation theory diagram for $\omega\rho\pi$ vertex. But, the interaction is unrenormalizable and the result would depend on a cut-off parameter. Besides, as the coupling is not particularly weak, higher orders must be taken into account. For these reasons we estimated F(x,y) calculating the sum of the infinite set of the "ladder"-diagrams (Fig.2) . Let us denote the vertex by

$$\Gamma^{\mu\nu}_{\omega\rho\pi}(p_\omega, p_\rho) = f_{\omega\rho\pi}\, \mathcal{E}^{\mu\nu\lambda\sigma}\, p_\lambda^{(\omega)}\, p_\sigma^{(\rho)}\, F\left(\frac{p_\omega^2}{m_\omega^2}, \frac{p_\rho^2}{m_\rho^2}\right) .$$

$F(x,y)$ satisfies the linear integral equation(Fig.2)($\Gamma_{\omega\rho\pi}$ is displayed on Fig.2 by the circle). To make the calculations as simple as possible, we set $p_\pi \simeq p_\omega + p_\rho \simeq 0$, $m_\pi \simeq 0$ (soft pions approximation). Then the equation for $F(x) \equiv F(x,x)$ coincides with that investigated in ref.[5]. By use of modified perturbation theory[5] the expansion of $F(x)$ in a series of powers of $f \equiv \frac{1}{12} \frac{f^2_{\omega\rho\pi}}{4\pi} m_\pi^2 \simeq 0.55$ and of $\log f$ may be found. Up to the terms of the order $\sim f^2$ we obtain $F(1,1)/F(0,0)=1.23$. This approximation works only if $f \lesssim 0.3$. For $f \sim 0.5$ the crude estimation of higher order gives $F(1,1)/F(0,0) \simeq 1.3$. So for the linear extrapolation (4) we have $\lambda = (0.12 \div 0.15)$. (The details of the calculations will be published in a subsequent paper). Our simple dynamical model is therefore qualitatively consistent with experiment. For the quantitative test of the theory the non-ladder diagrams with $p_\pi \neq 0$ should be considered.

In conclusion we would like to stress that λ is strongly dependent on $\Gamma(\pi^\circ \to 2\gamma)$. In fact, if we take the new value[1,7] $\Gamma(\pi^\circ \to 2\gamma) = (11 {}^{+1.6}_{-2.8})\,eV$ we will obtain: $\lambda_a^{(1)} = (0.18 \pm 0.18)$, $\lambda_b^{(1)} = (0.23 \pm 0.11)$, $\lambda_a^{(2)} = (0.32 \pm 0.17)$, $\lambda_b^{(2)} = (0.37 \pm 0.1?)$ in close agreement with our simplified theory.

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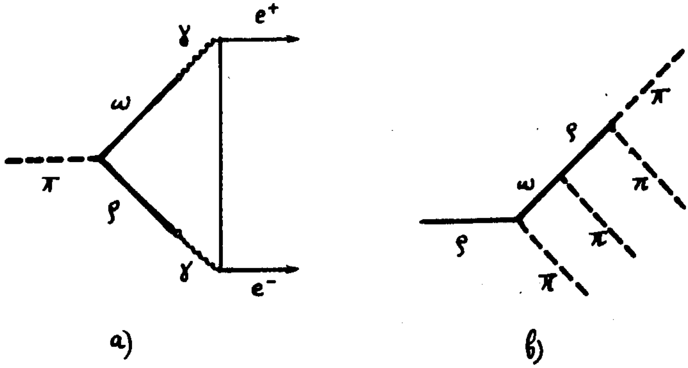

Fig. 1

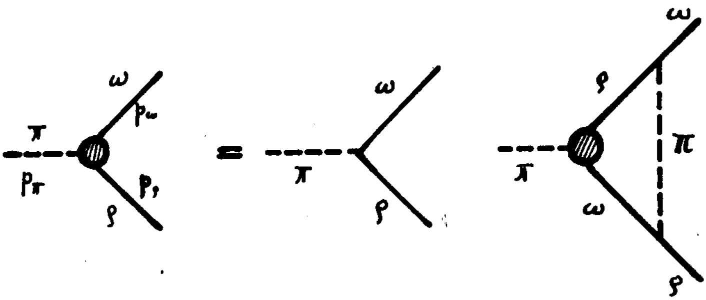

Fig. 2.