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HIGH-ENERGY ELASTIC pp- AND np- SCATTERING IN THE QUASIPOTENTIAL MODEL

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In recent papers[1,2] a relativistic model of particle scattering at high energies was proposed which was based on the quasipotential equation of Logunov and Tavkhelidze for the scattering amplitude in quantum field theory[3,4]. In the simplest case of scattering of two spinless particles of equal masses the quasipotential equation reads :

$$T(\vec{p},\vec{k};E) = V[(\vec{p}-\vec{k})^2;E] + \int \frac{d\vec{q}}{\sqrt{m^2+\vec{q}^{\,2}}} \frac{V[(\vec{p}-\vec{q})^2;E]\,T(\vec{q},\vec{k};E)}{\vec{q}^{\,2}+m^2-E^2-i0}, \tag{1.1}$$

where E is the energy, $\vec{p}$ and $\vec{k}$ are the center of mass system relative momenta of the initial and final states, respectively.

The physical relativistically invariant scattering amplitude $T(s,t)$ is defined by the condition

$$T(s,t) = 32\pi^3 T(\vec{p},\vec{k};E)\Big|_{\substack{s=4E^2=4(m^2+\vec{p}^{\,2})=4(m^2+\vec{k}^{\,2})\\ t=-(\vec{p}-\vec{k})^2}} \tag{1.2}$$

Notice, that eq. (1.1) is a relativistic analogue of the Lippmann-Schwinger equation for the scattering amplitude in non-relativistic quantum mechanics.

The quasipotential in eq. (1.1) is a complex function of energy, the imaginary part of which is due to the inelastic processes in two particle scattering.

An approach to the description of high energy particle scattering developed in papers /1,2/ was based on the phenomenological choice of the quasipotential in eq. (1.1).

Furthermore an assumption was explored that high energy hadron scattering can be described as the scattering on a smooth complex quasipotential $V(\vec{r}, E)$ /5/ which is a nonsingular function of the relative coordinate of two particles.

Such an assumption means essentially that hadron scattering at high energies may be considered as an interaction of two "friable" systems. As was shown /1,2/ the simplest nonsingular quasipotential of the Gaussian type

$$V(\vec{r}, E) = i s g_0 \left(\frac{\pi}{a}\right)^{3/2} e^{-\frac{\vec{r}^{\,2}}{4a}} \qquad (1.3)$$

allows the main features of the hadron scattering amplitude at high energies to be reproduced.

In describing the real physical processes it is necessary, generally speaking, to take into account the spin structure of the scattering amplitude.

In the case of elastic proton-proton scattering, for instance, there are five independent invariant amplitudes, which can be chosen in the helicity basis as follows:

$$T_1 = \left\langle \frac{1}{2}, \frac{1}{2} \middle| T \middle| \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$T_2 = \left\langle \frac{1}{2}, \frac{1}{2} \middle| T \middle| -\frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$T_3 = \left\langle \frac{1}{2}, -\frac{1}{2} \middle| T \middle| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$T_4 = \left\langle \frac{1}{2}, -\frac{1}{2} \middle| T \middle| -\frac{1}{2}, \frac{1}{2} \right\rangle \qquad (1.4)$$

$$T_5 = \left\langle \frac{1}{2}, \frac{1}{2} \middle| T \middle| \frac{1}{2}, -\frac{1}{2} \right\rangle .$$

However only two of them T_1 and T_3, which correspond to the spin-non-flip processes, give nonvanishing contributions to the forward scattering.

The relative magnitudes of the spin-flip amplitudes T_2, T_4, T_5 at nonzero scattering angles can be determined from the knowledge of the polarization parameter which does not exceed 10% at high energies and decreases with increasing energy /6/.

The remaining spin-non-flip amplitudes T_1 and T_3 are approximately equal to each other. This is a consequence of a "pure elastic" character of high energy hadron scattering, which is due to the exchange of zero quantum numbers in crossed channels.

Thus in the description of an unpolarized proton-proton scattering at high energies one can confine oneself to consider one amplitude $T \approx T_1 \approx T_3$ in the framework of the quasipotential equation (1.1) for spinless particles x/.

In the present work we give a comparison of the results obtained in papers /1,2/ on the basis of eq. (1.1) with quasipotential (1.3), with experimental data on the high-energy elastic pp-scattering at small and large angles. Besides, following the papers /1,2/ we analyse experimental data on the elastic np-backward scattering (or, what is the same, charge exchange $pn \to np$ scattering), taking into account exchange forces in the proton-neutron system.

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x/ This assumption, however, may turn out to be not correct in the region of large scattering angles $\theta \approx 90^{\circ}$, where the requirement of crossing symmetry makes it necessary to take into account the spin-non-flip amplitudes too.

2. Small Angle pp -Scattering

Following the remark made in the introduction, we shall give a description of the high-energy elastic pp scattering at small angles with the help of one amplitude, which obeyes the quasipotential eq. (1.1).

As was shown in refs./1,2/, the solution of eq. (1.1) with quasipotential (1,3) in the region of small scattering angles at high energies:

$$\left|\frac{t}{s}\right| \ll 1 , \quad a s \gg 1 \tag{2.1}$$

can be found as a convergent series of Born approximations:

$$T(\vec{q}^2, E) = is\, g_0 \sum_{n=1}^{\infty} \frac{e^{\frac{at}{n}}}{n\, n!} \left(-\frac{4\pi^2 g_0}{a}\right)^{n-1} \Bigg|_{t=-\vec{q}^2} . \tag{2.2}$$

Notice, that the series (2.2) is a sum of the main contributions to the scattering amplitude increasing as $s \approx 4p^2$ at high energies, and is pure imaginary. The real part of the scattering amplitude is determined by the contributions which increase not faster than $\sqrt{s} \approx 2p$ with increasing energy.

The expression (2.2) depends on the two real parameters a and g_0 entering the definition of the quasipotential (1.3). The numerical values of these parameters can be found from the experimental data at small and vanishing momentum transfers , i.e. from the total cross section σ_{tot} and the diffraction peak width $1/A$ in the following manner

$$\sigma_{tot} = 8\pi a\, I(x) \tag{2.3a}$$

$$A \doteq \frac{d}{dt}\left[\ln \frac{d\sigma}{dt}\right]_{t=0} = 2a \frac{1}{I(x)} \int_0^x \frac{d\xi}{\xi} I(\xi) \tag{2.3b}$$

$$I(x) = -\sum_{n=1}^{\infty} \frac{(-x)^n}{n\, n!} = \int_0^x \frac{d\xi}{\xi}(1-e^{-\xi}) \tag{2.4}$$

$$x = \frac{4\pi^2 g_0}{a} .$$

We have done the comparison of the results obtained above with experimental data on the elastic pp -scattering in the region (2.1) at p_L =8.5, 12.4 and 18.4 GeV/c./7,8/ The theoretical curves on Figs. 1 and 2 correspond to the following values of the parameters g_0 and a

$$p_L = 8.5 \frac{GeV}{c}, \quad g_0 = 0.13\left(\frac{GeV}{c}\right)^{-2}, \quad a = 2.6\left(\frac{GeV}{c}\right)^{-2}.$$

$$p_L = 12.4 \frac{GeV}{c}, \quad g_0 = 0.12\left(\frac{GeV}{c}\right)^{-2}, \quad a = 2.8\left(\frac{GeV}{c}\right)^{-2}$$

$$p_L = 18.4 \frac{GeV}{c}, \quad g_0 = 0.14\left(\frac{GeV}{c}\right)^{-2}, \quad a = 3.8\left(\frac{GeV}{c}\right)^{-2}$$

which were calculated using formulae (2.3) and (2.4) from the experimental values of the total cross section/9/ and the diffraction peak width /7/ at corresponding energies[x/]. As is seen from Figs.1 and 2 the theoretical curves, reproduce rather well the behaviour of the differential cross section of the elastic pp -scattering in the region (2.1), as well as the positions of diffraction minima and their energy

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x/ The numerical values of these parameters and others, calculated in this paper, contain uncertainties which are determined by the errors of corresponding experimental data.

dependence. Notice, that the qualitative analysis of the elastic pp-scattering at high energies has been done from various points of view in refs./10-14/ too. Near the points, where the sum (2.2) vanishes, it is necessary to take into account the following terms of expansion of the scattering amplitude in inverse powers of momentum p. This leads to the so called "filling of minima". Furthermore, on Fig.5 the behaviour of the differential cross section of the elastic pp-scattering at $p_L = 8.5$ GeV/c in the region $0 \le |t| < 0.6\,(\frac{GeV}{c})^2$ is shown. One can see from Fig.5 that the existence of small "shoulder" at $|t| \approx 0.3$ (GeV/c)^2 is in agreement with the results of theoretical calculations. A similar behaviour is observed at other energies as well.

3. Large Angle Elastic pp-Scattering

Let us consider high energy particle scattering at fixed scattering angles:

$$\left|\frac{t}{s}\right| \approx \sin^2\frac{\theta}{2} = \text{fixed} . \qquad (3.1)$$

In this case the series of Born approximations for the scattering amplitude has the following form:

$$T(\vec{q}^{\,2},E) \approx i s g_0 \sum_{n=1}^{\infty} \frac{n^{2n}}{(n!)^2} \frac{e^{\frac{at}{n}}}{n^{3/2}} \left(\frac{i s g_0 \pi\sqrt{\pi}}{t p a \sqrt{a}} \right)^{n-1} . \qquad (3.2)$$

When $a|t| \gg 1$ the main contributions to the sum (3.2) are given by the terms with $n \gg 1$. This allows the Stirling formula $n! \approx \sqrt{2\pi n}\,(\frac{n}{e})^n$ to be used. As a result we obtain the following expression for the scattering amplitude:

$$T(\vec{q}^{\,2},E) \underset{\substack{\theta = \text{fixed} \\ s\to\infty}}{\to} \frac{i s g_0}{2\pi} \sum_{n=1}^{\infty} \frac{e^{\frac{at}{n}}}{n^{5/2}} (-i\gamma)^{n-1} , \qquad (3.3)$$

where

$$\gamma = \frac{g_0 e^2}{p \sin^2\theta/2} \left(\frac{\pi}{a}\right)^{3/2} = \frac{s g_0 e^2}{|t|\, p} \left(\frac{\pi}{a}\right)^{3/2} . \qquad (3.4)$$

The series (3.3) is convergent under the condition $|\gamma| < 1$ or using the numerical values of the parameters $a = 3.0\,(\frac{GeV}{c})^{-2}$ and $g_0 = 0.13\,(\frac{GeV}{c})^{-2}$

$$|t| > \sqrt{s} \cdot 0.3\,\frac{GeV}{c} . \qquad (3.5)$$

Rewriting the series (3.3) in the integral form and taking the integral by means of the residue theorem we get:

$$T(\vec{q}^{\,2},E) \underset{\substack{\theta = \text{fixed} \\ s\to\infty}}{\to} -\frac{q p r_0^2}{12\pi^2} e^{i q r_0} , \qquad (3.6)$$

where the parameter r_0^2 is equal to

$$r_0^2 = -2\pi i a \left(1 + \frac{2i}{\pi} \ln\gamma\right) . \qquad (3.7)$$

At intermediate energies, when the second term in eq. (3.7) can be neglected, i.e. $r_0^2 \approx 2\pi i a$ we obtain the following expression for the differential cross section at large angles:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\pi a}{3}\right)^2 q^2 e^{-2q\sqrt{\pi a}} \quad ; q = \sqrt{|t|} . \qquad (3.8)$$

An interesting feature of the result (3.8) is the fact, that at fixed momentum transfers corresponding to large scattering angles $\frac{d\sigma}{d\Omega}$ weakly depends on energy. The only energy dependence of $\frac{d\sigma}{d\Omega}$ enters through the parameter a which is connected with the forward diffraction peak width. The theoretical curve on Fig. 3 corresponds to $a = 3.0$ (GeV/c)$^{-2}$ and reproduces the absolute value and character of the decrease of the differential cross section /15-17/ in the region of large scattering angles, restricted by the condition (3.5). We stress, that in accordance with the remark in page 5 the formula (3.8) can not be applied, generally speaking, at large scattering angles $\theta \approx 90^{\circ}$.

4. Elastic np-Backward Scattering

One can see from the foregoing consideration that the scattering amplitude at large angles (3.6) exponentially decreases with increasing energy. Thus, the solution of eq. (1.1) with quasipotential (1.3) leads to the exponentially small cross section for the backward scattering at high energies, what contradicts in a number of cases the experimental data. As was pointed out in ref./2/ this fact is due to the neglect of exchange forces in the two-particle system.

In what follows we shall show how the exchange forces can be included in the quasipotential equation and shall use the results obtained for the analysis of experimental data on np-backward scattering.

In the presence of exchange forces the scattering amplitude $T(\vec{p},\vec{k};E)$ can be represented as a sum of two quantities /18/:

$$T(\vec{p},\vec{k};E) = G(\vec{p},\vec{k};E) + H(\vec{p},\vec{k};E) \tag{4.1}$$

which obey the following system of quasipotential equations x/

$$G = g + g \times G + h \times H , \tag{4.2a}$$

$$H = h + h \times G + g \times H . \tag{4.2b}$$

Multiplication in formulae (4.2) implies an integration in the sense of eq. (1. 1). The quantities g and h are the Fourier transforms of the "direct" and "exchange" parts of the quasipotential, respectively:

$$g(s,t) = \frac{1}{(2\pi)^3} \int d\vec{r}\, e^{i\vec{p}\vec{r}}\, V(s,\vec{r})\, e^{-i\vec{k}\vec{r}} , \quad t = -(\vec{p}-\vec{k})^2, \tag{4.3a}$$

$$h(s,u) = \frac{1}{(2\pi)^3} \int d\vec{r}\, e^{i\vec{p}\vec{r}}\, V_e(s,\vec{r})\, \hat{P}\, e^{-i\vec{k}\vec{r}} ; \quad u = -(\vec{p}+\vec{k})^2, \tag{4.3b}$$

where $\hat{P}$ is the coordinate-exchange operator.

As a quasipotential of "direct" interaction we use the expression (1.3), or $g(s,t) = is\, g_0 e^{at}$.

The "exchange" part of the quasipotential is due to the crossed u-channel contributions.

Taking into account the condition

$$\left| \frac{h(s,0)}{s g_0} \right| \ll 1 \qquad \text{at} \qquad s \approx \infty \tag{4.4}$$

one can neglect the last term in eq. (4.2a). Iterating the obtained system of equations we get

$$H = h + h \times G + G \times h + G \times h \times G , \tag{4.5}$$

where G is determined by the solution of eq. (1.1) with quasipotential (1.3). The expression (4.5) for the amplitude H is pictured symbolically in Fig.6.

Let us assume now that the "exchange" quasipotential can be represented as a sum:

$$h(s,u) = \sum_i h_i(s) e^{b_i u} , \tag{4.6}$$

where $\left| \frac{h_i(s)}{s g_0} \right| \ll 1$ at high energies.

For this case the amplitude H in the region $\left| \frac{u}{s} \right| \ll 1$ can be found in the following form:

x/ The system of eq. (4.2) is equivalent to the pair of equations with definite parities which were considered in ref./3/

$$H(\vec{q}'^2, E) = \sum_i H_i(\vec{q}'^2, E), \tag{4.7}$$

where

$$H_i(q'^2, E) = h_i(s) \sum_{n=0}^{\infty} \frac{a\, e^{\frac{a b_i}{a+nb_i} u}}{(a+nb_i)\, n!} \left(-\frac{4\pi^2 g_0}{a}\right)^n . \tag{4.8}$$

These results were used for the analysis of the elastic np -backward scattering at $p_L = 8.0$ GeV/c and $|u| < 0.6 \left(\frac{GeV}{c}\right)^2$ /19/. Only two terms in expression (4.6) for the exchange quasipotential were taken into account. For the sake of simplicity the parameters h_1 and h_2 are assumed to be real; the cases of equal and opposite signs of h_i were considered.

The parameters a and g_0 entering the definition of the "direct" part of quasipotential were determined from the experimental data on elastic proton-proton scattering at

$$p_L = 8.5 \frac{GeV}{c}; \quad g_0 = 0.1 \left(\frac{GeV}{c}\right)^{-2}, \quad a = 2.6 \left(\frac{GeV}{c}\right)^{-2}.$$

The theoretical curves I and II on Fig.4, which correspond to the equal and opposite signs of the quantities h_1 and h_2, are calculated for the following values of the parameters h_i and b_i.

Equal signs (I)

$|h_1| = 0.07$, $\quad b_1 = 110.0 \left(\frac{GeV}{c}\right)^{-2}$

$|h_2| = 0.3$, $\quad b_2 = 1.8 \left(\frac{GeV}{c}\right)^{-2}$.

Opposite signs (II)

$|h_1| = 0.29$, $\quad b_1 = 34.0 \left(\frac{GeV}{c}\right)^{-2}$

$|h_2| = 0.30$, $\quad b_2 = 1.8 \left(\frac{GeV}{c}\right)^{-2}$.

On Fig.5 the same theoretical curves are plotted for comparison together with the curve of the differential cross section of the elastic pp -scattering at $p_L = 8.5$ GeV/c which is normalized to 1 mb/(GeV/c)^2 at $t = 0$. One can see from Figs. 4 and 5 that the case of equal signs is, apparently, more preferable.

We have shown that theoretical results obtained on the basis of the quasipotential equation with quasipotential of the simple Gaussian form with a small number of parameters are in good agreement with experimental data on high energy proton-proton and proton-neutron elastic scattering.

Notice that the choice of the quasipotential which gives an adequate description of hadron scattering at high energies is a problem of principle and at the same time a nontrivial one. As one can see, however, the physical assumption of nonsingular character of hadron interaction at high energies allows the main features of high-energy particles scattering to be reproduced.

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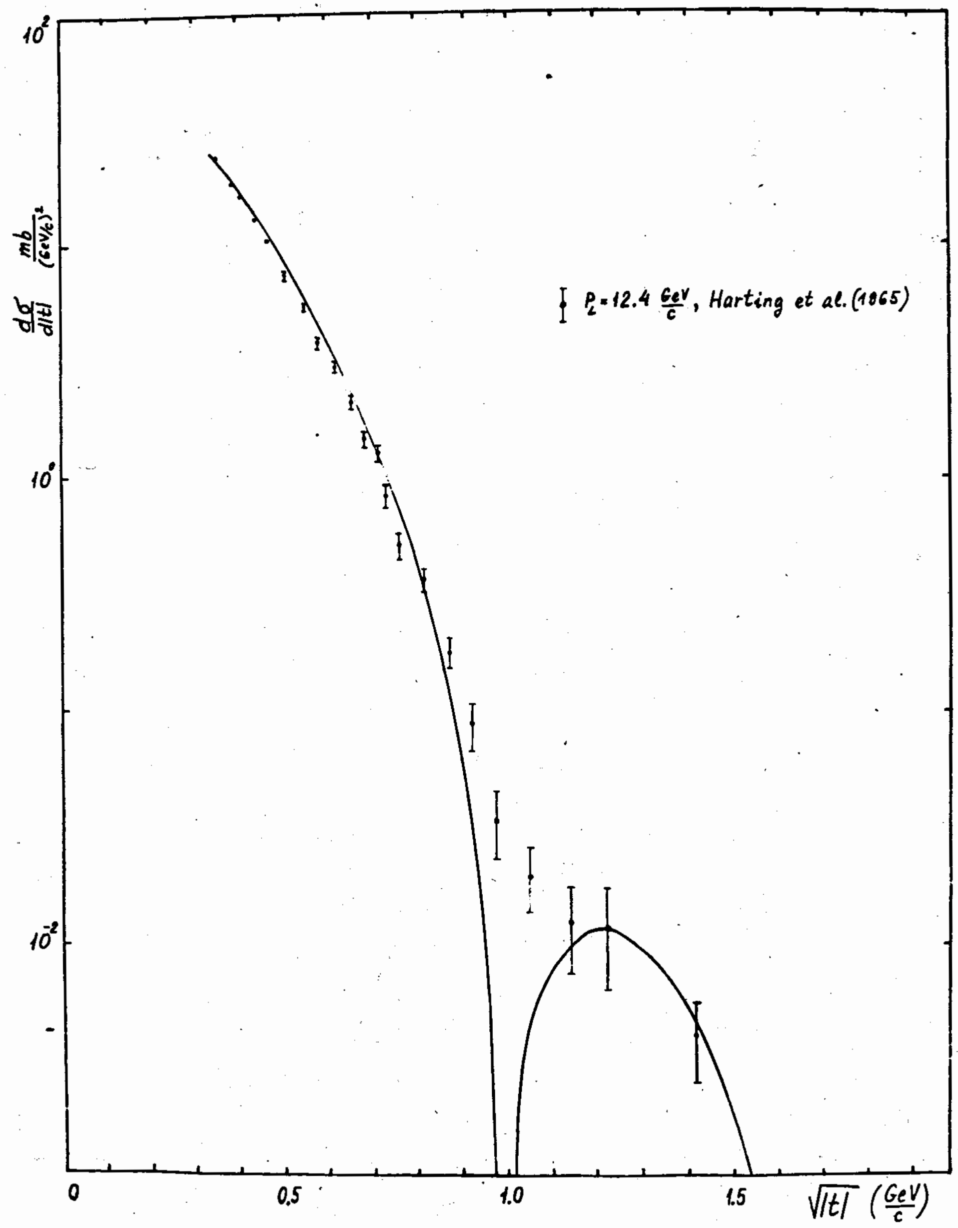

Fig.1. pp -scattering at p_L = 12.4 GeV/c.

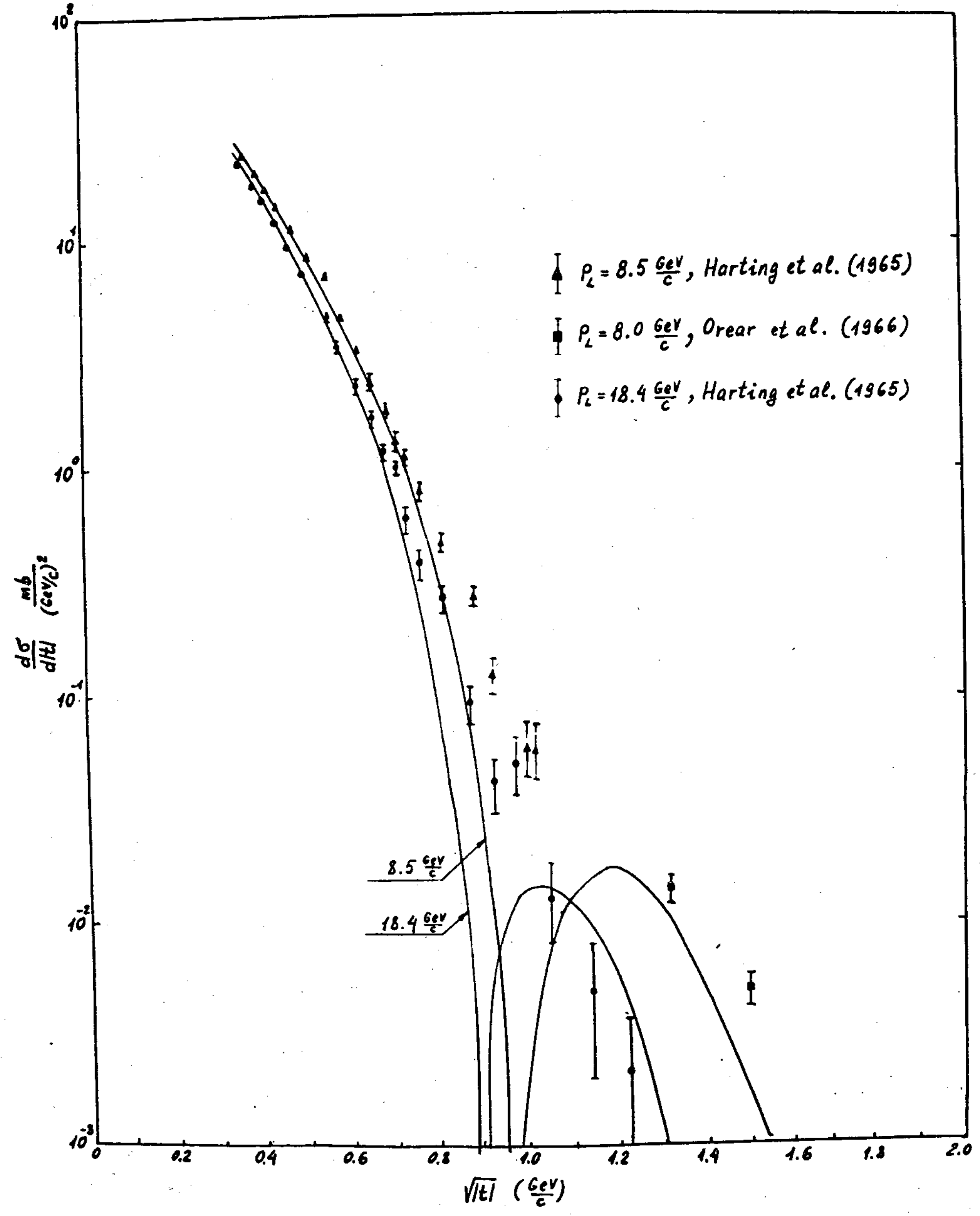

Fig.2. pp -scattering at p_L =8.5 GeV/c and 18.4 GeV/c.

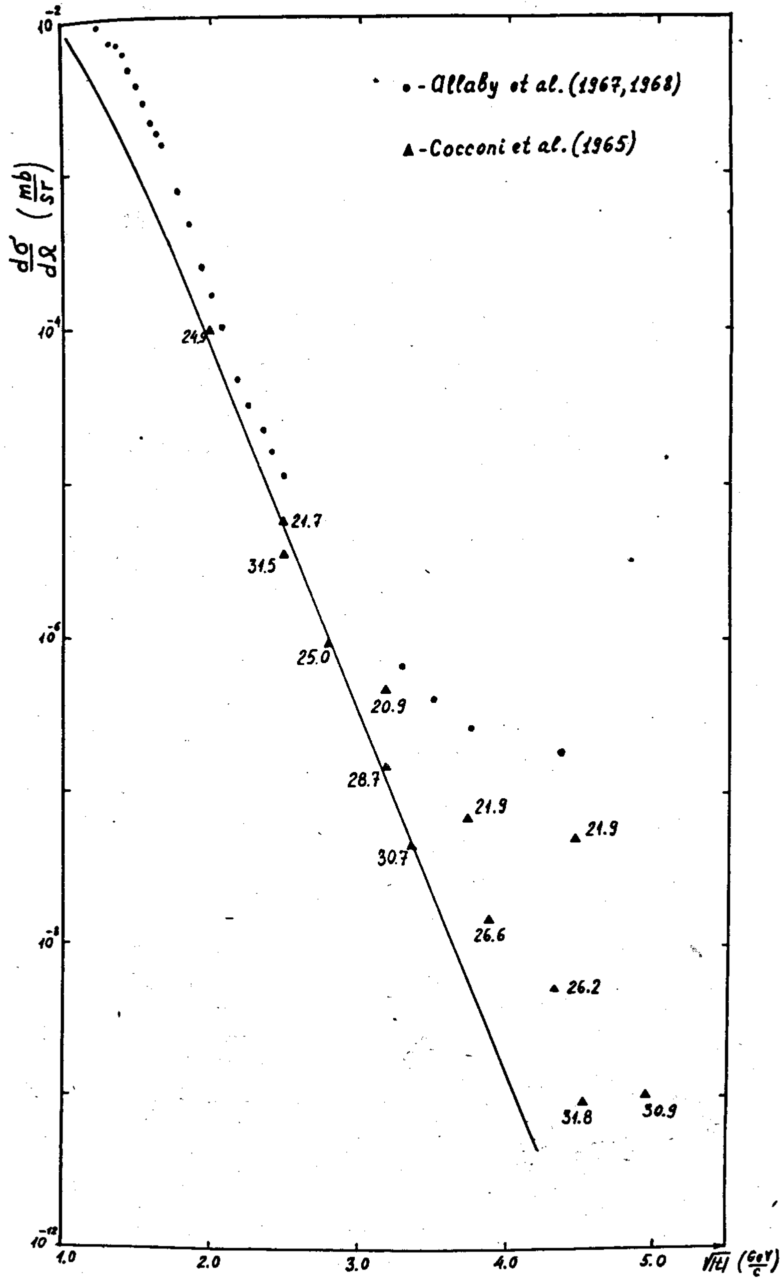

Fig.3. pp -scattering at large angles.

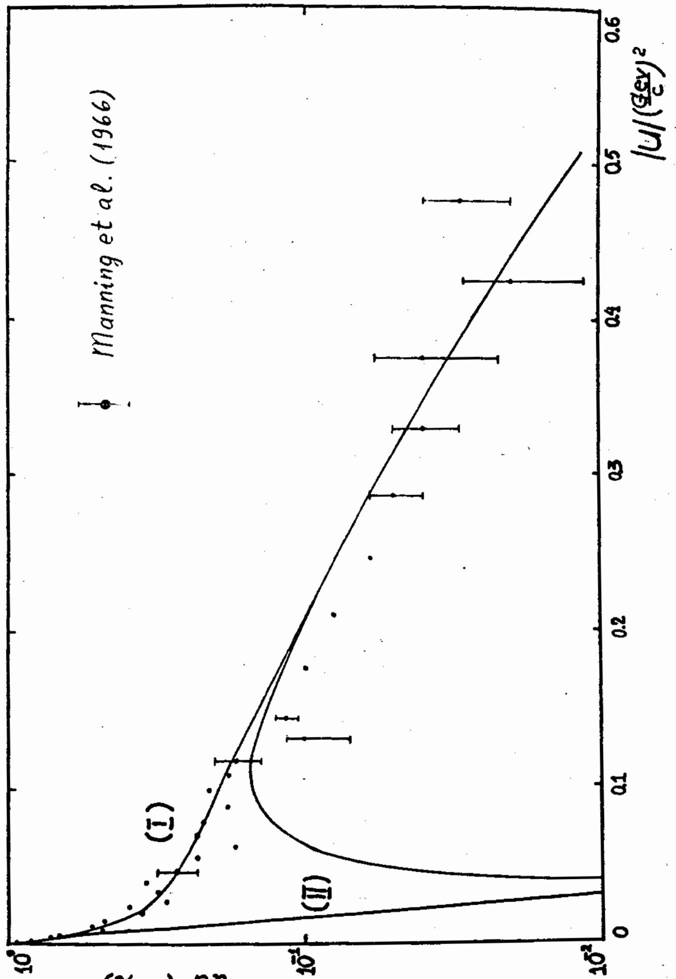

Fig.4. np- backward scattering at p_L=8.0 GeV/c.

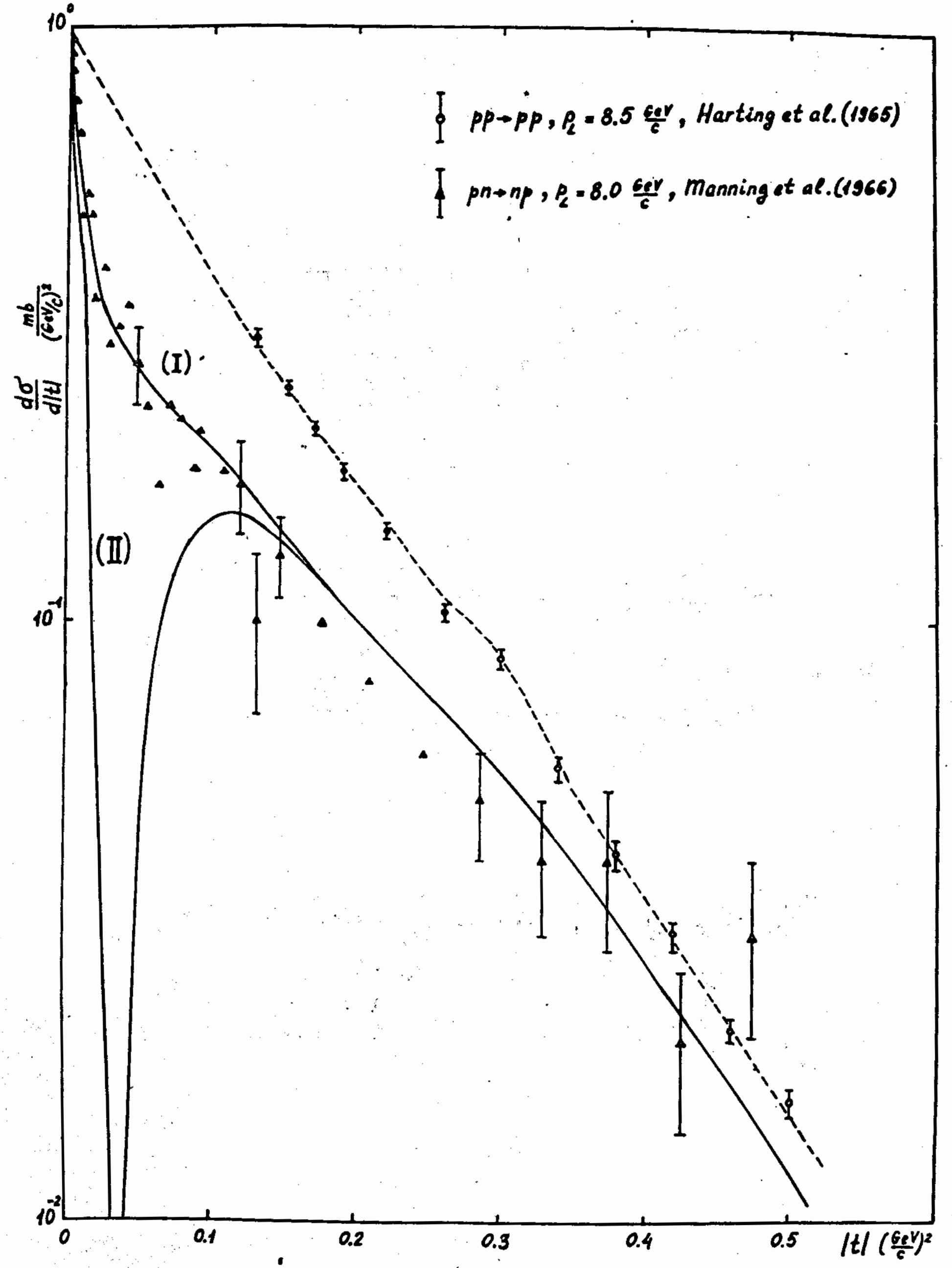

Fig.5. np -charge exchange at p_L =8.0 GeV/c and pp -elastic scattering at p_L =8.5 GeV/c. (Cross section of pp -scattering is normalized to 1mb/(GeV/c)^2 at t = 0).

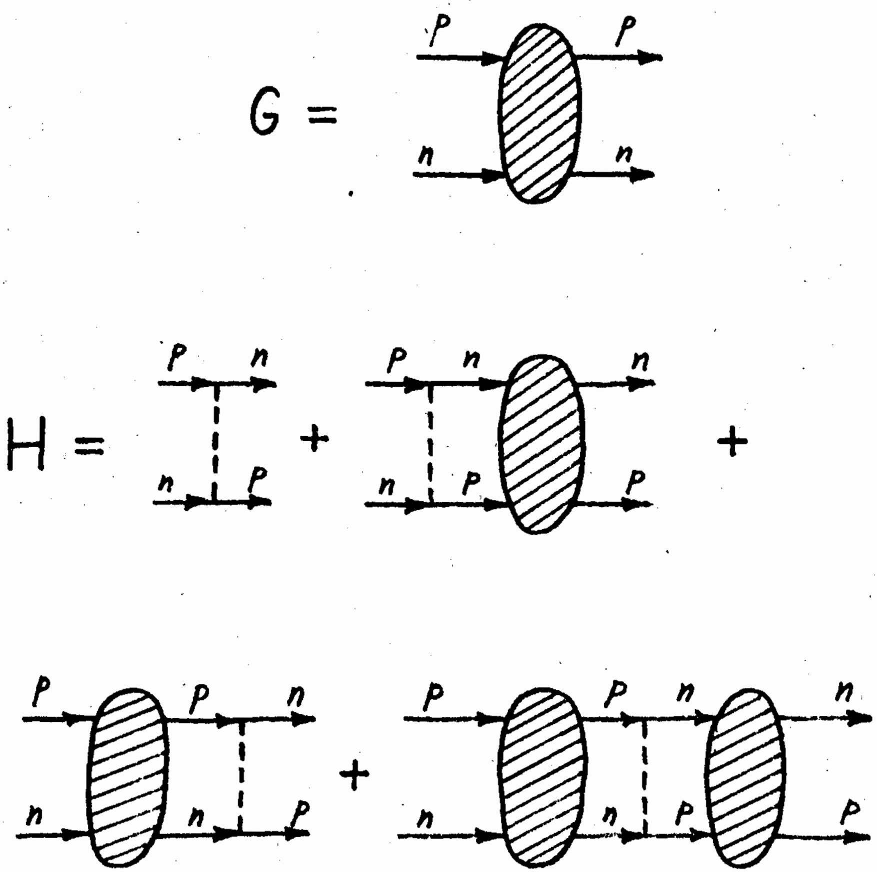

Fig. 6.