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RELATIVISTIC QUASIPOTENTIAL MODEL OF PARTICLE SCATTERING AT HIGH ENERGIES

In this note we consider the high-energy scattering of particles on the basis of the quasipotential equation /1/ for the scattering amplitude. For the sake of simplicity we shall study the case of an elastic scattering of two spinless particles of equal masses.

As was shown in ref. /1/, the scattering amplitude of two spinless particles in quantum field theory satisfies the following equation:

$$T(\vec{p},\vec{k};E) = V[(\vec{p}-\vec{k})^2;E] + \int \frac{d\vec{q}}{\sqrt{m^2+\vec{q}^{\,2}}} \frac{V[(\vec{p}-\vec{q})^2;E]\,T(\vec{q},\vec{k};E)}{\vec{q}^{\,2}+m^2-E^2-i0} . \tag{1}$$

Here E is the energy, and $\vec{p}$ and $\vec{k}$ are the centre-of-mass relative momenta of the initial and final states, respectively.

The physical scattering amplitude $T(s,t)$ is defined by the condition:

$$T(s,t) = 32\pi^3 T(\vec{p},\vec{k};E)\Big|_{\substack{s=4E^2=4(\vec{p}^{\,2}+m^2)=4(\vec{k}^{\,2}+m^2) \\ t=-(\vec{p}-\vec{k})^2}} . \tag{2}$$

The quasipotential in eq. (1) is a function of energy, the imaginary part of which is due to inelastic processes in the two-particle scattering. For the pure real potential the scattering amplitude satisfies the relativistic two-particle unitarity.

We note that in the case of weak coupling there exists a regular method for constructing the quasipotential using the perturbation expansion for the scattering amplitude in the framework of quantum field theory. For the strong interactions there are no general methods of constructing the quasipotential.

We shall consider a model of high-energy scattering of strongly interacting particles at small and large angles based on the phenomenological choice of the quasipotential in eq. (1).

It was shown in ref. /2/ that the high-energy large angle scattering of hadrons can be described as the scattering of a particle on the smooth complex quasi-potential in the "shadow" region. Such a model signifies essentially that the high-energy scattering of strongly interacting particles can be considered as an interaction of two "friable" systems /3/. From this point of view, we shall use in what follows the quasipotential of the Gaussian type:

$$V(s,\vec{r}) = i s g_0 \left(\frac{\pi}{a}\right)^{3/2} e^{-\frac{r^2}{4a}} , \tag{3}$$

where $g_0 > 0$. We note that the chosen potential provides for the constancy of the total cross section at high energies. The parameter a defines the effective range of the interaction region and, generally speaking, can increase with energy not faster than $\ln s$, i.e.

$$|a| \leq \ln s , \quad s \to \infty . \tag{4}$$

The quasipotential in eq. (1) being the Fourier transform of (3) is equal to

$$V(s,\vec{\Delta}^2) = \frac{1}{(2\pi)^3} \int d\vec{r}\, e^{i\vec{\Delta}\vec{r}}\, V(s,\vec{r}) = i s g_0\, e^{at} \tag{5}$$

$$t = -\vec{\Delta}^{\,2} .$$

Let us consider first the small angle scattering at high energies

$$a|t| < 1 ; \quad a s \gg 1 . \tag{6}$$

In this case eq. (1) can be solved by iteration procedure :

$$T(\vec{\Delta}^2 ; E) = V(\vec{\Delta}^2 ; E) + \delta T(\vec{\Delta}^2 ; E) + \dots \tag{7}$$

The expression for the first correction to the Born approximation is of the form:

$$\begin{aligned} \delta T(\vec{\Delta}^2 ; E) &= \int \frac{d\vec{q}}{\sqrt{\vec{m}^2 + \vec{q}^2}} \frac{V[(\vec{p}-\vec{q})^2 ; E]\, V[(\vec{q}-\vec{k})^2 ; E]}{(\vec{q}^2 + m^2 - E^2 - i0)} = \\ &= -\frac{\pi(s g_0)^2}{2a\lambda} e^{at/2} I(\vec{\Delta}^2 ; E), \end{aligned} \tag{8}$$

where

$$I(\vec{\Delta}^2 ; E) = \int_{-\infty}^{+\infty} \frac{q\,dq}{\sqrt{m^2 + q^2}} \frac{e^{-2a(q-\lambda)^2}}{q^2 + m^2 - E^2 - i0} ; \quad \lambda = \sqrt{\vec{p}^2 + t/4} . \tag{9}$$

Taking into account eq. (8) we get the following expression for the scattering amplitude:

$$T(E, \vec{\Delta}^2) = i s g_0 e^{at} - i s g_0 \frac{\pi^2 g}{a} e^{\frac{at}{2}} \tag{10}$$

The condition of validity of the Born approximation reads:

$$\frac{g_0 \pi^2}{a} < 1 ; \qquad a\,|t| < 2 \ln \frac{a}{\pi^2 g_0} . \tag{11}$$

The first term in eq. (10) describes the diffraction scattering at small momentum transfers with diffraction peak width $A \approx 2a$. Near the point

$$t = -\frac{2}{a} \ln \frac{a}{\pi^2 g_0} , \tag{12}$$

where the correction becomes comparable with the first Born approximation, a minimum can be observed in the differential cross section. From eq. (10) at $t=0$ we get for the total cross section

$$\sigma_{tot} = 32\,\pi^3 g_0 \left(1 - \frac{\pi^2 g_0}{a}\right). \tag{13}$$

If one assumes a logaritmical growth of the parameter a at high energies the total cross section (13) will tend to its asymptotic value $\sigma_{tot}(\infty) = 32\,\pi^3 g_0$ from below /4/.

Consider now the scattering outside the diffraction region, when

$$\left|\frac{t}{s}\right| \ll 1; \quad a|t| > 1. \tag{14}$$

In this case it can be shown that the solution of eq. (1) is given by a series

$$T = \sum_{n=1}^{\infty} T^{(n)}, \tag{15}$$

where

$$T^{(n)} = i s g_0 \frac{e^{at/n}}{n \cdot n!} \left(-\frac{4\pi^2 g_0}{a}\right)^{n-1} \tag{15'}$$

is the n-th perturbation term of eq. (1). We notice that the series (15) can be represented in the following form

$$T(s,t) = \frac{s}{(2\pi)^3} \int d^2\rho\, e^{i\vec{\Delta}_\perp \cdot \vec{\rho}} \left(\frac{e^{2i\chi} - 1}{2i}\right), \tag{16}$$

where

$$\chi = 2i \cdot \frac{\pi^2 g_0}{a}\, e^{-\vec{\rho}^{\,2}/4a} \tag{17}$$

and $\vec{\Delta}_\perp = (\vec{p} - \vec{k})$ varies in a plane, which is perpendicular to the vector $(\vec{p} + \vec{k})$. The function χ is related to the quasipotential (3) by

$$\chi = \frac{1}{s} \int_{-\infty}^{\infty} V(s, \vec{r})\, dz; \quad \vec{r} = (\vec{\rho}, z). \tag{18}$$

Thus, one can see, that the scattering amplitude, which is the solution of the eq. (1) with the quasipotential (3), satisfies the Glauber representation /7/ in the region (14). One can expect that this representation would be hold for the more wide class of smooth quasipotentials /8/. Let us consider an asymptotic behaviour of the series (15) at $a|t| \gg 1$.

Using the Sommerfeld-Watson transform we rewrite eqs.(15)(15') as follows

$$T(\vec{\Delta}^2; E) = \frac{as}{8\pi^2} \int_C \frac{dz\, e^{-\sigma z - \frac{\omega}{z}}}{2z \cdot \Gamma(z+1) \sin \pi z}, \tag{19}$$

where

$$\sigma = \ln\left(\frac{a}{4\pi^2 g_0}\right); \quad \omega = a|t|.$$

The integration contour C surrounds the positive real semi-axis in the clockwise direction and includes the integers $z = 1, 2, \ldots$

It may be shown that in the region (14) the main contribution to the integral (19) comes from the first term of the series expansion of the function $\frac{1}{\sin \pi z}$

$$\frac{1}{\sin \pi z} = \mp 2i \sum_{n=0}^{\infty} e^{\pm 2\pi i z (n + 1/2)} \tag{20}$$

on the upper and lower edges of the contour C respectively.

As a result we have

$$T(\vec{\Delta}^2, E) \underset{a|t| \gg 1}{\to} -\frac{isa}{4\pi^2} \frac{1}{\sqrt{a|t|}} \mathrm{Re} \left\{ \frac{e^{-2\sqrt{2a|t|}\, \mathrm{sh}\, \gamma + \frac{\gamma}{2}}}{\sqrt{\mathrm{ch}\, \gamma}} \right\} \tag{21}$$

where γ is determined by the equation

$$(e^{2\gamma}+2\gamma)=2(\sigma+\ln\sqrt{2\omega}-i\pi). \tag{22}$$

We see, that the scattering amplitude in the region (14) should have the Orear behaviour with oscillations /5,8/.

Now we study the high energy behaviour of the scattering amplitude at fixed scattering angles

$$\left|\frac{t}{s}\right| \approx \sin^2\frac{\theta}{2} = \text{fixed}. \tag{23}$$

In this case the Born series for the scattering amplitude reads:

$$T(\vec{\Delta}^2, E) \approx i s g_0 \sum_{n=0}^{\infty} \frac{(n+1)^{2n}}{(n!)^2} \frac{e^{\frac{at}{n+1}}}{(n+1)^{3/2}} \left(\frac{i s g_0 \pi\sqrt{\pi}\, e^{\phi(\theta)}}{at\cdot p\sqrt{a}} \right)^n , \tag{24}$$

where

$$\phi(\theta) = 1 + \mathrm{Re}\,\frac{(1-\beta)}{\beta}\ln(1-\beta)$$

$$\beta = 2i\sin\frac{\theta}{2}\,\bar{e}^{i\theta/2}.$$

Notice that in deriving the Born series terms (24) the main contribution is obtained by integrating in eq. (1) over the principle values i.e. by integrating off the energy shell. The function $\phi(\theta)$ is really small, for example

$$\beta(\theta=0)=0: \quad \phi(\theta=\pi/2)=0{,}215.$$

The asymptotic behaviour of (24) can be found by the saddle point method. Thus, neglecting $\phi(\theta)$ we get

$$T(\vec{\Delta}^2, E) \underset{\substack{s\to\infty \\ \theta=\text{fixed}}}{\to} C\cdot e^{-2\sqrt{a|t|}\cdot\sqrt{\ln\left(\frac{i a|t|\cdot p\sqrt{a}}{s g_0 \pi\sqrt{\pi}\, e^2}\right)}} \tag{25}$$

where

$$C = -\frac{p\sqrt{a}}{\pi^2}\sqrt{\ln\left(\frac{i a|t|\cdot p\sqrt{a}}{s g_0 \pi\sqrt{\pi}\, e^2}\right)}.$$

We note that eq. (24) differs from the corresponding result of ref/2,6/ only by an unessential factor $4/e^2$ under the logarithmic sign.

Thus in the model considered the behaviour of the small and large angle elastic scattering amplitude for spinless particles at high energies is determined by the two parameters g_0 and a entering the definition of the quasipotential.

In principle, these parameters can be determined from the experimental data at small and vanishing momentum transfer, i.e. from the total cross section σ_{tot} and the diffraction peak width A in the following manner

$$\sigma_{tot} = 8\pi a\, I(x); \tag{26}$$

$$A = \frac{d}{dt}\left[\ln\frac{d\sigma}{dt}\right]_{t=0} = 2a\,\frac{1}{I(x)}\int_0^x \frac{d\xi}{\xi} I(\xi), \tag{27}$$

where

$$I(x) = -\sum_{n=1}^{\infty}\frac{(-x)^n}{n\cdot n!} = \int_0^x \frac{d\xi}{\xi}(1-e^{-\xi})$$

$$x = \frac{4\pi^2 g_0}{a}. \tag{28}$$

Note that in order to describe the real physical processes it is necessary, generally speaking, to take into account the spin and isospin dependence of the scattering amplitude. Such a problem may be solved in the framework of the multi-channel quasipotential equation.

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