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FIELD OPERATOR FOR INTERACTING UNSTABLE ELEMENTARY SYSTEM

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FIELD OPERATOR FOR INTERACTING UNSTABLE ELEMENTARY SYSTEM [xx)]

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Usually one assumes that every intermediate state occuring in the collisions of elementary particles can be described in terms of free stable particles, with definite spins and masses. In recent years it has been shown, however, that one gets remarkably good fits to experimental data if one introduces the exchange of unstable objects, like resonances or Reggeons. It is interesting, therefore, to modify the conventional formalism of quantum field theory in such a way that the unstable objects may become the primary ones, and can be defined by means of the one-particle states.

In this lecture we wish to outline the quantum field theory of interacting resonances. We assume for simplicity that the resonances are characterized by a sharp value of spin, independent of interaction. This restriction should be dropped if we pass to the description of Reggeons, a case which we shall discuss in detail in another publication.

The resonances as one-particle states have been discussed already in the frame of Feynmann diagrams method /1/, by means of group theoretical approach /2/, and using the techniques of infinite-component wave equations /3/. All these considerations are, however, not sufficient for our purpose, because the following two basic problems, concerning unstable systems, remained unsolved[x]:

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[x] The difficulties in solving the problems a) and b) are caused by the instability property, and this explains why it is often assumed that the resonances are approximated by stable particles.

a) The relation between the S-matrix [x)] and field-theoretic definition of the elastic resonance.

b) The problem of asymptotic limits for the field operator describing interacting resonance, and the consistency with Haag-Ruelle theory of asymptotic states.

We intend to give the answer to these two questions in this lecture.

Let us discuss briefly the notion of resonant scattering. If the scattering develops a resonance this is an effect describing particular correlation of ingoing and outgoing wave packets. The resonance, contrary to the case of appearence of a bound state, is not observed asymptotically as a new object [xx)] but represents a name for a particular type of the scattering process. The field operator, describing resonance, should therefore describe by means of its one-particle wave function the space-time development of the resonant scattering, The main idea is to relate single field operator with the resonant scattering channel.

The interacting multiparticle states, defining scattering channel, are kinematically characterized by total mass s, total angular momentum J, and by some additional quantum numbers a which are called the degeneracy parameters/4/ [xxx)]

[x)] We define the resonance by the property that the scattering phase passes through $\pi/2$. The definition by means of the complex pole on the unphysical sheet seems to be less appropriate (see G. Callucci, L. Fonda and G.C. Ghirardi. Phys.Rev., 166, 1719 (1968)).

[xx)] The bound state is obtained as a limiting case, when the correlation between ingoing and outgoing states becomes so singular that we are forced to enlarge the space of asymptotic states, unless the unitarity conditions are violated.

[xxx)] One can say that the degeneracy parameters determine internal spins of the channel functions.

We shall consider here only elementary channels, defined by means of multi-particle states fully characterized by their total four-momentum and the angular momentum parameters J and α x). Such multiparticle states represent an unstable elementary system[4] because one cannot measure the observables characterizing its separate components without destroying the system. We attribute to every such elementary system a field operator. It should be stressed that the notion of unstable elementary system is much wider than the notion of a resonance, and describes also nonresonant ways of scattering.

Our main formal assumptions are the following ones:

1) The interacting elementary system, characterized kinematically by the choice of the mass spectrum $s \subseteq \Sigma$ ($\Sigma \subset R^1_+$) spin J and the degeneracy parameters α, is described by the field operator $\phi_{j;\alpha}(x;s)$ ($j = J, J-1 \ldots -J$).

2) We assume Lorentz invariance and the locality condition

$$[\phi_{j;\alpha}(x;s); \phi_{j;\alpha}(x';s')] = 0 \quad \begin{matrix} x-x' \text{ space-like} \\ s, s' \text{ arbitrary} \end{matrix}.$$

3) The interaction between the multiparticle states goes only through short range forces acting between the elementary systems described by the field operators $\phi_{j;\alpha}(x;s)$, and it is possible to introduce the free asymptotic fields $\phi^{in}_{j;\alpha}(x;s)$ and $\phi^{out}_{j;\alpha}(x;s)$, satisfying the following commutation relations

$$[\phi^{in}_{j,\alpha}{}^{\!\!out}(x;s), \phi_{j',\alpha'}(x',s')] = i\delta_{\alpha\alpha'}\delta_{JJ'}\hat{P}^{(J)}_{jj'}(\partial_\mu)\Delta(x-x';s)\delta(s-s'), \qquad (1)$$

where $\hat{P}^{(J)}_{jj'}(\partial_\mu)$ represents the projection operator defining commutator function for the spin J field[5].

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x) In N-particle elementary channel α denotes fixed set of eigenvalue of 3N-6 commuting angular momentum operators, forming together with the total four-momentum operator, total angular momentum and its third component-the complete set of commuting observables, fully characterizing the N-particle states.

The fields (1) and the asymptotic condition for the simplest case when the indices j and α are not present (S -wave two-particle subsystem) have been introduced by Licht[6]. It can be shown[7] that introducing suitably modified Wightman axioms for the field $\varphi_{j;\alpha}(x;s)$ one can prove rigorously the asymptotic conditions, leading to asymptotic fields (1), following the lines of Hepp's proof of LSZ formulation in the frame of Wightman formalism (see[8]).

Now we introduce the notion of free elementary unstable system. Such systems are defined by means of free field operator $\phi^0_{j;\alpha}(x;s)$ having c-number commutators:

$$[\phi^0_{j;\alpha}(x;s), \phi^0_{j',\alpha'}(x',s')] = i\int d\kappa^2 \rho^{(J)}_{\alpha\alpha'}(s,s';\kappa^2)\cdot \cdot P^{(J)}_{jj'}(\partial_\mu)\Delta(x-x';\kappa^2). \quad (2)$$

It is easy to see that every decoupled elementary channel, and particularly a channel developing elastic resonance, can be described by such free field with a parameter. Because from (2) follows that all truncated VEV of order higher than two vanish the field $\phi^0_{j;\alpha}(x;s)$ can lead only to the scattering of one-particle states. It should be stressed that the presence of the continuous parameter a causes the fact that the scattering of one-particle states is possible. We define the asymptotic one-particle states as follows

$$|\vec{p};s;j;\alpha>_{\substack{in\\out}} = a^{\substack{in\\out}}_{j;\alpha}(\vec{p};s)\,|0>, \quad (3)$$

where the creation operators, occuring in the definition (3), are obtained from the asymptotic fields (1) by means of the conventional formulae for spin J fields[5], and satisfy the following commutation relations:

$$[a^{+\substack{in\\out}}_{j;\alpha}(\vec{p};s), a^{\substack{in\\out}}_{j',\alpha'}(\vec{p}';s')] = \delta_{\alpha\alpha'}\delta_{jj'}\delta(\vec{p}-\vec{p}')\delta(s-s'). \quad (4)$$

Lorentz invariance and the unitarity condition imply that

$$|\vec{p};s;j;\alpha>_{out} = e^{2i\eta_{J;\alpha}(s)}\,|\vec{p};s;j;\alpha>_{in}, \quad (5)$$

where the function $\eta_{J;\alpha}(s)$ describes the phase shift in the decoupled channel (J,α). We see therefore that there is one-to-one correspondence between the elastic scattering and the scattering of one-particle states in our formalism. An example of such formulation has been provided by Thirring/6/ in his Lagrangian approach to the Zacharisen model/10/.

Now we are prepared to solve our problems. The first answer (see a)) follows from the relation (5). Every field operator $\phi^{0}_{j;\alpha}(x;s)$, having such asymptotic limits that the relation (5) remains valid, is a good candidate for the space-time description of an elastic resonance, present in the phase shift $\eta_{J;\alpha}(s)$. The consistency with Haag-Ruelle theory of asymptotic states (see b)) follows directly from the physical interpretation of the asymptotic fields (1). The asymptotic states

$$|\vec{p}_1 \ldots \vec{p}_n ; s_1 \ldots s_n ; j_1 \ldots j_n , \alpha_1 \ldots \alpha_n >_{\substack{in \\ out}} = \prod_{i=1}^{n} a^{\substack{in \\ out}}_{j_i,\alpha_i} (\vec{p}_i ; s_i) | 0 > \qquad (6)$$

represent only another way of description of conventional asymptotic states. Particularly if $n=1$ (see (3)) we obtain in the considered elementary channel (J,α) the description of ingoing and outgoing multiparticle states with a given value of total fourmomenta $(\vec{p}, p_0 = \sqrt{\vec{p}^{\,2}+s})$ and given polarization index. The physical in- and outgoing wave packets are obtained by smearing-out with some smooth function $f(s)$ with supp $f(s) \subset \Sigma$. Such asymptotic wave packets can be described by means of generalized free fields/11/. The intuition, that the propagators with continuous mass spectrum, substituted in Feynmann graphs, describe in relativistic quantum field theory the exchange of unstable objects, is already an old one/12/. In the formalism, presented here, such conclusion follows in a natural way.

The existence of our alternative description of interacting system and its asymptotic states is caused by a particular type of dynamics, which allowed to introduce for the field operators $\phi_{j;\alpha}(x;s)$ the asymptotic fields (1). If the interaction is such that it does not require the introduction of separate asymptotic particles, we can forget about the stable multiparticle asymptotic states, and discuss only the scattering of the free elementary unstable objects, described by the asymptotic fields with continuous mass parameter. The scattering matrix operator can be defined as follows:

$$\phi^{out}_{j;a}(x;s) = \hat{S}^{-1}\phi^{in}_{j;a}(x;s)\hat{S}\ . \qquad (7)$$

Further details are similar like in the LSZ scattering theory.

The simplest way of expressing the dynamics, consistent with our formulation, is to introduce local Lagrangeans of the field operators $\phi_{j;a}(x;s)$. It is also possible, however, to formulate the appropriate dynamics, using dispersion-theoretic approach, or, at least in some cases, by introducing suitably truncated set of conventional Feynmann graphs.

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