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RELATION BETWEEN THE SPIN AND ISOSPIN DEPENDENCE OF RADIATIVE TRANSITIONS $\gamma + N \rightarrow N^*$

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The aim of this note is to point out, that in the framework of the quark model one can obtain the following sum rule, which relates the spin and isospin dependence of the photoexcitation reactions $\gamma + N \to N^*$:

$$J_a^V\left(\frac{3}{2}\right) = J_p^V\left(\frac{1}{2}\right) - \frac{1}{3}\left(J_p^P - J_p^V\right). \qquad (1)$$

The left-hand side of eq. (1) is the integral over the bremsstrahlung-weighted excitation cross section of all Δ-resonances (with isospin $I = 3/2$) by circularly polarized photons with their spins antiparallel (subscript "a") to that of a nucleon. The right-hand side of eq.(1) includes the excitation of all N-resonances ($I = 1/2$) by photons with spins parallel (subscript "p") to that of the target nucleon. The superscript "P" denotes the proton and "V" corresponds to the "isovector" photons (for comparison see also eqs. (3)-(6)).

Our basic assumptions are as follows. Firstly, we assume that the (low-lying) baryonic states result from the binding of three quarks /1/ and the transition current governing the reaction $\gamma + N \to N^*$ is the operator function of the generalized coordinates and effective coupling constants of three quarks. The second important point is the explicit form of the commutator

$$[\rho^i(\vec{x},t), \rho^j(\vec{y},t)] = i\epsilon_{ijk}\, \rho^k(\vec{x},t)\,\delta^{(3)}(\vec{x}-\vec{y}), \qquad (2)$$

where $\rho^i(x)$ is the time component of the isospin current. Eq. (2)

implies the absence of the "non-minimal" terms in current e.g. anomalous magnetic moments of quarks. Hence, we follow Bogolubov et al.[2] in explaining large magnetic moments of quarks in terms of the small "effective" mass of quarks in the bound state .

Applying now the standard techniques of the dipole moment algebra at the infinite momentum [3-5], we obtain the following sum rules:

$$2\pi^2 \alpha\left(\frac{1}{3}\langle r_1^2\rangle_B^V - \frac{1}{4M^2}(\kappa^2)_B^V\right) = \int \frac{d\omega}{\omega}\left(2\sigma_{1/2}^V(\omega) - \sigma_{3/2}^V(\omega)\right) \equiv 2J^V\left(\tfrac{1}{2}\right) - J^V\left(\tfrac{3}{2}\right) \quad (3)$$

$$2\pi^2 \alpha \frac{1}{4M^2}(\kappa^2)_B^V = \int \frac{d\omega}{\omega}\left(\sigma_p^V(\omega) - \sigma_a^V(\omega)\right) \equiv J_p^V - J_a^V , \quad (4)$$

$$2\pi^2 \alpha \frac{1}{M^2}(\kappa^2)_B^P = \int \frac{d\omega}{\omega}\left(\sigma_p^P(\omega) - \sigma_a^P(\omega)\right) \equiv J_p^P - J_a^P , \quad (5)$$

$$4\pi^2 \alpha\left(\frac{1}{3}\langle r_1^2\rangle_B^P - \frac{1}{4M^2}(\kappa^2)_B^P\right) = \int \frac{d\omega}{\omega}\sigma^P(\omega) \equiv J^P , \quad (6)$$

where $\alpha^{-1} = 137$, M is the nucleon mass, $\sigma = \frac{1}{2}(\sigma_p + \sigma_a)$, $\sigma^V = \sigma_{1/2}^V + \sigma_{3/2}^V$, $-\frac{1}{6}\langle r_1^2\rangle = F_1'(0)$, $\kappa = F_2(0)$, $F_{1,2}$ are the Dirac form-factors and the meaning of J's in eqs. (3)-(6) is selfexplanatory. The subscript "B" is used to recall, that all cross sections in eqs. (3)-(6) are those of resonance photoexcitation of the three-quark states and $\langle r^2\rangle_B$ and $(\kappa^2)_B$ are generally different from $\langle r^2\rangle_{exp}$ and κ^2_{exp} due to the proper radii of the quarks and the exchange meson currents.

Eqs. (3) and (4), (5) are formally analogous to the general sum rules for the isovector radius[3] and the anomalous magnetic moments [6-8]. Eq. (6) is the peculiar quark model sum rule. It was derived by Gottfried [9] by a somewhat different method. Essential points here are the full symmetry (or full antisymmetry) of the radial wave function of nucleons and the specific charges of the quarks composing the proton. Making use of the symmetry property, we get for the neutron radius

$$\langle r_1^2\rangle_B^N = 0 \quad (7)$$

so that

$$\langle r_1^2\rangle_B^V = \langle r_1^2\rangle_B^P . \quad (8)$$

Eq. (1) follows immediately, if we substitute eq.(4) into eq.(3), eq.(5) into eq.(6) and take into account eq.(8). We turn now to the evaluation of sum rule (1). Define $r_{p(a)}$ by writting

$$\sigma_{p(a)} = r_{p(a)}\,\sigma . \tag{9}$$

For a given partial wave the r^n's take the form

$$r_p^{n\pm} = \frac{n(n+2)(1-\gamma_{n\pm})^2}{(n+1)[\,n+(n+2)\gamma^2_{n\pm}\,]} , \tag{10}$$

$$r_a^{n\pm} = \frac{[\,n+(n+2)\gamma_{n\pm}\,]^2}{(n+1)[\,n+(n+2)\gamma^2_{n\pm}\,]} , \tag{11}$$

where $n = J - 1/2$, J is the total angular momentum,

$\gamma_{n_+} = E_{n_+}/M_{n_+}$, $\gamma_{n_-} = -M_{(n+1)-}/E_{(n+1)-}$, $n \geq 1$.

The notations for $E_{\ell\pm}$ and $M_{\ell\pm}$ are those of CGLN/10/. To get eqs.(9) and (10) the use of some general formulas contained in the Walker's paper/11/ is most helpful. All but the $\Delta(1236)$ resonance contributions will be treated below in the narrow-width approximation

$$\int_{res} \frac{d\omega}{\omega}\,\sigma(\omega) = 4\pi^2(2J+1)\,\Gamma_\gamma\,[M^*/(M^{*2}-M^2)]^3 , \tag{12}$$

where Γ_γ is the radiative width of a given resonance with spin J and mass M^*. To calculate the Γ_γ's we take the resonance values of $\sigma(\gamma N \to \pi N)$ from the Walker's fit/11/ and other resonance parameters (M^*, Γ_{tot}, etc) as given by Rosenfeld et al./12/. The result is

$$\Gamma_\gamma[\Delta(1236)] \approx 0{,}62\ \text{MeV},\quad \Gamma_\gamma[N(1518)] \approx 0{,}47\ \text{MeV}, \tag{13}$$

$$\Gamma_\gamma[N(1680)] \approx 0{,}044\ \text{MeV},\quad \Gamma_\gamma[N(1690)] \approx 0{,}37\ \text{MeV}.$$

Finally, we have

$$J_a^V(\tfrac{3}{2}) = (80-105)[\Delta(1236)] + \ldots = (80-105)\,\mu b + \Delta , \tag{14}$$

$$J_p^V(\tfrac{1}{2}) = (67-70)[N(1518)] + 5[N(1680)] + 42[N(1690)] + \ldots = \tag{15}$$

$$= (114-117)\,\mu b + N ,$$

where Δ and N are still unknown contributions of higher excited states of a nucleon. The lower and upper bounds in eqs. (14) and (15) correspond to the variation of γ's in the limits[11]: $-0{,}04 \le E_{1+}/M_{1+} \le 0$ and $0{,}5 \ge M_{2-}/E_{2-} \ge 0{,}33$. We use also $M_{3-}/E_{3-} = -E_{2+}/M_{2+} = 0{,}5$. Thus, the "spin-isospin-flip" sum rule (1) is not inconsistent with available data. In conclusion we note, that the analogous consideration can be applied to other hadrons. For example, using the quark model relation

$$\langle \vec{D}^2 \rangle^{\pi^+} + \frac{4}{5}\langle \vec{D}^2 \rangle^{\pi^0} = \langle r^2 \rangle^{\pi^+} , \tag{16}$$

and combining the relativistic "dipole moment fluctuation" sum rule, such as eq.(6), and the Cabibbo- Radicati sum rule[3] for pions, we get

$$9J(\gamma + \pi^+ \to I^G = 1^+) = J(\gamma + \pi^0 \to I^G = 0^-), \tag{17}$$

with the obvious notations.

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