СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
Дубна

E2 - 4455

S.P.Kuleshov, V.A.Matveev, A.N.Sissakia

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

GLAUBER REPRESENTATION FOR HIGH ENERGY SCATTERING OF THE DIRAC PARTICLES ON SMOOTH POTENTIALS

1969

E2 - 4455

S.P.Kuleshov, V.A.Matveev, A.N.Sissakian

GLAUBER REPRESENTATION FOR HIGH ENERGY SCATTERING OF THE DIRAC PARTICLES ON SMOOTH POTENTIALS

1. Introduction

Recently in papers[1,2] an approach to the description of high energy particle scattering was developed which was based on the quasipotential equation of Logunov and Tavkhelidze in quantum field theory[3,4].

The approach is supported on an essential assumption of a nonsingular or smooth behaviour of the local quasipotential $V(E,\vec{r})$ as a function of the relative coordinate of two particles $\vec{r}$. Under this assumption the two-particle scattering has the semi-classical character for small angles[1], as well as for large ones[5]. In particular, in paper[1] it was shown that for smooth quasipotentials the amplitude of high energy scattering of two spinless particles at small angles obeys an integral representation, which is closely connected with the Glauber representation for the scattering amplitude of rapid particles on nuclei in the eikonal approximation[6].

It should be stressed, that a rigorous derivation of the Glauber representation in the framework of quantum field theory supposes the proof of smooth behaviour of quasipotential at high energies of hadrons.

It is interesting to investigate spin effects in particle scattering at high energies. In the present work we give a derivation of the

Glauber representation for the amplitude of the spin 1/2 particle scattering on smooth potentials at high energies of incident particles. The consideration is given in the framework of the two-component description as well as on the basis of the Dirac equation. The study of the Dirac equation allows one to separate relativistic kinematical effects in scattering of the rapid spin particles.

2. Glauber Representation for the Scattering of Spin 1/2 Particles in the Two-Component Description

Consider the scattering of the rapid spin 1/2 particle on the basis of the Klein-Gordon type equation

$$(p^{2} + \vec{\nabla}^{2})\psi(\vec{r}) = V(\vec{r})\,\psi(\vec{r}), \tag{2.1}$$

where the potential includes the contribution of spin-orbital forces

$$V(\vec{r}) = V_{0}(r) + V_{1}(r)(\vec{\sigma}\,\vec{L}); \quad \vec{L} = -i\,[\,\vec{r}\times\vec{\nabla}\,]. \tag{2.2}$$

We assume here that $V_0(r)$ and $V_1(r)$ are scalar smooth functions of $\vec{r}$, which depend, generally speaking, on the particle energy $E = \sqrt{m^2 + p^2}$.

The solution of eq. (2.1) can be represented in the form

$$\psi(\vec{r}) = e^{ipz}\,\phi(\vec{r}), \tag{2.3}$$

where the two-component spinor $\phi(\vec{r})$ satisfies the boundary condition

$$\phi(\vec{r})\,\big|_{z\to-\infty} = \phi_{0}, \tag{2.4}$$

that corresponds to an incident particle with spin wave function ϕ_0 and large momentum p along the z-axis.

Under the condition of smooth behaviour of the potential and at sufficiently large energies, when

$$| V / p^2 | \ll 1 ; \quad | \dot{V} / p V | \ll 1 , \tag{2.5}$$

the spinor $\phi(\vec{r})$ is a slow varying function and approximately obeys the equation

$$2ip \frac{\partial \phi}{\partial z} = (V_0 + V_1 p [\vec{\sigma} \times \vec{r}]_z) \phi . \tag{2.6}$$

The solution of (2.6) under the boundary condition (2.4) has the form

$$\phi(\vec{r}) = e^{\chi_0 (\vec{\rho}, z) + i [\vec{n} \times \vec{\sigma}]_z \chi_1 (\vec{\rho}, z)} \phi_0 , \tag{2.7}$$

where

$$\chi_0 (\vec{\rho} , z) = \frac{1}{2ip} \int_{-\infty}^{z} dz' V_0 (\vec{\rho} , z'), \tag{2.8a}$$

$$\chi_1 (\vec{\rho}, z) = \frac{\rho}{2} \int_{-\infty}^{z} dz' V_1 (\vec{\rho} , z'). \tag{2.8b}$$

Here $\vec{r} = (\vec{\rho} ; z) ; \quad \vec{n} = \frac{\vec{\rho}}{\rho} = (\cos \Phi , \sin \Phi) ,$

where Φ is an azimuthal angle in the (x, y) -plane, so that

$$[\vec{\sigma} \times \vec{n}]_z = \cos \Phi \cdot \sigma_y - \sin \Phi \cdot \sigma_c$$
$$[\vec{\sigma} \times \vec{n}]_z^2 = 1 . \tag{2.9}$$

Using eqs. (2.7) and (2.8) we get for the scattering amplitude the Glauber representation

$$f(p,\Delta)=-\frac{1}{4\pi}\int d\vec{r}\, e^{-i\vec{p}\,'\vec{r}}\,\phi_0^*(\vec{p}\,')(V_0+p[\vec{\sigma}\times\vec{r}\,]_z V_1)\,\psi(\vec{r}\,)=$$

$$=\frac{p}{2\pi i}\int d^2\rho\, e^{-i\vec{\rho}\vec{\Delta}}\,\phi_0^*(\vec{p}\,)\{e^{\chi_0+i[\vec{n}\times\vec{\sigma}]_z\chi_1}-1\}\phi_0(\vec{p}\,), \tag{2.10}$$

where

$$\vec{\Delta}=(\vec{p}\,'-\vec{p}\,)\ ;\quad \vec{\Delta}\vec{p}\approx 0$$

$$\chi_0(\vec{\rho}\,)=\chi_0(\vec{\rho},+\infty)\ ;\quad \chi_1(\vec{\rho}\,)=\chi_1(\vec{\rho},+\infty).$$

Using the formulae for the integral representations of the Bessel functions

$$J_0(x)=\frac{1}{2\pi}\int_0^{2\pi} d\Phi\, e^{ix\cos\Phi}, \tag{2.11a}$$

$$J_1(x)=-\frac{i}{2\pi}\int_0^{2\pi} d\Phi\, e^{ix\cos\Phi}\cos\Phi, \tag{2.11b}$$

we receive the following expression for the scattering amplitude

$$f(p,\Delta)=\phi_0^*(\vec{p}\,')\,[\,a+\sigma_y b\,]\,\phi_0(\vec{p}\,), \tag{2.12}$$

where

$$a=-ip\int_0^\infty \rho\, d\rho\, J_0(\rho\Delta)\{e^{\chi_0}\cos\chi_1-1\}, \tag{2.13a}$$

$$b=-ip\int_0^\infty \rho\, d\rho\, J_1(\rho\Delta)\, e^{\chi_0}\sin\chi_1. \tag{2.13b}$$

The wave functions of spin particles with the helicity values $\lambda=\frac{\vec{\sigma}\vec{p}}{p}$ and $\lambda'=\frac{\vec{\sigma}\vec{p}\,'}{p'}$ has the form/7/

$$\phi_0(\vec{p}) = \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \begin{pmatrix}0\\1\end{pmatrix}, \text{ at } \lambda = +1/2 \text{ and } -1/2; \tag{2.14a}$$

$$\phi_0(\vec{p}\,') = \begin{pmatrix}\cos\theta/2\\ -\sin\theta/2\end{pmatrix} \text{ and } \begin{pmatrix}\sin\theta/2\\ \cos\theta/2\end{pmatrix}, \text{ at } \lambda' = +1/2 \text{ and } -1/2; \tag{2.14b}$$

where θ is the scattering angle and

$$\left.\begin{aligned} \cos\theta/2 &= \sqrt{1-\frac{\Delta^2}{4p^2}} \to 1 \\ \sin\theta/2 &= \frac{\Delta}{2p} \to 0 \end{aligned}\right\} \text{ at } p \to \infty. \tag{2.15}$$

Thus, the quantities a and b defined by eqs. (2.12) and (2.13) in the limit $p \to \infty$ at fixed moments Δ will coincide with the non-spin-flip and spin-flip amplitudes, respectively.

3. Glauber Representation for the Scattering of the Dirac Particles

Consider now the scattering of the rapid spin particle on the scalar potential on the basis of the Dirac equation

$$(E + i\vec{a}\,\vec{\nabla} - \beta m - \beta V(\vec{r}))\,\psi(\vec{r}) = 0. \tag{3.1}$$

We shall seek the solution of equation (3.1) in the following form

$$\psi(\vec{r}) = e^{ipz} F^{(+)}(\vec{r}) + e^{-ipz} F^{(-)}(\vec{r}), \tag{3.2}$$

where

$$E = \sqrt{m^2 + p^2} \approx p_z = p .$$

Substituting the expression (3.2) into eq. (3.1) and requiring the term which contain as a factor the large parameter p to be vanish, we get

$$a_z F^{(\pm)} = \pm F^{(\pm)} . \tag{3.3}$$

From eq. (3.3) it follows, that

$$F^{(\pm)}(\vec{r}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm\sigma_z \end{pmatrix} \phi^{(\pm)}(\vec{r}), \tag{3.4}$$

where $\phi^{(\pm)}(\vec{r})$ are the two-component spinors.

As one can see from formulae (3.2-4) the two terms in eq.(3.2) describe the propagation along z -axis of the incident and reflected waves, respectively. The following boundary conditions hold

$$\phi^{(+)}(\vec{r})\big|_{z\to-\infty} = \phi_0 \;; \quad \phi^{(-)}(\vec{r})\big|_{z\to-\infty} = 0, \tag{3.5}$$

where ϕ_0 is the constant two-component spinor of an incident particle. Using formula (3.4) one can find that the spinors $\phi^{(\pm)}$ satisfy the equations

$$i\frac{\partial\phi^{(+)}}{\partial z} = e^{-2ipz}\left(W - [\vec{\sigma}\times\vec{\nabla}]_z\right)\phi^{(-)}, \tag{3.6a}$$

$$i\frac{\partial\phi^{(-)}}{\partial z} = -e^{2ipz}\left(W + [\vec{\sigma}\times\vec{\nabla}]_z\right)\phi^{(+)}, \tag{3.6b}$$

where

$$W = m + V(\vec{r})$$

The differential equations (3.6) with the boundary conditions (3.5) are equivalent to the following pair of the integral equations

$$\phi^{+}(z,\vec{\rho}) = \phi_0 - i\int_{-\infty}^{z} dz' e^{-2ipz'}\left(W(z',\vec{\rho}) - [\vec{\sigma}\times\vec{\nabla}_\perp]_z\right)\phi^{(-)}(z',\vec{\rho}), \tag{3.7a}$$

$$\phi^{(-)}(z,\vec{\rho}) = i\int_{-\infty}^{z'} dz'' e^{2ipz''}\left(W(z'',\vec{\rho}) + [\vec{\sigma}\times\vec{\nabla}_\perp]_z\right)\phi^{(+)}(z'',\vec{\rho}), \tag{3.7b}$$

where

$$\vec{r} = (r, \vec{\rho}); \qquad \vec{\nabla}_\perp = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right).$$

The analysis of eqs. (3.7) in the limit of large momenta p under the condition of smooth behaviour of the potential $V(\vec{r})$ shows that the spinor $\phi^{(+)}$ is a slow varying function of z, whereas the $\phi^{(-)}$ oscillates rapidly with variation of z.

So, eq. (3.7b) in the limit $p \approx \infty$ gives

$$\phi^{(-)}(z,\vec{\rho}) = \frac{1}{2p} e^{2ipz} \left(W(z,\vec{\rho}) + [\vec{\sigma} \times \vec{\nabla}_\perp]_z \right) \phi^{(+)}(z,\vec{\rho}) + O(1/p^2) \tag{3.8}$$

Substituting the expression (3.8) in eq. (3.7a), we obtain the following integral equation for the spinor $\phi^{(+)}$

$$\phi^{(+)}(z,\vec{\rho}) = \phi_0 + \frac{1}{2ip} \int_{-\infty}^{z} dz' \left(W^2(z',\vec{\rho}) - \vec{\nabla}_\perp^2 - [\vec{\sigma} \times \vec{n}]_z \frac{dW}{d\rho} \right) \phi^{(+)}(z',\vec{\rho}) \tag{3.9}$$

In eq. (3.9) the term which includes $\frac{m^2 - \vec{\nabla}_\perp^2}{2p}$ should be dropped. This term is similar to that one which we neglect in derivation of eq. (3.6)[x/]. As a result we obtain the following solution

$$\phi^{(+)}(z,\vec{\rho}) = e^{\frac{1}{2ip} \int_{-\infty}^{z} dz' \left(V^2 + 2mV - [\vec{\sigma} \times \vec{n}]_z \frac{dV}{d\rho} \right)} \phi_0 . \tag{3.10}$$

The expression for the scattering amplitude can be found by quadrating the Dirac equation (3.1)

$$(p^2 + \vec{\nabla}^2 - W^2 + m^2 - i\vec{\gamma}\vec{\nabla} W)\, \psi(\vec{r}) = 0, \tag{3.11}$$

[x/] This quantity is nothing else than the second term in expansion of the particle energy in powers of the large momentum p_z

$$E = p_z + \frac{m^2 + p_\perp^2}{2p_z} + O(1/p_z^2).$$

from where it follows that

$$f(p,\Delta) = -\frac{1}{4\pi}\int d\vec{r}\,\psi^{(0)\,*}_{\vec{p}\,'}(\vec{r})\,(V^2 + 2m\,V + i\,\vec{\gamma}\vec{\nabla}\,V)\,\psi\,(\vec{r}). \tag{3.12}$$

Here

$$\psi^{(0)}_{\vec{p}\,'}(\vec{r}) = e^{i\vec{p}\,'\vec{r}}\,\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \sigma_z\end{pmatrix}\phi'_0 \tag{3.13}$$

is the spinor, which describes an outgoing particle with large component momentum along z-axis.

Using the formulae (3.2), (3.4) and (3.10) we get finally for the scattering amplitude the expression of the type (2.12), where

$$\chi_0(\vec{\rho}) = \frac{1}{2ip}\int_{-\infty}^{+\infty} dz\,(V^2(z,\vec{\rho}) + 2mV(z,\vec{\rho})) \tag{3.14a}$$

$$\chi_1(\vec{\rho}) = -\frac{1}{2ip}\int_{-\infty}^{+\infty} dz\,\frac{dV(z,\vec{\rho})}{d\rho}. \tag{3.14b}$$

It is easy to see that the spin-flip part of the scattering amplitude of rapid Dirac particles on a scalar potential is due to relativistic kinematical effects.

The method developed here allows one to derive also the Glauber representation for the scattering amplitude of the rapid Dirac particles on smooth potentials of other types.

The results obtained here, can be used in the description of the scattering of rapid spin particles on heavy nuclei as well as in the phenomenological analysis of the pion-nucleon scattering at high energies. We should notice, however, that more rigorous consideration of spin effects in the two-particle interactions at high energies is possible in the framework of the quasipotential equations.

The authors express their deep gratitude to N.N.Bogolubov, B.M.Barbashov,A.N.Tavkhelidze for helpful discussions and critical remarks.

References

1. *V.R.Garsevanishvili, V.A.Matveev, L.A.Slepchenko, A.N.Tavkhelidze. Preprint E2-4251, Dubna (1969); Talk given at the Coral Gables Conference, Miami (1969).*
2. *V.R.Garsevanishvili, S.V.Goloskokov, V.A.Matveev, L.A.Slepchenko. Preprint E2-4361, Dubna (1969).*
3. *A.A.Logunov, A.N.Tavkhelidze. Nuovo Cim., 29, 380 (1963).*
4. *A.N.Tavkhelidze. Lectures on Quasipotential Method in Field Theory., Tata Institute of Fundamental Research, Bombey (1964). V.G.Kadyshevsky, A.N.Tavkhelidze. Three-Dimensional Formulation of Relativistic Two-Body Problem, Proceedings of the International Seminar on Elementary Particle Theory, Varna (1968).*
5. *S.P.Alliluyev, S.S.Gershtein, A.A.Logunov. Phys. Lett., 18, 195 (1965).*
6. *R.J.Glauber. in "Lectures in Theoretical Physics", vol.1, p.315, N.Y. (1959).*
7. *A.S.Davydov. Quantum Mechanics, Fizmatgiz , Moscow (1963).*

Received by Publishing Department
on April 25, 1969.