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A LOWER BOUND OF THE ELASTIC SCATTERING AT FIXED MOMENTUM TRANSFER II

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Нижняя граница амплитуды упругого рассеяния при фиксированной передаче импульса

На основе аксиоматической аналитичности получена нижняя граница амплитуды упругого рассеяния, справедливая для всех конечных передаваемых импульсов.

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A Lower Bound of the Elastic Scattering at Fixed Momentum Transfer II

On the basis of axiomatic analyticity we establish some lower bound of the elastic scattering amplitude for all finite momentum transfer.

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A LOWER BOUND OF THE ELASTIC SCATTERING AT FIXED MOMENTUM TRANSFER II

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In the previous work by one of us [1] it was shown, that on the basis of the rigorously proved analytical properties of the absorptive part $A(s,t)$ of the elastic scattering amplitude, we can prove some lower bound for this amplitude in some interval of the momentum transfer.

In this work we generalize the results obtained in ref. [1] and establish some lower bound which holds for all finite momentum transfer $t \leq 0$.

We know that $A(s,t)$ is an analytic function of t in the Martin ellipse E_t with the foci at $t=0$ and $t=-4k^2$ and the major semiaxis $2k^2+\gamma$ for some constant $\gamma > 0$ (k- is the three dimensional momentum in c.m.s.) and for all values of t in this ellipse $A(s,t)$ grows at $s \to \infty$ no faster than const. $s^{1+\epsilon}$, $\epsilon \leq 1$. We consider two points $t_+ = -a$ and $t_- = -4k^2 + a$ for some $a > 0$ and denote by $w(t)$ such an analytic function which maps conformally the ellipse E_t in the t-plane into some ellipse E_w in the w -plane with the foci at $w = \pm 1$, that the points $t_\pm$ are transformed into the foci ± 1 , resp. We denote by w_0 , the image of the point $t = 0$, and by a, the major semiaxis of the ellipse E_w, resp. The magnitudes of w_0 and a depend on s, α and γ.

The conformal mappings of the ellipse into the half-plane and of the half-plane into the ellipse are studied in book /2/. Using the expression given in /2/ it can be shown, that at $s \to \infty$

$$w_0 \approx 1 + \frac{8}{\pi^2} \frac{\gamma}{s} \{ \arccos [ch^{-1}(-\frac{\pi}{2}\sqrt{\frac{\alpha}{\gamma}})]\}$$

$$a \approx 1 + \frac{2\gamma}{s} .$$

Now by means of the conformal mapping

$$\xi = w + \sqrt{w^2 - 1}$$

we transform the ellipse E_w into the ring with the internal radius 1 and the external one R

$$R = a + \sqrt{a^2 - 1}$$

following Cerulues and Martin /3/ (see also /1,4/). Further applying the Hadamard's theorem on three circles, we obtain the lower bound for the ratios $f(s,t) = \frac{A(s,t)}{A(s,0)}$

$$\max_{-4k^2+\alpha \leq t \leq -\alpha} |f(s,t)| \geq s^{-(\epsilon-\rho)\phi(\frac{\pi}{2}\sqrt{\alpha/\gamma})} ,$$

where

$$\phi(x) = \frac{\arccos (ch^{-1} x)}{\frac{\pi}{2} - \arccos (ch^{-1} x)}$$

and ρ is such a constant that at $s \to \infty$

$$A(s,o) \geq \text{const } s^{1+\rho} .$$

In particular if the amplitude has the Regge behaviour [5] then for the trajectory $\beta(t)$ we have the following lower bound

$$\max_{-4k^2+a\leq t\leq -a} \beta(t) \geq 1 + \rho - (\epsilon - \rho)\,\phi\left(\frac{\pi}{2}\sqrt{a/\gamma}\right) \quad .$$

For πN scattering the analyticity on t was proved for $\gamma = 1{,}83\ m_\pi^2$. However one can hope, that γ achieves value $4\,m_\pi^2$.

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