ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 4189

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PARTIAL SYMMETRY, EQUIVALENCE THEOREMS AND CALCULATION OF HIGHER ORDERS IN NONRENORMALIZABLE FIELD THEORIES

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1968

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The classification of field theories into renormalizable (R) and non-renormalizable (N-ones) was essentially introduced by W.Heisenberg in 1936 /1/. This classification is based on a substantially different dependence of the higher order perturbation theory terms on a cut-off parameter. Logarithmic divergences in R -theories may be eliminated by a renormalization procedure while power law divergences in N -theories allow no cut-off independent perturbation theory calculations /2/. So the first attemps of calculations in N -theories started from assumption of some self-damping on the so-called "unitary limit" /1/, i.e. for values of momenta $|p^2| \gg D^2 = \frac{1}{G}$, where is a dimensional coupling constant (we shall, for definiteness, speak of the weak interaction). The unitary cut-off is however completely inconsistent with perturbation theory because the ratio between two successive terms of the perturbation expansion is of the order $GD^2 \approx 1$. For this reason attemps were undertaken to find experimental limitations on D the result being /3/ $GD^2 \ll 1$. The origin of such a low cut-off was completely unconceivable /4/ and for this reason different ways out of this unpleasant situation were searched for.

The essentially two possibilities were discussed up to now:

1) the theory of weak interactions should be modified in such a way that a cut-off was intrinsically low (e.g. through a strong interaction of the intermediate W-bosons) /5/;

2) a new theory of weak interactions should be constructed so that the unitary cut-off contributions to (up to now) observable quantities were small/6/. In both cases the present theoretical scheme of the weak interactions should be abandoned, its compactness and aesthetic attraction/7/ being lost.

It should be particularly emphasized that there exists the third possibility of avoiding the cut-off difficulties in the theory of the weak interaction: the theory should not be modified, the self-damping begins above the unitary limit, but the usual perturbation theory is completely inadequate because the amplitudes have a logarithmic branch point at $G=0$. In this case the higher order corrections may be small and of the order $G^2 \log^m G^2$, $m=0,1,..$ This possibility was first pointed out by T.D.Lee/8/ and since then different models of such a kind were invented/9,10/. We want to stress that this third possibility may be realized not in any theory. (For example, in the Feinberg-Pais/11/ model the higher order corrections are not small for some processes). We shall see later that a more refined (than one into R and N theories) classification of the field theories should be considered to incorporate this circumstance. To find a criterion for such a classification we consider first one highly typical mathematical model of the N-theory, the physical meaning of it being established later.

The model is defined by the invariant function $f(p^2)$, where p_μ is Euclidean 4-momentum $(p^2 \equiv p_\mu^2 \equiv p_0^2 + \vec{p}^{\,2})$. Consider the equation for this function

$$f(p^2) = Z^{-1} + g^2 \int \frac{d^4q}{(2\pi)^4} \, \frac{q^2 - (qp)}{q^2 (q-p)^2} \, f(q^2). \tag{1}$$

Equations of such a type are characterisric of many N-theories. One theory giving exactly Eq. (1) will be considered later. After integration over the angle variables in a four-dimensional spherical coordinate system Eq. (1) can be written in the form $(\lambda \equiv g/4\pi)$:

$$f(x) = Z^{-1} + \frac{\lambda^2}{2} \int_0^\infty dy \left[\frac{y}{x}\,\theta(x-y) + \left(2 - \frac{x}{y}\right)\theta(y-x) \right]. \tag{2}$$

Making a cut-off on the upper limit $y = D \gg x$ and solving Eq. (2) by iterations we find

$$f(x) = Z^{-1} \Big\{ 1 + \frac{\lambda^2}{2} \left[2D - \frac{3}{2}x - x \log\frac{D}{x} \right] +$$
$$+ \frac{\lambda^4}{4} \left[2D^2 - 2Dx \log\frac{D}{x} - \frac{1}{2}Dx - \frac{x^2}{3}\log\frac{D}{x} - \frac{10}{9}x^2 \right] + \ldots \Big\} . \tag{3}$$

If D is of the order of $1/\lambda^2$ (unitary limit) then the higher order corrections are not small as compared to lower order ones. So we attemp to make a partial summation of the series (3). With this aim we use at first the fundamental idea of the "peratization", i.e. we neglect in Eq. (1) all the terms but "most divergent" ones. Denoting the corresponding approximate f-function by f_{FP} we find for it the equation

$$f_{FP}(p^2) = Z^{-1} + g^2 \int \frac{d^4q}{(2\pi)^4} \frac{q^2 f_F(q^2)}{q^2(q-p)^2} \tag{4}$$

or after integration over the angle variables

$$f_{FP}(x) = Z^{-1} + \lambda^2 \int_0^\infty dy\, f_{FP}(y) [\frac{y}{x} \theta(x-y) + \theta(y-x)]. \tag{5}$$

Iterative cut-off solution of Eq. (5) is

$$f_{FP}(x) = Z^{-1}\{1 + \frac{\lambda^2}{2}[2D - x] + \frac{\lambda^4}{4}[3D^2 - 2Dx + \frac{1}{3}x^2] + \ldots\}. \tag{6}$$

So we see that Eq. (4), which is often considered as an equation giving correctly all the most divergent terms in Eq. (3), gives us in fact correctly only the exponent of the most divergent terms but not its coefficient. We have pointed out earlier/12/ that all the three terms in Eq.(2) are essential for finding the correct asymptotic behaviuor of $f(x)$ for $x \to \infty$. Now we see that all these terms should be taken into account if we want to find correctly the most divergent terms in the perturbative series. Therefore we may conclude that the fundamental assumption of "peratization" method is essentially groundless. Nevertheless, it is instructive to investigate the question of the existence of the exact solution to Eq. (5). The equation is reduced to the boundary value problem

$$\phi''(x) + \frac{\lambda^2}{x}\phi(x) = 0; \quad \phi'(\infty) = Z^{-1}; \quad [\phi(x) - x\phi'(x)] \underset{x\to\infty}{\to} 0; \tag{7}$$

$$\phi(x) \underset{x\to\infty}{\to} 0 \quad ,$$

where $\phi(x) = x f(x)$.(We point out that in this case the conditions near $x=0$ may not be taken into account because the approximate equation (5) roughly disturbs the behaviour of $f(x)$ at this point). It is not hard to verify that the boundary condition at $x=0$ can be satisfied but the condition at $x=\infty$ cannot be satisfied. If we change λ^2 in sign then there exists a solution decreasing for $x\to\infty$ and it has the form $\phi_{FP}(x) = c\sqrt{|\lambda^2|x}\ K_1(2\sqrt{|\lambda^2|x})$. The boundary condition at $x=0$ is not satisfied and that one at $x=\infty$ is satisfied if $Z^{-1}=0$. It follows from here and from dimensionality considerations that the solution, if it exists, must have the form $f_{FP}(x)=c'(|\lambda^2|x)^{-1/2}\ K_1\ [2\sqrt{|\lambda^2|x}]$, where c' is some number. We obtained therefore the strong singularity in λ^2 at the point $\lambda^2=0$ (not only logarithmic branch point but also the pole). Turning to the exact equation, we will see shortly that it bears no resemblance to the approximate FP-equation (5). In fact, eq. (1) can be reduced to the boundary value problem

$$x^3 f''' + 3x^2 f'' = \lambda^2 x f ; \quad x(xf)'' - 2x(xf)' + 2(xf) \underset{x\to 0}{\to} 0 ; \tag{8}$$

$$x(xf)'' - (xf)' \underset{x\to\infty}{\to} 0 ; \qquad (xf)'' \underset{x\to\infty}{\to} Z^{-1} .$$

It is not hard to verify that this problem has a solution if and only if $xf \underset{x\to 0}{\to} 0$, $xf \underset{x\to\infty}{\to} 0$; $Z^{-1} = 0$. This solution normalized by the condition $f(0) = 1$ may be written in the form

$$f(x) = G^{20}_{03}(\lambda^2 x \mid 1,0,-1) \equiv 1 + \lambda^2 x \log\lambda^2 x \sum_{n=0}^{\infty} \frac{(\lambda^2 x)^n}{n!(n+1)!(n+2)!} +$$

$$+ (-\lambda^2 x) \sum_{n=0}^{\infty} \frac{(\lambda^2 x)^n [\psi_n + \psi_{n+1} + \psi_{n+2}]}{n!(n+1)!(n+2)!} . \tag{9}$$

Here $\psi_n = \frac{\Gamma'(n+1)}{\Gamma(n+1)}$ and G^{30}_{03} is the Meijer function /13/. The solution has the following asymptotic behaviour in the complex z-plane ($z \equiv \lambda^2 x$) for $|z| \to \infty$ /13/

$$f(z) \approx -\frac{i}{\sqrt{3}} H(z e^{i\pi}), \quad 0 < \arg z < 3\pi;$$

$$f(z) \approx \frac{i}{\sqrt{3}} H(z e^{-i\pi}), \quad -3\pi < \arg z < 0 \tag{10}$$

$$f(z) \approx \frac{-i}{\sqrt{3}} H(z e^{i\pi}) + \frac{i}{\sqrt{3}} H(z e^{-i\pi}), \quad \arg z = 0$$

$$H(z) \equiv z^{-1/3} e^{-3z^{1/3}} \{1 + \frac{a_1}{z^{1/3}} + \frac{a_2}{z^{2/3}} + \ldots\}.$$

Under close examination of the example considered above the interesting general conclusion may be inferred (but not proved!): 1) the most singular at infinity FP-equation has no direct connection with the exact equation 2) up to an arbitrary factor, defined by physical normalization conditions, the solution can be expanded in a series of powers of the coupling constant λ^2 and of $\log \lambda^2$; this series contains no divergent terms and the higher order corrections are small for λ^2 small enough; 3) the self-damping cut-off is characterized by the parameter $D^2 \approx 1/\lambda^2$ and only for $p^2 \gtrsim D^2 \approx 1/\lambda^2$ the higher order corrections become significant; 4) the mathematical origin of the divergences, occurring in the perturbative solution and of the impossibility of removing them by usual methods, lies in the presence of the logarithmic branch point.

There are many reasons to believe that the similar features are characteristic of any theory with some restrictions on the behaviour of $f(x)$ at infinity ($p^2 \to \infty$). The nature of such restrictions may be demonstrated by a comparison between the asymptotic behaviour of $f(x)$ and of $f_{FP}(x)$.

The conditions of the localizability of the theory (cf. Jaffe[14] or the earlier approach by Meyman[15]) require the Green's function to have such an asymptotic behaviour that

$$\int_1^\infty dx \frac{\log^+|f(x)|}{x^{3/2}} < \infty , \quad \log^+|x| \equiv \begin{cases} \log|x| , & |x| > 1 \\ 0 & |x| \leq 1. \end{cases} \tag{11}$$

Our exact solution f satisfies this criterion but f_{FP} does not satisfy it. Investigations of other examples (see e.g.[16]) allow one to assume that the features 1) -4) are characteristic of localizable N-theories but not of nonlocalizable ones. So we suggest as a likely hypothesis: in localizable theories the higher order corrections are small and can be calculated by a modified perturbation theory which takes into account the presence of the logarithmic branch point at $\lambda^2 = 0$; the usual perturbation expansion with a unitary cut-off gives, in this case, quite incorrect impression of the significance of the higher order corrections.

The highly non-trivial nature of the localizability condition should be particularly emphasized. As a matter of fact, we have considered above the model in which a simple modification of Eq.(1) does not change the powers of divergences but principally changes the asymptotic behaviour (so the "localizable" equation is converted into the "nonlocalizable" one). We shall formulate a simple theorem about localizability of the equations of the Eq.(2) type. Let $f(x)$ satisfy the linear integral equation like (2). with M independent integrals in the right-hand side. Let the coupling constant g^2 have the dimension of $[m^2]^{-N}$. Then $f(x)$ is localizable if $N/M < 1/2$; if $N/M \geq 1/2$ it is nonlocalizable. The proof is as follows. The differential equation has the form $\sum_{n=0}^{M} x^n f^{(n)}(x) = g^2 x^N f(x)$ and its asymptotic solutions (obtained by WKBJ method) are $f(x) \sim \exp\{g^{2/M} \times x^{N/M}\}$ Q.E.D . Now the fundamental problem is to calculate M in different N-theories. We hope that M can be find by more careful investigation of perturbation theory but we do not know yet any simple rules for this. Now we turn to a discussion of general methods for treatment of N-theories. If the problem is reduced to some integral equation in the Euclidean momentum space then all the necessary calculations seem to be straightforward (some complications occuring in cases of increasing (for $p^2 \to \infty$) solutions will be discussed later). Our aim is, therefore, to find such integral equations for Green's functions which give us the correct asymptotic behaviour or, at least, are localizable or nonlocalizable simultaneously with exact equations. By considering simple exactly solvable models we infer that the "ladder-type" equations probably satisfy the last requirement /17/ (the corresponding approximation for one-particle Green's function is usually called the "string-approximation" /18/. Therefore, the summation of "ladder" diagrams could provide us with the effective tool for calculating higher order corrections in localizable N-theories /19/. For example, to find the g^2 order correction for a vertex part in some R-theory we need to calculate the diagram Fig.1b. In an N-theory the diagrams Fig.1c should be summed and then the expansion of the sum in a series of powers of g^2 and $\log g^2$ will give a correct expression for $g^2 \log^m g^2$ order terms. The diagrams Fig.1d give analogously correct $g^4 \log^m g^2$ order terms and do not change $g^2 \log^m g^2$ terms. Being quite general, the method of summation of ladder diagrams, however, has some serious shortcomings. Firstly, it is impossible to find the asymptotic behaviour of the exact Green's functions. Secondly, in theories with some gauge-type symmetry it destroys the corresponding gauge invariant structure. The existence of such a symmetry enables, however, to develop another approximation (cf. /20/). We present here some related ideas originated from results obtained by Umezawa, Kamefuchi, O'Raifertaigh Salam et al. /21/ for partial symmetries and equivalence theorems (Nelson, Dyson /22/).

Let us consider a theory with a Lagrangian represented in a form $L = L_s + L_M$. Here L_s contains nonrenormalizable terms but has such a symmetry that it can be transformed by some change of variables to a renormalizable Lagrangian. The Lagrangian L_M violates the symmetry but it has very simple form and in terms of the old variables is renormalizable. The case when L_M contains only the mass terms $-M\bar{\psi}\psi$, $-\frac{m^2}{2}\phi^2$, $-m^2\phi^+\phi$ etc. is of the particular interest and we shall refer this case as the partial symmetry of L . The most simple situation occurs if L_s itself defines the trivial S-matrix on the mass shell. We shall speak then of a partial supersymmetry. We shall consider here the simple model with the partial supersymmetry:

$$L_S = \bar{\psi}[\partial_\mu + ig(a + b\gamma_5)\partial_\mu\phi]\gamma_\mu\psi$$

$$L_M = -M\bar{\psi}\psi \quad . \tag{12}$$

Here ψ is the spinor field, ϕ is the neutral scalar or pseudoscalar field. By the change of variables

$$\psi = e^{-ig(a+b\gamma_5)\phi}\psi_0, \quad \bar{\psi} = \bar{\psi}_0 e^{ig(a-b\gamma_5)\phi} \tag{13}$$

L_s is reduced to the free Lagrangian for the fields ψ_0 and ϕ and this results in the identity $S=1$ (on the mass shell) for this Lagrangian[21]. The standard method for making calculations in this theory consists in exploiting of perturbation theory for the new field variables. So it requires a construction of perturbation theory for the essentially nonlinear Lagrangian $-M\bar{\psi}_0 e^{-2igb\gamma_5\phi}\psi_0$. We exploit here another method (see also the earlier paper[16]). For the beginning we will find and solve (in principle) the exact equations for the Green's functions for $M=0$ and then we will demonstrate how these functions can be used as the first approximation for the perturbation theory expansion in powers of M.

Let us introduce the generating functional

$$Z=\langle T^*\exp\{i\int d^4x[\bar{\eta}(x)\psi(x)+\bar{\psi}(x)\eta(x)+\phi(x)j(x)]\}\rangle. \quad (14)$$

Here $\eta, \bar{\eta}, j$ are external sources and the symbol T^* implies that partial derivatives of field variables should operate on the T-production as a whole (this rule can be verified by use of perturbation theory in the form presented in the books[2], see especially the book by Bogolubov and Shirkov). The equation for the field variables have the form

$$\begin{aligned} &i\gamma_\mu\partial_\mu\psi - M\psi + gO_\mu\psi\partial_\mu\phi = -\eta \\ &-i\partial_\mu\bar{\psi}\gamma_\mu - M\bar{\psi} + g\bar{\psi}O_\mu\partial_\mu\phi = -\bar{\eta} \\ &(\partial^2+m^2)\phi = j - g\partial_\mu(\bar{\psi}O_\mu\psi) = \\ &= j + ig(\eta\bar{O}\psi - \bar{\psi}O\eta) - 2Mgb\bar{\psi}\gamma_5\psi, \end{aligned} \quad (15)$$

where

$$O_\mu \equiv (a+b\gamma_5)\gamma_\mu, \quad O \equiv a+b\gamma_5, \quad \bar{O} \equiv a-b\gamma_5$$

For $M=0$ the equation for ϕ has the form

$$(\partial^2 + m^2)\phi = j + ig(\bar{\eta} O \psi - \bar{\psi} O \eta) \tag{16}$$

and the infinite system of the identities for the Green's functions, very much alike the generalized Ward identities in quantum electrodynamics /25/, may be deduced from here (in this deduction we follow the method used by E.S.Fradkin/24/). Rewriting Eq.(16) in the form

$$(\partial_x^2 + m^2)\frac{\delta Z}{\delta j(x)} = iZj(x) + ig\left[\bar{\eta}(x) O \frac{\delta Z}{\delta \bar{\eta}(x)} - \frac{\delta Z}{\delta \eta(x)} O \eta(x)\right] \tag{17}$$

we deduce from here the system of identities connecting the n-boson, $2m$ fermion vertex with the $(n-1)$-boson, $2m$ - fermion vertex. For the simple vertex Green's function $G(xy|\xi)$ we find (for example)

$$\begin{aligned} G(x\, y\,|\,\xi) &\equiv \langle T^* \psi(x)\bar{\psi}(y)\phi(\xi)\rangle \Big|_{j=\eta=\bar{\eta}=0} = \\ &= ig\left[D_0(\xi - x) O G(x-y) - D_0(\xi - y) G(x-y) O\right], \end{aligned} \tag{18}$$

where

$$G(x-y) \equiv \langle T^* \psi(x)\psi(y)\rangle\Big|_{j=\eta=\bar{\eta}=0};\ D_0(\xi-\eta) \equiv \langle T^* \phi(\xi)\phi(\eta)\rangle\Big|_{j=\eta=\bar{\eta}=0}$$

Any amplitude with n -boson and two fermion ends can be written by use of these identities in terms of one particle Green's function $G(x-y)$, which satisfies some linear equation. Consider for simplicity the case $a \neq 0,\ b=1$. Then the equation for $G(x-y)$ is reduced to Eq.(1) with the opposite sign of g^2 $(g^2 < 0)$. It is not hard to verify that the corresponding boundary value problem has no solution and so we need some new recipe for the construction of the solution.

The integral equation is not well defined without regularizations. Making use of some regularization one can reduce the integral equation to the boundary problem for the differential equation. Then the regularization may be removed, the boundary condition at $x = 0$ being unchanged where - as the boundary condition at $x = \infty$ essentially changes. In a case, when the condition at $x = \infty$ does not make sense, we are forced to give up this condition and to seek for some substitution of it by some new asymptotic condition. One possibility is to reverse the sign of λ^2. Then the boundary value problem coincides with that of Eq.(7) which makes sense and has the unique solution. But trying to come back to the physical sign of λ^2, we meet the difficulty of the logarithmic branch point at $\lambda = 0$ which does not allow one to make the analytic continuation without a violation of the reality condition for $f^*(x), (x>0)$. (This point was missed in the paper of Arnowitt and Deser/27/ see corresponding remarks in the paper of Barbashov and Efimov/27/).

Let us try another consideration which incidentally will enable us to find the natural recipe of connecting $\lambda < 0$ with $\lambda > 0$. It is not a difficult task to verify that the differential equation uniquely determines the imaginary part of $f^*(x) (\mathrm{Im}\, f^*(x))$ on the cut $x < 0$ (with the bounda-

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In the Minkowski momentum space we have $G(p) = -\hat{p}/p^2 f^*(p^2)$ and $f^*(p^2)$ satisfies the Eq.(2) with λ^2 changed in sign.

ry condition at $x=0$ taken into account). Therefore we try to require the asymptotic behaviour of $f^*(x)$ on the cut to be uniquely determined by that of $\mathrm{Im}\, f^*(x)$ i.e. essentially by the inelastic processes contribution. This condition may be written as $\mathrm{Re}\, f^*(x)/\mathrm{Im}\, f^*(x) \underset{x\to-\infty}{\to} 0$. It is easy to prove that this condition gives the unique solution

$$f^*(x)=\frac{1}{2}[G^{20}_{03}(\lambda^2 e^{i\pi}x|1,0,-1)+G^{20}_{03}(\lambda^2 e^{-i\pi}x|1,0,-1)]. \qquad (19)$$

At the same time we found the recipe for connecting $\lambda^2>0$ with $\lambda^2<0$. In fact, for the solution $f_{\lambda^2}(x)\equiv f(x)$ decreasing for $x\to+\infty$, $\lambda^2>0$ we have the relations

$$\begin{aligned} f^*(x)&\equiv f_{-\lambda^2}(x)=\frac{1}{2}[f_{\lambda^2 e^{i\pi}}(x)+f_{\lambda^2 e^{-i\pi}}(x)] \\ f(x)&\equiv f_{\lambda^2}(x)=\frac{1}{2}[f_{-\lambda^2 e^{i\pi}}(x)+f_{-\lambda^2 e^{-i\pi}}(x)]. \end{aligned} \qquad (20)$$

The solution so constructed is single-valued function of λ^2 in the whole complex λ^2 plane but it is not analytic in λ^2 variable. If we would use no boundary condition at $x=\infty$ then the solution was not unique, the series $c\sum_{n=0}^{\infty}\frac{(-\lambda^2 x)^{n+1}}{n!(n+1)!(n+2)!}$ with an arbitrary real factor c mignt be added. The consideration were discussed /14/ concerning the physical irrelevance of such an arbitraryness but we do not know any proof of this assertion. Any way, our boundary condition for $x\to\infty$ proves to be very useful in the quite another aspect, for it can be shown that it is convenient for constructing the Green's functions in coordinate space. We shall not consider here this problem because we do not need the coordinate representation. (For not time ordered functions the problem is solved in /14,38/).

By use of the identities found above we can find in the case $M=0$ the linear equations for the Green's function with an arbitrary number of ends. So we may consider all the Green's functions to be known for $M=0$. These functions can be used for calculations of the observable quantities for $M=0$. (by use of perturbation theory with the term L_M as the perturbation). For example, in the first order (in powers of M) the one meson Green's function is expressed in terms of the meson-fermion scattering amplitude for $M=0$ and the last one is expressed in terms of G by use of our identities. A possibility of the consistent development of such a new perturbation theory is in principle obvious but requires a lot of calculations. We mention here one simple approximation without any attempt of proving it. Let us find all the irreducible $M=0$ vertex functions , i.e. all the Green's functions with the dressed external lines being removed. In the case $M=0$ they define the trivial S-matrix on the mass shell. However, if we substitute into external lines the real physical particles with the real physical mass $M=0$, we shall find some nontrivial expression for the S-matrix. It is interesting that this expression (for M being small enough) coincides with that one derived by use of equivalence theorems[21,22]. So, it is possible that this simple approximation is not so stupid as it seems for the first sight.

In conclusion we would like to mention some applications of the methods developed here. The domain of the applicability of these methods seems to be large enough but we consider as being most prominent the applications to theories with partial symmetry. Many examples of such theories were considered in the literature[21] and one of the most interesting applications is derivation of the contribution of the scalar part of the Yang-Mills field (in the

Schtueckelberg representation). These ideas were clearly outlined by Umezawa and Kamefuchi [21] and we hope that now it is possible to calculate some simplest observable effects in theories with a partial symmetry.

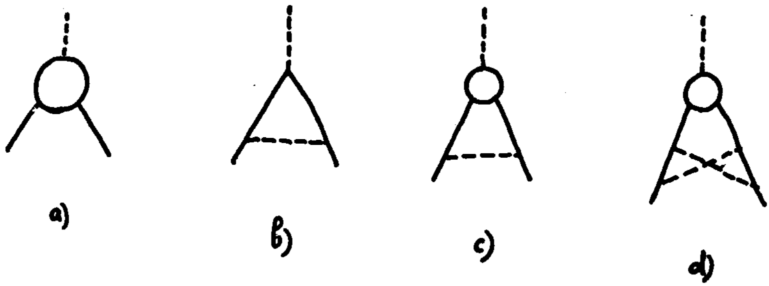

Fig.1

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Received by Publishing Department
on December 8, 1968.