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PHOTOPRODUCTION OF  $\Delta$  (1236)  
AND  $\pi$ -CONSPIRACY

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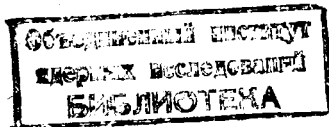
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## 1. Introduction

The problem of  $\pi$ -conspiracy in the nucleon-nucleon scattering from the standpoint of the Regge pole theory was discussed five years ago<sup>/1/</sup>. This problem is simply connected with the fact, that due to conservation of angular momentum in the forward direction in the s-channel only three t-channel amplitudes are independent at  $t=0$ . Recently this problem has been discussed in many papers<sup>/2-6/</sup>. In particular, in<sup>/7/</sup> it is proved, that in the reactions of the type  $\pi + N \rightarrow M + B$  where  $M$  denotes mesons with  $J^P = 1^-$  or  $2^+$   $B$  are baryons, in the absence of conspiracy all amplitudes with non-zero helicity transfer between  $M$ -mesons and pions show a minimum in the forward direction. For this reason the photoproduction processes provide a good test for the inspection of the existence of the  $\pi$ -conspirator. Of course, in the photoproduction of pions there are no amplitudes without helicity flip in the gamma-pi-reggeon vertex, therefore in the absence of conspiracy the differential cross section must show a strong minimum in the forward direction. By

means of conspiracy Frautchi and Jones<sup>/8/</sup> explain successfully the occurrence of a peak in the photoproduction of charged pions. From the analysis of the experimental data for the processes  $\pi + N \rightarrow M + B$  Jones<sup>/7/</sup> concludes that only the  $\pi$ -trajectory can have a conspirator. The narrow peaks in the processes  $n + p \rightarrow p + n$  (nucleon-nucleon charge exchange scattering) and  $\gamma + p \rightarrow \pi^+ + n$  at small  $t$  were explained also by  $\pi$ -conspiracy<sup>/3,9/</sup>. The normalization of  $\frac{d\sigma}{dt}$  at the optical point  $t=0$  requires a strong variation of the residue functions of the  $\pi$ -trajectory in comparison with the pion-nucleon coupling constant. Assuming linear dependence on the square of the momentum transfer, we conclude that these residue functions must vanish at  $t = -0,02 \left(\frac{GeV}{c}\right)^2$ <sup>/3,9/</sup>. From the factorization and real analyticity of the Regge-pole residues it follows that all pion vertex functions must vanish at this point. The residue functions in the process  $n + p \rightarrow p + n$  vanish at the point  $t = -0,05$ , which differs from the point  $t = -0,02$  in the process  $\gamma + p \rightarrow \pi^+ + n$ . The investigation of the processes of the type  $NN \rightarrow N\Delta$  and  $\gamma + p \rightarrow \pi\Delta$  can help to clarify the situation as noted by Froggat<sup>/4/</sup>.

In this article we shall consider the isobar photoproduction processes  $\gamma + p \rightarrow \pi(V) + \Delta(1236)$  where  $\Delta$  is the 3,3-resonance and  $V$  the vector meson. We assume, that only the  $\pi$ -trajectory has a conspirator. In 2 we consider the kinematics of the processes, in 3 the conspiracy relations, and the threshold and pseudothres-

hold relations are obtained. In 4 the factorization is discussed and some information about the solution of conspiracy equations is obtained. In 5 the procedure of reggeization is performed and the properties of the residue functions are discussed. In 6 we give the expressions for the differential cross sections and for the spin density matrix of isobars in terms of the Regge-pole amplitudes. In 7 some mechanisms for explaining the appearance of forward peaks in these processes are discussed. Comparison with experimental data and some predictions will be given in a next publication.

## 2. Kinematics

The direct channel (s-channel) process is

$$a + b \rightarrow c + d \quad (2.1)$$

In the s-channel c.m. system the partial wave expansion of the general helicity amplitude reads /12/ x)

$$f_{cd;ab}^{\Delta}(\theta, t) = \sum_J (2J+1) F_{cd;ab}^J(\theta) d_{\lambda\mu}^J(\theta), \quad (2.2)$$

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<sup>3</sup>We denote by a, b, c, d, A, D the helicity of particles a, b, ... A, D, respectively.

where  $\lambda = a - b$ ,  $\mu = c - d$ .

The t-channel process is

$$D + \beta \rightarrow c + A \quad (2.3)$$

where D and A are the antiparticles of d and a respectively.

In the t-channel o.m. system the helicity amplitude  $f_{cA;D\beta}^t(s,t)$  has the following partial-wave expansion

$$f_{cA;D\beta}^t(s,t) = \sum_J (2J+1) F_{cA;D\beta}^J(t) d_{\lambda'\mu'}^J(\theta_t), \quad (2.4)$$

where  $\lambda' = D - \beta$ ,  $\mu' = c - A$ .

These amplitudes are related by the following crossing relation<sup>[13]</sup>

$$f_{cA;D\beta}^t(s,t) = \sum_{c',D',A',\beta'} d_{A'a}^{J_a}(\chi_a) d_{\beta'b}^{J_b}(\chi_b) d_{c'c}^{J_c}(\chi_c) d_{D'd}^{J_d}(\chi_d) \cdot f_{c'A';D'\beta'}^t(s,t). \quad (2.5)$$

The conspiracy relations connect the kinematic singularities free helicity amplitudes and we shall construct these amplitudes. The following amplitudes do not contain kinematic singularities on the variable  $s$ . [14,15]

$$\overline{f}_{cA;D\beta}^t = \left(\sin \frac{\theta_t}{2}\right)^{-|\lambda' - \mu'|} \left(\cos \frac{\theta_t}{2}\right)^{-|\lambda' + \mu'|} f_{cA;D\beta}^t; \quad (2.6)$$

Following Wang and Hara <sup>16,17</sup> we can introduce the combinations of the amplitudes  $\overline{f}_{CA;D\delta}^{-t}$  with definite parity, which are free of any kinematic singularities, namely

$$\overline{f}_{CA;D\delta}^{-t\pm} = \overline{f}_{CA;D\delta}^{-t} \pm \overline{f}_{-C-A;D\delta}^{-t} = K_{CA;D\delta}^{\pm} \widetilde{f}_{CA;D\delta}^{-t}, \quad (2.7)$$

where  $\widetilde{f}_{CA;D\delta}^{-t}$  contains only dynamical singularities on  $s$  and  $t$  and is a suitable object for reggeization by the method of Gell-Mann et al. <sup>14</sup>. The quantities  $K_{CA;D\delta}^{\pm}$  are the known factors, which contain all kinematic singularities on  $t$  and can be found by the methods of refs. <sup>18 19</sup>. These factors for the processes  $\gamma + p \rightarrow \pi + \Delta$  and  $\gamma + p \rightarrow V + \Delta$  are given in Tables 1 and 2 respectively. Here we should like to make a comment, concerning the zero mass of the photon. Till now in all papers on photoproduction <sup>8,10,19</sup> all factors  $K_{CA;D\delta}^{\pm}$  were calculated by the method of Wang for  $m_\gamma \neq 0$  and then taking the limit  $m_\gamma \rightarrow 0$ . However, recently <sup>20</sup> Gostman and Maor have pointed out, that the situation is not so simple. The point is that in the crossing matrix ( see formula III.11 in <sup>16</sup>) the factor  $\left( \frac{1 - \cos \chi_a}{\sin \chi_a} \right)^2 \cos \chi_a$  at  $m_\gamma \neq 0$  behaves as  $\varphi_{ab}^{-1}$  near  $\varphi_{ab} = 0$ , where  $\varphi_{ab}^2 = [t - (m_\gamma - m_b)^2][t - (m_\gamma + m_b)^2]$ . If we take  $m_\gamma = 0$ , then  $\cos \chi_a = -1$  and near  $\varphi_{ab} = 0$  no singularity of the type  $\varphi_{ab}^{-1}$  can exist. For this reason in some  $K_{CA;D\delta}^{\pm}$  functions calculated by the Wang method it is necessary to add a factor  $(t - \mu^2)$ , where  $\mu$  is the pion or vector meson mass.

Table 1

 $K^\pm$  - functions of the process  $\gamma + p \rightarrow \pi + \Delta$  (x)

Notations	Amplitudes	$K^\pm$ (Wang)	$\widetilde{K}^\pm$ (Crossing)	Dominant parity
$2 F_1^t$	$\overline{F}_{0,1}^{t+}; \frac{1}{2} \frac{1}{2}$	$\varphi_2 \psi^{-1} t^{-\frac{1}{2}}$	$\varphi_2^2 \psi^{-1} t^{-\frac{1}{2}}$	$(-1)^J$
$2 F_2^t$	$\overline{F}_{0,1}^{t-}; \frac{1}{2} \frac{1}{2}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$\varphi_2 \varphi_1^{-1} t^{-\frac{1}{2}}$	$(-1)^{J+1}$
$2 F_3^t$	$\overline{F}_{0,1}^{t-}; \frac{1}{2} -\frac{1}{2}$	$\varphi_2 \psi^{-1} t^{-1}$	$\varphi_2^2 \psi^{-1} t^{-1}$	$(-1)^J$
$2 F_4^t$	$\overline{F}_{0,1}^{t+}; \frac{1}{2} -\frac{1}{2}$	$\varphi_1^{-1} t^{-1}$	$\varphi_2 \varphi_1^{-1} t^{-1}$	$(-1)^{J+1}$
$2 F_5^t$	$\overline{F}_{0,1}^{t-}; \frac{3}{2} \frac{1}{2}$	$\varphi_2 \psi^{-1} t^{-1}$	$\varphi_2^2 \psi^{-1} t^{-1}$	$(-1)^J$
$2 F_6^t$	$\overline{F}_{0,1}^{t+}; \frac{3}{2} \frac{1}{2}$	$\varphi_1^{-1} t^{-1}$	$\varphi_2 \varphi_1^{-1} t^{-1}$	$(-1)^{J+1}$
$4 F_7^t$	$\overline{F}_{0,1}^{t-}; \frac{3}{2} -\frac{1}{2}$	$\varphi_2^2 \varphi_1 t^{-\frac{3}{2}}$	$\varphi_2^3 \varphi_1 t^{-\frac{3}{2}}$	$(-1)^J$
$4 F_8^t$	$\overline{F}_{0,1}^{t+}; \frac{3}{2} -\frac{1}{2}$	$\varphi_2 \psi t^{-\frac{3}{2}}$	$\varphi_2^2 \psi t^{-\frac{3}{2}}$	$(-1)^{J+1}$

x)  $\varphi_2 = (t - \mu^2)$ ,  $\varphi_1 = [t - (M+m)^2]^{\frac{1}{2}}$ ,  $\psi = [t - (M-m)^2]^{\frac{1}{2}}$  where  $\mu$  is pion

mass in Table 1 and vector meson mass in Table 2,  $M$  - isobar mass,  $m$  proton mass,  $t$  - square of transferred momentum in the  $S$  - channel.



Table 2

 $K^\pm$  - functions for the process  $\gamma + p \rightarrow V + \Delta$  x)

Notations	Amplitudes	$K^\pm$ (Wang)	$\widetilde{K}^\pm$ (Crossing)	Dominant parity
$2 H_1^t$	$\overline{f}_{01; \frac{1}{2} \frac{1}{2}}^{-t-}$	$\psi^{-1} \varphi_2^{-1} t^{-\frac{1}{2}}$	$\psi^{-1} t^{-\frac{1}{2}}$	$(-1)^J$
$2 H_2^t$	$\overline{f}_{01; \frac{1}{2} \frac{3}{2}}^{-t+}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$(-1)^{J+1}$
$2 H_3^t$	$\overline{f}_{01; \frac{1}{2} -\frac{1}{2}}^{-t+}$	$\psi^{-1} \varphi_2^{-1} t^{-1}$	$\psi^{-1} t^{-1}$	$(-1)^J$
$2 H_4^t$	$\overline{f}_{01; \frac{1}{2} -\frac{3}{2}}^{-t-}$	$\varphi_1^{-1} t^{-1}$	$\varphi_1^{-1} t^{-1}$	$(-1)^{J+1}$
$2 H_5^t$	$\overline{f}_{01; \frac{3}{2} \frac{3}{2}}^{-t+}$	$\psi^{-1} \varphi_2^{-1} t^{-1}$	$\psi^{-1} t^{-1}$	$(-1)^J$
$2 H_6^t$	$\overline{f}_{01; \frac{3}{2} \frac{1}{2}}^{-t-}$	$\varphi_1^{-1} t^{-1}$	$\varphi_1^{-1} t^{-1}$	$(-1)^{J+1}$
$4 H_7^t$	$\overline{f}_{01; \frac{3}{2} -\frac{1}{2}}^{-t+}$	$\varphi_1 t^{-\frac{3}{2}}$	$\varphi_1 \varphi_2 t^{-\frac{3}{2}}$	$(-1)^J$
$4 H_8^t$	$\overline{f}_{01; \frac{3}{2} -\frac{3}{2}}^{-t-}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$(-1)^{J+1}$
$H_9^t$	$\overline{f}_{11; \frac{1}{2} \frac{1}{2}}^{-t+}$	$\varphi_1^{-1} \psi^{-2} \varphi_2^{-2}$	$\varphi_1^{-1} \psi^{-2} \varphi_2^{-1}$	$(-1)^J$
$H_{10}^t$	$\overline{f}_{11; \frac{1}{2} \frac{3}{2}}^{-t-}$	$\varphi_1^{-2} \psi^{-1} \varphi_2^{-1}$	$\varphi_1^{-2} \psi^{-1} \varphi_2^{-1}$	$(-1)^{J+1}$
$2 H_{11}^t$	$\overline{f}_{11; \frac{1}{2} -\frac{1}{2}}^{-t+}$	$\psi^{-1} \varphi_2^{-1} t^{-\frac{1}{2}}$	$\psi^{-1} t^{-\frac{1}{2}}$	$(-1)^J$
$2 H_{12}^t$	$\overline{f}_{11; \frac{1}{2} -\frac{3}{2}}^{-t-}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$(-1)^{J+1}$

x) See footnote below Table 1.

Table 2 ( continuation)

Notations	Amplitudes	$K^\pm$ (Wang)	$\widetilde{K}^\pm$ (Crossing)	Dominant parity
$2 H_{13}^t$	$\overline{F}_{11}^{t+}; \frac{3}{2} \frac{1}{2}$	$\psi^{-1} \varphi_2^{-1} t^{-\frac{1}{2}}$	$\psi^{-1} t^{-\frac{1}{2}}$	$(-1)^J$
$2 H_{14}^t$	$\overline{F}_{11}^{t-}; \frac{3}{2} \frac{1}{2}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$\varphi_1^{-1} t^{-\frac{1}{2}}$	$(-1)^{J+1}$
$4 H_{15}^t$	$\overline{F}_{14}^{t+}; \frac{3}{2} -\frac{1}{2}$	$\varphi_1 t^{-1}$	$\varphi_1 \varphi_2 t^{-1}$	$(-1)^J$
$4 H_{16}^t$	$\overline{F}_{11}^{t-}; \frac{3}{2} -\frac{1}{2}$	$\psi \varphi_2 t^{-1}$	$\psi \varphi_2 t^{-1}$	$(-1)^{J+1}$
$4 H_{17}^t$	$\overline{F}_{-11}^{t+}; \frac{1}{2} \frac{1}{2}$	$\varphi_1 t^{-1}$	$\varphi_1 \varphi_2 t^{-1}$	$(-1)^J$
$4 H_{18}^t$	$\overline{F}_{-11}^{t-}; \frac{1}{2} \frac{1}{2}$	$\psi \varphi_2 t^{-1}$	$\psi \varphi_2 t^{-1}$	$(-1)^{J+1}$
$4 H_{19}^t$	$\overline{F}_{-11}^{t-}; \frac{1}{2} -\frac{1}{2}$	$\varphi_1 t^{-\frac{3}{2}}$	$\varphi_1 \varphi_2 t^{-\frac{3}{2}}$	$(-1)^J$
$4 H_{20}^t$	$\overline{F}_{-11}^{t+}; \frac{1}{2} -\frac{1}{2}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$(-1)^{J+1}$
$4 H_{21}^t$	$\overline{F}_{-11}^{t-}; \frac{3}{2} \frac{1}{2}$	$\varphi_1 t^{-\frac{3}{2}}$	$\varphi_1 \varphi_2 t^{-\frac{3}{2}}$	$(-1)^J$
$4 H_{22}^t$	$\overline{F}_{-11}^{t+}; \frac{3}{2} \frac{1}{2}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$\psi \varphi_2 t^{-\frac{3}{2}}$	$(-1)^{J+1}$
$4 H_{23}^t$	$\overline{F}_{-11}^{t+}; \frac{3}{2} -\frac{1}{2}$	$\varphi_1 t^{-2}$	$\varphi_1 \varphi_2 t^{-2}$	$(-1)^J$
$4 H_{24}^t$	$\overline{F}_{-11}^{t-}; \frac{3}{2} -\frac{1}{2}$	$\psi \varphi_2 t^{-2}$	$\psi \varphi_2 t^{-2}$	$(-1)^{J+1}$

In Tables 1 and 2 these corrected K-functions are denoted by  $\widetilde{K}^{\pm}$  (crossing).

### 3. Conspiracy relations, threshold and pseudothreshold constraints

The conspiracy relations give additional kinematic zeros at  $t=0$  in some combinations of the parity-conserving amplitudes. The physical meaning of the conspiracy relations in processes with unequal masses is explained in<sup>/5/</sup>. For the derivation of conspiracy relations we follow the method, discussed in<sup>/18/</sup>, and also in Appendix A of<sup>/5/</sup> and obtain:  
For the reaction  $\gamma + p \rightarrow \pi + \Delta$

$$t \left( \overline{F}_{01, \frac{1}{2}-\frac{1}{2}}^t + \overline{F}_{0-1, \frac{1}{2}-\frac{1}{2}}^t \right) = t \left( \overline{F}_{01, \frac{1}{2}-\frac{1}{2}}^t - \overline{F}_{0-1, \frac{1}{2}-\frac{1}{2}}^t \right), \quad (3.1)$$

$$t \left( \overline{F}_{01, \frac{3}{2}\frac{1}{2}}^t + \overline{F}_{0-1, \frac{3}{2}\frac{1}{2}}^t \right) = t \left( \overline{F}_{01, \frac{3}{2}\frac{1}{2}}^t - \overline{F}_{0-1, \frac{3}{2}\frac{1}{2}}^t \right), \quad (3.2)$$

$$t^{\frac{3}{2}} \left( \overline{F}_{01, \frac{3}{2}-\frac{1}{2}}^t + \overline{F}_{0-1, \frac{3}{2}-\frac{1}{2}}^t \right) = t^{\frac{3}{2}} \left( \overline{F}_{01, \frac{3}{2}-\frac{1}{2}}^t - \overline{F}_{0-1, \frac{3}{2}-\frac{1}{2}}^t \right). \quad (3.3)$$

For the reaction  $\gamma + p \rightarrow V + \Delta$ :

$$t \left( \overline{F}_{01, \frac{1}{2}-\frac{1}{2}}^t + \overline{F}_{0-1, \frac{1}{2}-\frac{1}{2}}^t \right) = t \left( \overline{F}_{01, \frac{1}{2}-\frac{1}{2}}^t - \overline{F}_{0-1, \frac{1}{2}-\frac{1}{2}}^t \right), \quad (3.4)$$

$$t \left( \overline{F}_{01, \frac{3}{2}\frac{1}{2}}^t + \overline{F}_{0-1, \frac{3}{2}\frac{1}{2}}^t \right) = t \left( \overline{F}_{01, \frac{3}{2}\frac{1}{2}}^t - \overline{F}_{0-1, \frac{3}{2}\frac{1}{2}}^t \right), \quad (3.5)$$

$$t^{\frac{3}{2}} \left( \overline{F}_{01, \frac{3}{2}-\frac{1}{2}}^t + \overline{F}_{0-1, \frac{3}{2}-\frac{1}{2}}^t \right) = t^{\frac{3}{2}} \left( \overline{F}_{01, \frac{3}{2}-\frac{1}{2}}^t - \overline{F}_{0-1, \frac{3}{2}-\frac{1}{2}}^t \right), \quad (3.6)$$

$$t^{\frac{3}{2}} \left( \overline{F}_{1-11, \frac{1}{2}-\frac{1}{2}}^t + \overline{F}_{1-1, \frac{1}{2}-\frac{1}{2}}^t \right) = t^{\frac{3}{2}} \left( \overline{F}_{1-11, \frac{1}{2}-\frac{1}{2}}^t - \overline{F}_{1-1, \frac{1}{2}-\frac{1}{2}}^t \right), \quad (3.7)$$

$$t^{\frac{3}{2}} \left( \bar{f}_{-11, \frac{3}{2} \frac{1}{2}}^t + \bar{f}_{1-1, \frac{3}{2} \frac{1}{2}}^t \right) = t^{\frac{3}{2}} \left( \bar{f}_{-11, \frac{3}{2} \frac{1}{2}}^t - \bar{f}_{1-1, \frac{3}{2} \frac{1}{2}}^t \right), \quad (3.8)$$

$$t^2 \left( \bar{f}_{-11, \frac{3}{2} - \frac{1}{2}}^t + \bar{f}_{1-1, \frac{3}{2} - \frac{1}{2}}^t \right) = t^2 \left( \bar{f}_{-11, \frac{3}{2} - \frac{1}{2}}^t - \bar{f}_{1-1, \frac{3}{2} - \frac{1}{2}}^t \right), \quad (3.9)$$

From these relations and Tables 1 and 2 we see, that this conspiracy belongs to the class III conspiracy relations according to the group-theory classification<sup>[21-23]</sup>. This result was established in<sup>[24]</sup> by means of group-theoretic methods. The conspiracy relations (3.1)-(3.9) can be satisfied by two different ways: either all residues of amplitudes in (3.1 - 3.9) have an additional kinematic factor  $t$  (evasion), or every amplitude  $\bar{f}$  retains its singular behaviour near  $t=0$ , but both sides in (2.1) and (3.9) must approach the same limit (conspiracy). In this case each trajectory will have the corresponding conspirator trajectory with opposite parity. Factorization gives also some information on the behaviour of Regge-pole amplitudes at  $t=0$  and can help in the choice of the solutions of the conspiracy relations. Factorization will be considered in the next section. Here we give conspiracy relations for the processes, which take part in factorization. Namely, for  $\gamma + \gamma \rightarrow \pi + \pi$

$$t \left( \bar{f}_{01, 01}^t + \bar{f}_{0-1, 01}^t \right) = -t \left( \bar{f}_{01, 01}^t - \bar{f}_{0-1, 01}^t \right), \quad (3.10)$$

for  $\gamma + \gamma \rightarrow V + V$  :

$$t^2(\bar{F}_{1,-1;1,-1}^t + \bar{F}_{-11;1,-1}^t) = -t^2(\bar{F}_{1,-1;1,-1}^t - \bar{F}_{-11;1,-1}^t), \quad (3.11)$$

$$t(\bar{F}_{01;01}^t + \bar{F}_{0-1;01}^t) = -t(\bar{F}_{01;01}^t - \bar{F}_{0-1;01}^t), \quad (3.12)$$

for  $\bar{N} + N \rightarrow \bar{\Delta} + \Delta$  :

$$t(\bar{F}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}^t + \bar{F}_{-\frac{3}{2}-\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t) = -t(\bar{F}_{\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t - \bar{F}_{-\frac{3}{2}-\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t), \quad (3.13)$$

$$t(\bar{F}_{-\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t + \bar{F}_{\frac{1}{2}-\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t) = t(\bar{F}_{-\frac{1}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t - \bar{F}_{\frac{1}{2}-\frac{1}{2};\frac{3}{2}\frac{1}{2}}^t), \quad (3.14)$$

$$t^2(\bar{F}_{\frac{3}{2}-\frac{1}{2};\frac{3}{2}-\frac{1}{2}}^t + \bar{F}_{-\frac{3}{2}\frac{1}{2};\frac{3}{2}-\frac{1}{2}}^t) = -t^2(\bar{F}_{\frac{3}{2}-\frac{1}{2};\frac{3}{2}-\frac{1}{2}}^t - \bar{F}_{-\frac{3}{2}\frac{1}{2};\frac{3}{2}-\frac{1}{2}}^t), \quad (3.15)$$

$$t(\bar{F}_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t + \bar{F}_{-\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t) = -t(\bar{F}_{\frac{3}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t - \bar{F}_{-\frac{3}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t), \quad (3.16)$$

$$t(\bar{F}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t + \bar{F}_{-\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t) = -t(\bar{F}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t - \bar{F}_{-\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}}^t), \quad (3.17)$$

Relation (3.10) was found in<sup>/5/</sup> and relations (3.11), and (3.12) in<sup>/10/</sup>. The helicity amplitudes must obey also the so-called threshold<sup>/25/</sup> and pseudothreshold<sup>/18,26/</sup> constraints equations. In photoproduction the threshold and pseudothreshold  $t = (m_\gamma \pm \mu)^2$  coincide. We represent here threshold constraints at  $t = \mu^2$  for the process  $\gamma + p \rightarrow \pi + \Delta$ . For the process  $\gamma + p \rightarrow V + \Delta$  these constraints do not play an important role, because the point  $t = m_V^2$  are far from the

physical region of the s-channel. But for both processes the pseudothreshold relations at  $t = (M-m)^2 \simeq 0,09 \left(\frac{\text{GeV}}{c}\right)^2$  are very important because this point lies near the physical region of the s-channel. For derivation of these constraints we use the Cohen-Tannoudji-Morel-Nevelet method, based on the simple behaviour of the transversality amplitudes<sup>[27]</sup> near thresholds and pseudothresholds.

The constraint equations read

at  $t = \mu^2$  for  $\gamma + p \rightarrow \pi + \Delta$  \*)

$$\varphi_2 \left\{ F_{(5,6)}^t + \sqrt{3} F_{(3,4)}^t - iy F_{(7,8)}^t + i\sqrt{3} y F_2^t \right\} \simeq \varphi_2^3, \quad (3.18)$$

$$\varphi_2 \left\{ -\sqrt{3} F_{(5,6)}^t + F_{(3,4)}^t - i\sqrt{3} y F_{(7,8)}^t - iy F_2^t \right\} \simeq \varphi_2^3, \quad (3.19)$$

$$\varphi_2 \left\{ F_{(5,6)}^t + \sqrt{3} F_{(3,4)}^t + iy F_{(7,8)}^t - i\sqrt{3} y F_2^t \right\} \simeq \varphi_2^3, \quad (3.20)$$

$$\varphi_2 \left\{ -\sqrt{3} F_{(5,6)}^t + F_{(3,4)}^t + i\sqrt{3} y F_{(7,8)}^t + iy F_2^t \right\} \simeq \varphi_2^3, \quad (3.21)$$

\*) We use the notation  $F_{(i,j)}^t = F_i^t - x F_j^t$  where  $x = \cos \theta_t$ ,  $y = \sin \theta_t$ .  $\theta_t$  is the scattering angle in the t-channel cm.system

$$\varphi_1 \varphi_2 \psi \cos \theta_t = 2st + t^2 - t(M^2 + m^2 + \mu^2) + \mu^2(M^2 - m^2),$$

$$\varphi_1 \varphi_2 \psi \sin \theta_t = 2 [t \phi(s, t)]^{\frac{1}{2}}, \quad \text{where}$$

$$\phi(s, t) \equiv st (M^2 + m^2 + \mu^2 - s - t) - s\mu^2(M^2 - m^2) - tm^2(M^2 - \mu^2) + \mu^2 m^2 (M^2 - m^2 - \mu^2).$$

at  $t = (M-m)^2$  for  $\gamma + \rho \rightarrow \pi + \Delta$  :

$$\psi^2 \left[ F_{(6,5)}^t - \sqrt{3} F_{(4,3)}^t + iy \left( F_{(8,7)}^t + \sqrt{3} F_1^t \right) \right] \simeq \psi^4, \quad (3.22)$$

$$\psi^2 \left[ F_{(5,6)}^t + \sqrt{3} F_{(3,4)}^t - iy \left( F_{(7,8)}^t - \sqrt{3} F_2^t \right) \right] \simeq \psi^3, \quad (3.23)$$

$$\psi^2 \left[ -\sqrt{3} F_{(5,6)}^t + F_{(3,4)}^t - iy \left( \sqrt{3} F_{(7,8)}^t + F_2^t \right) \right] \simeq \psi^3, \quad (3.24)$$

$$\psi^2 \left[ \sqrt{3} F_{(6,5)}^t + F_{(4,3)}^t - iy \left( \sqrt{3} F_{(8,7)}^t - F_1^t \right) \right] \simeq \psi^2, \quad (3.25)$$

$$\psi^2 \left[ -\sqrt{3} F_{(6,5)}^t - F_{(4,3)}^t - iy \left( \sqrt{3} F_{(8,7)}^t - F_1^t \right) \right] \simeq \psi^2; \quad (3.26)$$

at  $t = (M-m)^2$  for  $\gamma + \rho \rightarrow V + \Delta$  :

$$\begin{aligned} \psi^2 \left[ -H_9^t + \sqrt{6} H_{(6,5)}^t + \sqrt{2} H_{(4,3)}^t + iy \left( \sqrt{3} H_{13}^t + H_{11}^t + \sqrt{2} H_1^t - \sqrt{6} H_{(8,7)}^t \right. \right. \\ \left. \left. - \sqrt{3} H_{(22,21)}^t - H_{(20,19)}^t \right) + y^2 \left( \sqrt{3} H_{15}^t + H_{17}^t \right) + 2\sqrt{3} x H_{24}^t \right. \\ \left. - \sqrt{3} (1+x^2) H_{23}^t \right] \simeq \psi^2, \end{aligned} \quad (3.27)$$

$$\begin{aligned} \psi^2 \left[ -\sqrt{3} H_9^t + \sqrt{2} H_{(6,5)}^t - \sqrt{6} H_{(4,3)}^t + iy \left( H_{13}^t - \sqrt{3} H_{11}^t + \sqrt{6} H_1^t + \right. \right. \\ \left. \left. + \sqrt{2} H_{(8,7)}^t - H_{(22,21)}^t + \sqrt{3} H_{(20,19)}^t \right) - y^2 \left( H_{15}^t - \sqrt{3} H_{17}^t \right) \right. \\ \left. - 2x H_{24}^t + (1+x^2) H_{23}^t \right] \simeq \psi^4, \end{aligned} \quad (3.28)$$

$$\begin{aligned} \psi^2 \left[ -H_9^t - \sqrt{6} H_{(6,5)}^t - \sqrt{2} H_{(4,3)}^t + i\gamma \left( -\sqrt{3} H_{13}^t - H_{11}^t + \sqrt{2} H_1^t \right. \right. \\ \left. \left. - \sqrt{6} H_{(8,7)}^t + \sqrt{3} H_{(22,21)}^t + H_{(20,19)}^t \right) + \gamma^2 \left( \sqrt{3} H_{15}^t + H_{17}^t \right) \right. \\ \left. + 2\sqrt{3}\alpha H_{24}^t - \sqrt{3} (1+\alpha^2) H_{25}^t \right] \simeq \psi^2, \end{aligned} \quad (3.29)$$

$$\begin{aligned} \psi^2 \left[ H_9^t + \sqrt{6} H_{(6,5)}^t + \sqrt{2} H_{(4,3)}^t + i\gamma \left( -\sqrt{3} H_{13}^t - H_{11}^t + \sqrt{2} H_1^t \right. \right. \\ \left. \left. - \sqrt{6} H_{(8,7)}^t + \sqrt{3} H_{(22,21)}^t + H_{(20,19)}^t \right) - \gamma^2 \left( \sqrt{3} H_{15}^t + H_{17}^t \right) \right. \\ \left. - 2\sqrt{3}\alpha H_{24}^t + \sqrt{3} (1+\alpha^2) H_{23}^t \right] \simeq \psi^2, \end{aligned} \quad (3.30)$$

$$\begin{aligned} \psi^2 \left[ \sqrt{3} H_9^t + \sqrt{2} H_{(6,5)}^t - \sqrt{6} H_{(4,3)}^t + i\gamma \left( -H_{13}^t + H_{11}^t + \sqrt{6} H_1^t \right. \right. \\ \left. \left. + \sqrt{2} H_{(8,7)}^t + H_{(22,21)}^t - \sqrt{3} H_{(20,19)}^t \right) + \gamma^2 \left( H_{15}^t - \sqrt{3} H_{17}^t \right) \right. \\ \left. + 2\alpha H_{24}^t - (1+\alpha^2) H_{23}^t \right] \simeq \psi^4, \end{aligned} \quad (3.31)$$

$$\begin{aligned} \psi^2 \left[ H_9^t - \sqrt{6} H_{(6,5)}^t - \sqrt{2} H_{(4,3)}^t + i\gamma \left( \sqrt{3} H_{13}^t + H_{11}^t + \sqrt{2} H_1^t \right. \right. \\ \left. \left. - \sqrt{6} H_{(8,7)}^t - \sqrt{3} H_{(22,21)}^t - H_{(20,19)}^t \right) - \gamma^2 \left( \sqrt{3} H_{15}^t + H_{17}^t \right) \right. \\ \left. - 2\sqrt{3}\alpha H_{24}^t + \sqrt{3} (1+\alpha^2) H_{23}^t \right] \simeq \psi^2 \end{aligned} \quad (3.32)$$

$$\begin{aligned} \psi^2 \left[ -H_{10}^t + i\gamma \left( \sqrt{3} H_{14}^t - H_{12}^t + \sqrt{3} H_{(21,22)}^t - H_{(19,20)}^t \right) - \right. \\ \left. - \gamma^2 \left( \sqrt{3} H_{16}^t + H_{18}^t \right) + 2\sqrt{3}\alpha H_{23}^t - \sqrt{3} (1+\alpha^2) H_{24}^t \right] \simeq \psi^3, \end{aligned} \quad (3.33)$$



$$\psi^2 \left[ \sqrt{3} H_{10}^t + iy \left( -H_{14}^t - \sqrt{3} H_{12}^t - H_{(21,22)}^t - \sqrt{3} H_{(14,20)}^t \right) - iy^2 \left( H_{16}^t - \sqrt{3} H_{18}^t \right) + 2x H_{23}^t - (1+x^2) H_{24}^t \right] \simeq \psi^3, \quad (3.34)$$

$$\psi^2 \left[ -H_{10}^t + i\bar{6} H_{(5,6)}^t - i\bar{2} H_{(3,4)}^t + iy \left( \sqrt{3} H_{14}^t - H_{12}^t + i\bar{2} H_2^t + i\bar{6} H_{(7,8)}^t - \sqrt{3} H_{(21,22)}^t + H_{(19,20)}^t \right) + y^2 \left( -\sqrt{3} H_{16}^t + H_{18}^t \right) - 2\sqrt{3}x H_{23}^t + i\bar{3} (1+x^2) H_{24}^t \right] \simeq \psi^3, \quad (3.35)$$

$$\psi^2 \left[ \sqrt{3} H_{10}^t - i\bar{2} H_{(5,6)}^t - i\bar{6} H_{(3,4)}^t + iy \left( -H_{14}^t - \sqrt{3} H_{12}^t - \sqrt{6} H_2^t + \sqrt{2} H_{(7,8)}^t + H_{(21,22)}^t + \sqrt{3} H_{(19,20)}^t \right) - y^2 \left( H_{16}^t + \sqrt{3} H_{18}^t \right) - 2x H_{23}^t + (1+x^2) H_{24}^t \right] \simeq \psi^3, \quad (3.36)$$

$$\psi^2 \left[ H_{10}^t + i\bar{6} H_{(5,6)}^t - i\bar{2} H_{(3,4)}^t + iy \left( -\sqrt{3} H_{14}^t + H_{12}^t + i\bar{2} H_2^t + \sqrt{6} H_{(7,8)}^t + \sqrt{3} H_{(21,22)}^t - H_{(19,20)}^t \right) + y^2 \left( \sqrt{3} H_{16}^t - H_{18}^t \right) + 2\sqrt{3}x H_{23}^t - \sqrt{3} (1+x^2) H_{24}^t \right] \simeq \psi^3, \quad (3.37)$$

$$\psi^2 \left[ -\sqrt{3} H_{10}^t - i\bar{2} H_{(5,6)}^t - i\bar{6} H_{(3,4)}^t + iy \left( H_{14}^t + \sqrt{3} H_{12}^t - i\bar{6} H_2^t + \sqrt{2} H_{(7,8)}^t - H_{(21,22)}^t - \sqrt{3} H_{(19,20)}^t \right) + y^2 \left( H_{16}^t + \sqrt{3} H_{18}^t \right) + 2x H_{23}^t - (1+x^2) H_{24}^t \right] \simeq \psi^3, \quad (3.38)$$

$$\psi^2 \left[ H_9^t + iy \left( -\sqrt{3} H_{13}^t - H_{11}^t - \sqrt{3} H_{(22,21)}^t - H_{(20,19)}^t \right) - y^2 \left( \sqrt{3} H_{15}^t - H_{17}^t \right) + 2\sqrt{3} x H_{24}^t - \sqrt{3} (1+x^2) H_{23}^t \right] \simeq \psi^2 \quad (3.3)$$

$$\psi^2 \left[ \sqrt{3} H_9^t + iy \left( -H_{11}^t + \sqrt{3} H_{11}^t - H_{(22,21)}^t + \sqrt{3} H_{(20,19)}^t \right) + y^2 \left( H_{15}^t + \sqrt{3} H_{17}^t \right) - 2x H_{24}^t + (1+x^2) H_{23}^t \right] \simeq \psi^2, \quad (3.40)$$

$$\psi^2 \left[ H_9^t + iy \left( \sqrt{3} H_{13}^t + H_{11}^t + \sqrt{3} H_{(22,21)}^t + H_{(20,19)}^t \right) - y^2 \left( \sqrt{3} H_{15}^t - H_{17}^t \right) + 2\sqrt{3} x H_{24}^t - \sqrt{3} (1+x^2) H_{23}^t \right] \simeq \psi^2, \quad (3.41)$$

#### 4. Factorization

In the next section we shall see that the asymptotic behaviour of reggeized parity-conserving amplitudes has the following form

$$\frac{2\alpha^\pm(t) + 1}{\sin \pi \alpha^\pm(t)} E_{\lambda' \mu'}^{\alpha^\pm} + (\cos \theta_t) \widetilde{K}_{CA, DL}^\pm \left( \frac{p_{DL} p_{CA}}{s_0} \right)^{\alpha^\pm - M} \gamma_{CA, DL}'^\pm \quad (4.1)$$

$$\sim \widetilde{K}_{CA, DL}^\pm(t) \gamma_{CA, DL}^\pm(t) \left( \frac{s}{s_0} \right)^{\alpha^\pm - M}$$

where  $\tilde{K}_{CA;D\delta}^{\pm}$  is the crossing factor from Tables 1 and 2,  $\mu = \max \{ |\lambda'|, |\mu'| \}$ ,  $\gamma_{CA;D\delta}^{\pm}(t)$  is the dynamical part of the residue functions. As is known [2,5,28] factorization gives some information about the  $t$ -dependence of the residue functions which could be useful in choosing evasion or conspiracy. The general form of the factorization condition of the Regge-poles residue functions reads [29,30]

$$\left[ \beta^{ab \rightarrow cd}(t) \right]^2 = \beta^{ab \rightarrow ab}(t) \cdot \beta^{cd \rightarrow cd}(t). \quad (4.2)$$

In our notations this equation can be rewritten as follows

$$\left[ \gamma_2^{\pm}(t) \tilde{K}_2^{\pm}(t) (P_{CA} P_{D\delta})^{\alpha - M_2} \right]^2 = \quad (4.3)$$

$$\left[ \gamma_1^{\pm}(t) \tilde{K}_1^{\pm}(t) (P_{D\delta})^{2(\alpha - M_1)} \right] \cdot \left[ \gamma_3^{\pm}(t) \tilde{K}_3^{\pm}(t) (P_{CA})^{2(\alpha - M_3)} \right],$$

where the subscript 1 represents the reaction  $D+b \rightarrow D+b$  2 corresponds to  $D+b \rightarrow c+A$  and 3 to  $c+A \rightarrow c+A$ . This equation near  $t=0$  is given in Tables 3-6. The solutions for the factorization conditions with lowest  $t$ -dependence are given in Tables 7 and 8.

Table 3

Factorization condition for $\bar{Z}+N \rightarrow \pi+\delta$	Factorization satisfied
$t(\gamma_{01; \frac{1}{2} \frac{1}{2}}^{\pm})^2 = \gamma_{\frac{1}{2} \frac{1}{2}; \frac{1}{2} \frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no
$(\gamma_{01; \frac{1}{2} -\frac{1}{2}}^{\pm})^2 = \gamma_{\frac{1}{2} -\frac{1}{2}; \frac{1}{2} -\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	Yes
$(\gamma_{01; \frac{3}{2} \frac{1}{2}}^{\pm})^2 = \gamma_{\frac{3}{2} \frac{1}{2}; \frac{3}{2} \frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	yes
$t(\gamma_{01; \frac{3}{2} -\frac{1}{2}}^{\pm})^2 = \gamma_{\frac{3}{2} -\frac{1}{2}; \frac{3}{2} -\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no

Table 4 \*)

Factorization condition for $V+\gamma \rightarrow V+\gamma$	Factorization satisfied
$t^2(\gamma_{11; 1-1}^{\pm})^2 = \gamma_{11; 11}^{\pm} \cdot \gamma_{1-1; 1-1}^{\pm}$	no
$t(\gamma_{11; 01}^{\pm})^2 = \gamma_{11; 11}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no
$t(\gamma_{-11; 01}^{\pm})^2 = \gamma_{-11; -11}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no

\*) This table can be found in  $h^{10}$ .

Table 5

Factorization condition for $\bar{\Delta}N \rightarrow \bar{\Delta}N$	Factorization satisfied
$t\left(\gamma_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm}$	no
$t\left(\gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{\frac{3}{2}-\frac{1}{2}; \frac{3}{2}-\frac{1}{2}}^{\pm}$	no
$\left(\gamma_{-\frac{1}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{-\frac{1}{2}\frac{1}{2}; -\frac{1}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm}$	yes
$\left(\gamma_{\frac{3}{2}\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm}$	yes

Table 6

Factorization condition for $\bar{\Delta}N \rightarrow V\gamma$	Factorization satisfied
$t\left(\gamma_{01; \frac{1}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no
$\left(\gamma_{01; \frac{1}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	yes
$\left(\gamma_{01; \frac{3}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	yes
$t\left(\gamma_{01; \frac{3}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}-\frac{1}{2}; \frac{3}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{01; 01}^{\pm}$	no
$\left(\gamma_{11; \frac{1}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{11; 11}^{\pm}$	yes
$t\left(\gamma_{11; \frac{1}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{11; 11}^{\pm}$	no
$t\left(\gamma_{11; \frac{3}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{11; 11}^{\pm}$	no
$t^2\left(\gamma_{11; \frac{3}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}-\frac{1}{2}; \frac{3}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{11; 11}^{\pm}$	no
$t^2\left(\gamma_{-11; \frac{1}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{-11; -11}^{\pm}$	no
$t\left(\gamma_{-11; \frac{1}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{-11; -11}^{\pm}$	no
$t\left(\gamma_{-11; \frac{3}{2}\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}\frac{1}{2}; \frac{3}{2}\frac{1}{2}}^{\pm} \cdot \gamma_{-11; -11}^{\pm}$	no
$\left(\gamma_{-11; \frac{3}{2}-\frac{1}{2}}^{\pm}\right)^2 = \gamma_{\frac{3}{2}-\frac{1}{2}; \frac{3}{2}-\frac{1}{2}}^{\pm} \cdot \gamma_{-11; -11}^{\pm}$	yes

Table 7

$\pi\delta \rightarrow \pi\delta$	$\bar{\Delta}N \rightarrow \bar{\Delta}N$	$\bar{\Delta}N \rightarrow \pi\delta$
$\chi_{01,01}^+ \sim \text{const}$ $\chi_{01,01}^- \sim \text{const}$	$\chi_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \sim t$ $\chi_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm} \sim t$ Other residues approach the const. at $t=0$	All residues approach the const. at $t=0$

Table 8

$V\delta \rightarrow V\delta$	$\bar{\Delta}N \rightarrow \bar{\Delta}N$	$\bar{\Delta}N \rightarrow V\delta$
$\delta_{11,11}^{\pm} \sim t$ $\chi_{-11,1-1}^{\pm} \sim t$	$\chi_{\frac{1}{2}\frac{1}{2}; \frac{1}{2}\frac{1}{2}}^{\pm} \sim t$ $\delta_{\frac{1}{2}-\frac{1}{2}; \frac{1}{2}-\frac{1}{2}}^{\pm} \sim t$	$\delta_{11; \frac{1}{2}\frac{1}{2}}^{\pm} \sim t$ $\chi_{-11; \frac{1}{2}-\frac{1}{2}}^{\pm} \sim t$
Other residues approach the Const. at $t=0$		

We conclude from Tables 7 and 8 that

- 1) Relations (3.9) (3.11) and (3.15) can have only evasive solutions
- 11) Other relations can have conspiratorial solutions

We see that conspiracy in the principal processes requires conspiracy in processes connected through factorization.

### 5. Reggeization

For reggeization of the parity-conserving helicity amplitudes we follow the method given in <sup>[14]</sup>. We start from the partial-wave expansion

$$\begin{aligned}
 f_{cA, D\ell}^{\pm t} &= (\sqrt{2})^{-|\lambda'| + |\mu'| - |\lambda' - \mu'|} \left\{ \begin{array}{l} \bar{f}_{cA, D\ell}^{\pm t} \\ \pm \eta_c \eta_A (-1)^{\lambda' + M + J_c + J_A - \nu} \bar{f}_{-c-A, D\ell}^{\pm t} \end{array} \right\} = \\
 &= \sum_J (2J+1) e^{J\pm} F_{cA, D\ell}^{J\pm} + \text{unimportant terms for } \cos\theta_t \rightarrow \infty
 \end{aligned} \tag{5.1}$$

where  $\eta_i$  is the intrinsic parity,  $J_i$  is the spin  $M = \max\{|\lambda'|, |\mu'|\}$ ,  $\nu = 0$  if  $J_c + J_A = \text{integer}$ , and  $\nu = \frac{1}{2}$  if  $J_c + J_A = \text{half-integer}$ . After reggeization

<sup>[14]</sup> we have x) Instead of  $\alpha_i(t)$  and  $\beta^\pm(\alpha_i, t)$  we write  $\alpha_i$  and  $\beta^\pm$ .

$$(F_1^t, F_2^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{1}{2}\frac{1}{2}}^+, \beta_{01, \frac{1}{2}\frac{1}{2}}^-) E_{01}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(F_3^t, F_4^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{1}{2}-\frac{1}{2}}^-, \beta_{01, \frac{1}{2}-\frac{1}{2}}^+) E_{11}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(F_5^t, F_6^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{3}{2}\frac{1}{2}}^-, \beta_{01, \frac{3}{2}\frac{1}{2}}^+) E_{11}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(F_7^t, F_8^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{3}{2}-\frac{1}{2}}^-, \beta_{01, \frac{3}{2}-\frac{1}{2}}^+) E_{12}^{\alpha_i, +}(\omega \epsilon_t)$$

and

(5.2)

$$(H_1^t, H_2^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{1}{2}\frac{1}{2}}^-, \beta_{01, \frac{1}{2}\frac{1}{2}}^+) E_{01}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_3^t, H_4^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{1}{2}-\frac{1}{2}}^+, \beta_{01, \frac{1}{2}-\frac{1}{2}}^-) E_{11}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_5^t, H_6^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{3}{2}\frac{1}{2}}^+, \beta_{01, \frac{3}{2}\frac{1}{2}}^-) E_{11}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_7^t, H_8^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{01, \frac{3}{2}-\frac{1}{2}}^+, \beta_{01, \frac{3}{2}-\frac{1}{2}}^-) E_{12}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_9^t, H_{10}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{11, \frac{1}{2}\frac{1}{2}}^+, \beta_{11, \frac{1}{2}\frac{1}{2}}^-) E_{00}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_{11}^t, H_{12}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{11, \frac{1}{2}-\frac{1}{2}}^+, \beta_{11, \frac{1}{2}-\frac{1}{2}}^-) E_{01}^{\alpha_i, +}(\omega \epsilon_t),$$

$$(H_{13}^t, H_{14}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{11, \frac{3}{2}\frac{1}{2}}^+, \beta_{11, \frac{3}{2}\frac{1}{2}}^-) E_{01}^{\alpha_i, +}(\omega \epsilon_t).$$



$$(H_{17}^t, H_{17}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{11, \frac{1}{2} - \frac{1}{2}}^+, \beta_{11, \frac{1}{2} - \frac{1}{2}}^-) E_{02}^{\alpha_i, +}(\cos \theta_4),$$

$$(H_{17}^t, H_B^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{-11, \frac{1}{2} \frac{1}{2}}^+, \beta_{-11, \frac{1}{2} \frac{1}{2}}^-) E_{02}^{\alpha_i, +}(\cos \theta_4),$$

$$(H_{19}^t, H_{20}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{-11, \frac{1}{2} - \frac{1}{2}}^-, \beta_{-11, \frac{1}{2} - \frac{1}{2}}^+) E_{12}^{\alpha_i, +}(\cos \theta_4), \quad (5.3)$$

$$(H_{21}^t, H_{22}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{-11, \frac{3}{2} \frac{1}{2}}^-, \beta_{-11, \frac{3}{2} \frac{1}{2}}^+) E_{02}^{\alpha_i, +}(\cos \theta_4),$$

$$(H_{23}^t, H_{24}^t) = \sum_i (2\alpha_i + 1) g(\alpha_i) (\beta_{-11, \frac{1}{2} - \frac{1}{2}}^+, \beta_{-11, \frac{1}{2} - \frac{1}{2}}^-) E_{22}^{\alpha_i, +}(\cos \theta_4),$$

where

$$g(\alpha_i) = \frac{1 + \tau \exp(-i\pi\alpha_i)}{2\sin\pi\alpha_i}$$

$\tau$  is signature of the Regge-pole. From the kinematic for processes with unequal masses we know that  $\sin \theta_4 \rightarrow 0$  and  $|\cos \theta_4| \rightarrow 1$  in the forward direction. This difficulty can be overcome by the introduction of daughter trajectories<sup>[31]</sup>. Using the asymptotic forms of E-functions

$$E_{00}^{\alpha, +}(\cos \theta_4) \approx \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \left( \frac{1}{\beta_{\pi(N)} \beta_{\Delta N}} \right)^{\alpha},$$

$$E_{01}^{\alpha, +}(\cos \theta_4) \approx \frac{2\alpha}{[\alpha(\alpha+1)]^{\frac{1}{2}}} \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{\frac{1}{2}} \Gamma(\alpha + 1)} \left( \frac{1}{\beta_{\pi(N)} \beta_{\Delta N}} \right)^{\alpha-1}, \quad (5.4)$$

$$E_{c2}^{\alpha,+}(c_9\theta_4) \approx \frac{7\alpha(\alpha-1)}{[(\alpha-1)\alpha(\alpha+1)(\alpha+2)]^{\frac{1}{2}}} \frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(\alpha+1)} \left(\frac{\omega}{\beta_{\pi(V)}\beta_{\Delta N}}\right)^{\alpha-2},$$

$$E_{11}^{\alpha,+}(c_9\theta_4) \approx \frac{2\alpha^2}{\alpha(\alpha+1)} \frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(\alpha+1)} \left(\frac{\omega}{\beta_{\pi(V)}\beta_{\Delta N}}\right)^{\alpha-1},$$

$$E_{12}^{\alpha,+}(c_9\theta_4) \approx \frac{4\alpha^2(\alpha-1)}{\alpha(\alpha+1)[(\alpha-1)(\alpha+2)]^{\frac{1}{2}}} \frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(\alpha+1)} \left(\frac{\omega}{\beta_{\pi(V)}\beta_{\Delta N}}\right)^{\alpha-2},$$

$$E_{22}^{\alpha,+}(c_9\theta_4) \approx \frac{4\alpha^2(\alpha-1)^2}{(\alpha-1)\alpha(\alpha+1)(\alpha+2)} \frac{\Gamma(\alpha+\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(\alpha+1)} \left(\frac{\omega}{\beta_{\pi(V)}\beta_{\Delta N}}\right)^{\alpha-2};$$

Now the residue functions  $\beta_{CA, D6}^{\pm}$  in (52) and (53) can be rewritten in the form

$$\beta_{CA, D6}^{\pm}(\alpha_i) \approx \tilde{h}_{CA, D6}^{\pm}(\alpha_i) \left(\frac{\beta_{\pi(V)}\beta_{\Delta N}}{s_0}\right)^{\alpha_i^{\pm}-N} \tilde{K}_{CA, D6}^{\pm}(t) \gamma_{CA, D6}^{\pm}(\alpha_i(t)), \quad (5.5)$$

where  $\tilde{K}_{CA, D6}^{\pm}$  are the kinematic factors from Tables I and 2,  $\tilde{h}_{CA, D6}^{\pm}$  are functions depended on  $\alpha_i$ . Choosing the Gell-Mann's<sup>/33/</sup> ghost Killing mechanism we assume that the residue functions vanish in nonsense-sense channels as the square root of the trajectory (as well as at the point symmetrical with respect to  $\alpha = -\frac{1}{2}$ ) at the integers of  $\alpha$ .

For example, if  $\alpha = 0$  is such a point, then  $\beta_{\rho n}(\alpha) \sim [\alpha(\alpha+1)]^{\frac{1}{2}}$  where  $\wedge$  means sense and  $n$  nonsense. From the factorization  $[\beta_{\rho n}]^2 = \beta_{\rho 0} \cdot \beta_{nn}$  we can obtain two solutions: either  $\beta_{\rho 0} \sim 1$ ,  $\beta_{nn} \sim \alpha(\alpha+1)$  or  $\beta_{\rho 0} \sim \alpha(\alpha+1)$ ,  $\beta_{nn} \sim 1$ . This is the dynamical problem. The form of the functions  $h^{\pm}(\alpha_i)$  depends on the choice of the possible solutions. In our case from the conservation of isospin and P and  $P_J$  parity only  $\pi, \rho, A_1, A_2, \beta$  and  $\pi_c$  (conspirator of  $\pi$ ) trajectory can give a contribution. The unnatural parity trajectories  $\pi, \beta, A_1$  contribute to the amplitudes with even index (i.e.  $F_2^t, F_4^t, \dots, H_2^t, H_4^t, \dots$ ) the remaining trajectories  $\pi_c, \rho, A_2$  (natural parity) contribute to the amplitudes with odd index (i.e.  $F_1^t, F_3^t, \dots, H_1^t, H_3^t, \dots$ ) in formulas (5.2) and (5.3). Assuming that  $\pi, \rho, \beta$  are connected with the sense-sense channel, and  $\pi_c, A_1, A_2$  with the nonsense-nonsense channel it is possible to find all  $h_{cA, D\ell}^{\pm}(\alpha_i)$  in (5.2) and (5.3). These functions are given in Tables 9 and 10. Finally, introducing the reduced residue functions<sup>28,32</sup> also denoted by  $\gamma_{cA; D\ell}^{\pm}$  we can write the reggeized amplitudes in the form<sup>x)</sup>

$$F_j^t(H_k^t) = \sum_i \frac{g(\alpha_i)}{\Gamma(\alpha_i+1)} h_{cA, D\ell}^{\pm}(\alpha_i) \widetilde{K}_{cA; D\ell}^{\pm}(t) \gamma_{cA; D\ell}^{\pm}(t) \left(\frac{s}{s_0}\right)^{\alpha_i - M} \quad (5.6)$$

$$j = 1, 2, \dots, l, \quad k = 1, 2, \dots, 2l.$$

<sup>x)</sup> In (5.6)  $\tau_{\pi} = \tau_{\pi_c} = \tau_{A_2} = 1$ , and  $\tau_{\rho} = \tau_{\beta} = \tau_{A_1} = -1$

Table 9 x)

Functions  $h(\alpha)$  for the process  $\gamma + p \rightarrow \pi + \Delta$ 

Trajectory	$\pi, \rho, B$	$\pi, A_1, A_2$
$(01; \frac{1}{2} \frac{1}{2})$	$\alpha$	$\alpha$
$(01; \frac{1}{2} - \frac{1}{2})$	$\alpha^2$	$\alpha/(\alpha+1)$
$(01; \frac{3}{2} \frac{1}{2})$	$\alpha^2$	$\alpha/(\alpha+1)$
$(01; \frac{3}{2} - \frac{1}{2})$	$2\alpha^2(\alpha-1)$	$2\alpha(\alpha-1)/(\alpha+1)$

x) Here instead of  $h_{CA;D\delta}$  we write simply  $(CA;D\delta)$

Table 10 x)

Functions  $h(\alpha)$  for the process  $\gamma + p \rightarrow \nu + \Delta$ 

Trajectories	$\pi, \rho, \beta$	$\pi_c, A_1, A_2$
$(01; \frac{1}{2} \frac{1}{2})$	$\alpha$	$\alpha$
$(01; \frac{1}{2} - \frac{1}{2})$	$\alpha^2$	$\alpha/(\alpha+1)$
$(01; \frac{3}{2} \frac{1}{2})$	$\alpha^2$	$\alpha/(\alpha+1)$
$(01; \frac{3}{2} - \frac{1}{2})$	$2\alpha^2(\alpha-1)$	$2\alpha(\alpha-1)/(\alpha+1)$
$(11; \frac{1}{2} \frac{1}{2})$	$\frac{1}{2}$	$\frac{1}{2}$
$(11; \frac{1}{2} - \frac{1}{2})$	$\alpha$	$\alpha$
$(11; \frac{3}{2} \frac{1}{2})$	$\alpha$	$\alpha$
$(11; \frac{3}{2} - \frac{1}{2})$	$2\alpha(\alpha-1)$	$2\alpha(\alpha-1)$
$(-11; \frac{1}{2} \frac{1}{2})$	$2\alpha(\alpha-1)$	$2\alpha(\alpha-1)$
$(-11; \frac{1}{2} - \frac{1}{2})$	$2\alpha^2(\alpha-1)$	$2\alpha(\alpha-1)/(\alpha+1)$
$(-11; \frac{3}{2} \frac{1}{2})$	$2\alpha^2(\alpha-1)$	$2\alpha(\alpha-1)/(\alpha+1)$
$(-11; \frac{3}{2} - \frac{1}{2})$	$2\alpha^2(\alpha-1)^2$	$2\alpha(\alpha-1)/(\alpha+1)(\alpha+2)$

x) Here instead of  $h_{CA; D\delta}$  we write simply  $(CA; D\delta)$

## 6. Observable quantities

For the process  $a + b \rightarrow c + d$  the differential cross section with no polarizations is

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{ab}^2} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{c,d,a,b} |f_{cd;a,b}^s(s,t)|^2 \quad (6.1)$$

From the orthogonality of the crossing matrix  $d\sigma/dt$  can be expressed in terms of the t-channel amplitudes, namely

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s p_{ab}^2} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{c,A,D,b} |f_{cA;D,b}^t(s,t)|^2 \quad (6.2)$$

Using (2.6) and conservation of parity  $^{1/2}$  we can express (6.2) by means of the parity-conserving amplitudes (2.7). We have for the process  $\gamma + p \rightarrow \pi + \Delta$ :

$$\begin{aligned} 6\pi s p_{N\gamma}^2 \frac{d\sigma}{dt} = & \gamma^2 (|F_1^t|^2 + |F_2^t|^2) + \\ & + (1+x^2) [ |F_3^t|^2 + |F_4^t|^2 + |F_5^t|^2 + |F_6^t|^2 + \gamma^2 (|F_7^t|^2 + |F_8^t|^2) ] \\ & - 4x \cdot \text{Re} (F_3^t F_4^{t*} + F_5^t F_6^{t*} + \gamma^2 F_7^t F_8^{t*}). \end{aligned} \quad (6.3)$$

For the process  $\gamma + p \rightarrow V + \Delta$ :

$$\begin{aligned} 6\pi s p_{N\gamma}^2 \frac{d\sigma}{dt} = & |H_9^t|^2 + |H_{10}^t|^2 + (1+x^2 + 6x) (|H_{23}^t|^2 + |H_{24}^t|^2) + \\ & + \gamma^2 [ |H_4^t|^2 + |H_2^t|^2 + |H_{11}^t|^2 + |H_{12}^t|^2 + |H_{13}^t|^2 + |H_{14}^t|^2 + \\ & + \gamma^2 (|H_{15}^t|^2 + |H_{16}^t|^2 + |H_{17}^t|^2 + |H_{18}^t|^2) ] + \end{aligned} \quad (6.4)$$

$$\begin{aligned}
 & + (1+x^2) \left[ |H_3^t|^2 + |H_4^t|^2 + |H_5^t|^2 + |H_6^t|^2 + y^2 (|H_7^t|^2 + |H_8^t|^2 + \right. \\
 & \quad \left. + |H_{19}^t|^2 + |H_{20}^t|^2 + |H_{21}^t|^2 + |H_{22}^t|^2) \right] - \\
 & - 4x \operatorname{Re} \left[ H_3^t H_4^{t*} + H_5^t H_6^{t*} + 2(1+x^2) H_{23}^t H_{24}^{t*} \right. \\
 & \quad \left. + y^2 (H_7^t H_8^{t*} + H_{19}^t H_{20}^{t*} + H_{21}^t H_{22}^{t*}) \right].
 \end{aligned}$$

By means of (5.6) we can express  $d\sigma/dt$  through the Regge parameters  $\alpha_i$  and  $\gamma_{CA;DB}^{\alpha_i}$ . The spin density matrix of the  $\Delta$ -isobar in Jackson frame is equal<sup>[34]</sup>

$$\rho_{m'm} = \frac{\sum_{CA,B} f_{CA;m'B}^{t*} f_{CA;mB}^t}{\sum_{CA,DB} |f_{CA,DB}^t|^2}. \quad (6.5)$$

From (2.6) and (5.6) we can express  $\rho_{m'm}$  through the Regge parameters. The matrix elements  $\rho_{m'm}$  can be found from the angular distribution of the decay products<sup>[34]</sup>

$$\begin{aligned}
 W(\theta, \varphi) \simeq c \left\{ \rho_{33} \sin^2 \theta + \left( \frac{1}{2} - \rho_{33} \right) \left( \frac{1}{3} + \cos^2 \theta \right) \right. \\
 \left. - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{3-1} \sin^2 \theta \cdot \cos 2\varphi - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{31} \sin 2\theta \cdot \cos \varphi \right\} \quad (6.6)
 \end{aligned}$$

## 7. Conclusion

It is known, that the explanation of the forward peaks in the reactions  $\gamma + p \rightarrow \pi^+ + n$  and  $p + n \rightarrow \pi + p$  by means of  $\pi$ -conspiracy, requires a very strong dependence on  $t$  of the pion residue. Amati et al.<sup>[36]</sup>

suggested to explain this peaks by contribution of the cuts with an evasive pion Regge pole. In a recent paper Froyland and Gordon<sup>/37/</sup> proposed a mixed model with evading  $\pi, \rho$  trajectories and conspiring  $\mathcal{P}_1 - \rho, \rho - \rho$  cuts, where  $\rho$  is the Pomeranchuk trajectory. Then the conspiracy relations can be satisfied by the contribution of the cut, which plays in some sense the role of a conspirator trajectory. The results of this model are in agreement with experiment for a wide range of the momentum transfer ( for  $|t|$  from  $10^{-4}$  to  $0.2 (\frac{\text{GeV}}{c})^2$  ).

The preliminary experimental data on the differential cross section of the reaction  $\gamma + p \rightarrow \pi + \Delta$  /35/ do not show peak in the forward direction. At momentum transfer  $|t| \geq 0.2 (\frac{\text{GeV}}{c})^2$  the  $\Delta$  production cross section is almost coincides with single  $\pi^+$  production cross section. At small momentum transfer the differential cross section rises as  $e^{12t}$  and in maximum is equal to six time  $\pi^+$  cross section at small momentum transfer. Then  $\Delta$  cross section show a dip as the momentum transfer goes to  $t_{\text{min}}$ . But it is not clear whether the cross section extrapolates to finite value or to zero in the forward direction.

The numerical analysis can decide is the data consistent with factorization of the pion exchange (  $\pi$  conspiracy) or more complicated non-factorizable singularities (  $\pi - \rho$  out for example) in the complex angular momentum are present.

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