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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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SUMMATION OF ASYMPTOTICS IN PERTURBATION THEORY AND HIGH ENERGY BEHAVIOUR

Submitted to the XIVth International Conference on High Energy Physics, Vienna, 1968

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1. *Introduction*

Till now the perturbation theory with renormalization is the only regular method in QFT. It proves to be good for quantum electrodynamics, but for the only renormalizable mesodynamics Lagrangian[x)]

$$L_{int} = g \bar{\psi} \gamma^5 \psi \phi + h \phi^4 \tag{1.1}$$

there exists a well-known difficulty because of the large values of the coupling constants g and h. Nevertheless for high-energy region, where the small parameters m^2/s, t/s, $\log(m^2/s)$ and $\log(t/s)$ naturally appear, the situation is greatly simplified. (As usual 4 external momenta p are on the mass shell (fig.1), the squared c.m. energy $s=(p_1+p_2)^2$, the squared momentum transfers $t=(p_1-p_1')^2$, $u=(p_2-p_2')^2$ and $s+t+u=m_1^2+m_2^2+m_1'^2+m_2'^2$)

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x) *For the sake of definitness we speak here about pions and nucleons but all the considerations are valid for meson and baryon octets of SU_3 scheme, broken by the mass term alone.*

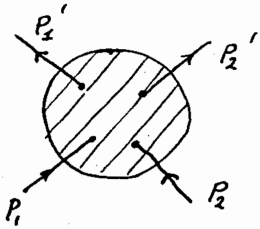

Fig.1

In previous papers[1] many pages were devoted to the detailed consideration of the $/s/ \to \infty$ asymptotical behaviour of the Feynman graphs in the theory (1.1). (The results of the investigation were collected in paper [2]). The perturbation theory in this limit seemed to us to be so simple that there appeared a natural temptation to sum up the asymptotics of all graphs. This resulted in the appearance of this paper. But in the course of the work we had understood that our intension would be reached more easily if we supposed the bare charge to be a finite number; so we did. We should confess that the statement of the problem in itself contains a groundless assumption that the asymptotic of a sum is the sum of asymptotics. But we trust on the smallness of the parameters for the asymptotical terms and that in the region $t \approx 0$, junior logarithm will remain junior in the sum. In other words, we hope for the so-called "doublelogarithmic situation". The comparison with experiment shows that we are right.

As to the asymptotics, one can say that any graph with integer angular momentum in the crossing t-channel, increases as

$\log^H$, the power H being determined by the number of two-particle separation in this channel and being independent of the remaining details of the structure. So, the Bethe-Salpeter scheme is especially suitable in searching for the asymptotics of amplitudes in the theory (1.1.). Symbolically the B-S equation can be written as a system for the $T_{\pi\pi}$, $T_{\pi n}$ and T_{nn} amplitudes (see fig.1).

$$T_{\pi\pi} = V_{\pi\pi} + V_{\pi\pi} \times T_{\pi n} + V_{\pi n} T_{\pi n}\,;\; T_{\pi n} = V_{\pi n} + V_{\pi n} \times T_{\pi n} + V_{\pi n} \times T_{nn} \tag{1.2}$$

$$T_{nn} = V_{nn} + V_{n\pi} \times T_{\pi n} + V_{nn} \times T_{nn}$$

the amplitudes T being here the sum of all the graphs of the process and the "potentials" V being the sum of the graphs without two-particle separations in crossing channel. In practice , this system breaks on the subsystems with definite quantum numbers in crossing channel, which conserve in the course of iteration. The iterated amplitudes are the usual linear combinations of the physical amplitudes of the s-channel. For example, the amplitude T_{nn} in (1.2) is a linear combination of the nucleon-nucleon and nucleon-antinucleon scattering amplitudes.

As in the usual Regge-pole theory the devision of the amplitudes into parts with definite signatures T^+ and T^- (symmetric and antisymmetric under the exchange $s \to u \approx -s$) appears to be useful for us. For the connection of this division with the topology of graphs see below.

In this paper we restrict ourselves to the amplitudes with integer angular momentum in the crossing channel.

2. Structure of Amplitudes and Potentials

The structure of the amplitudes is well known. The pion-pion scattering amplitude $T_{\pi\pi}$ is simply a scalar: $T_{\pi n}$ and T_{nn} have the usual form

$$T_{\pi n} = A + \frac{p_2 - p_2'}{2} B$$

$$T_{nn} = (I_{(1)} \times I_{(2)}) F_S + (\gamma^5_{(1)} \times \gamma^5_{(2)}) F_P + (\gamma^{\mu}_{(1)} \times \gamma^{\mu}_{(2)}) \frac{F_V}{2s} + \tag{2.1}$$

$$\times (\gamma^5_{(1)} \gamma^{\mu}_{(1)} \times \gamma^5_{(2)} \gamma^{\mu}_{(2)}) \frac{F_A}{2s} + (\sigma^{\mu\nu}_{(1)} \times \sigma^{\mu\nu}_{(2)}) \frac{F_T}{s}$$

(The isospin structure of these amplitudes is trivial and we do not write it explicitly). In the perturbation theory these amplitudes are built up according to the usual Feynman rules [3] and the general normalization is such that, for instance

$$\mathrm{Im}\ T_{\pi\pi} (t = 0) = \frac{s}{16 \pi^2} \sigma_{tot} \quad .$$

In fact, we know from the analysis of graphs [1,2] that amplitudes A, F_S and F_P do not contain senior logarithmic terms which agree with the absence of spin-flip for πn -scattering in high energy region. The amplitude F_T corresponds to the Regge-pole of the β -type [4] and will be considered elsewhere. (This amplitude is composed of senior logarithms of so-called "odd graphs" [1,2] i.e. of the graphs with an odd-pion intermediate state in the crossing channel). The remaining amplitudes can be distinguished by the quantum numbers of the crossing channel (see table 1). According to this table the system (1.1) often degenerates into one equation. We consider the asymptotic of the potential in (1.2) as a sum of the kernel asymptotics, i.e. the graphs without two-particle separation in the crossing channel. But the asymptotic of any kernel V_1 consists of the parts of positive and negative

Table 1

Amplitudes / Quantum numbers and all that	F_V, B, $T_{\pi\pi}$ "vacuum"	group	$T_{\pi\pi}$	F_V		F_A			
Isospin I	0	1	2	0	1	1	1	0	0
G-parity	+	+	+	−	−	−	−	+	+
Signature (τ)	+	−	+	−	+	+	−	+	−
"Internal parity" ($\tau \cdot P$)	+	+	+	+	+	−	−	−	−
Particles	P, P', f, f', η_V	ρ, ρ_V	?	ω, φ	π_V, A_2	π	A_1	(E)	$\mathcal{D}$
Potentials									

signatures which have the universal form (up to $1/S$)

$$V_i^+ = \frac{a_i}{2} [\log(-s) + \log(s)] \qquad \text{for } \pi\pi\text{-kernels,}$$

$$V_i^+ = a_i \qquad \text{for the other kernels} \quad (2.2)$$

and

$$V_i^- = -i\pi b_i = b_i(\log(-s) + \log(s))$$

(the isospin and G-parity indices are omitted here). If a kernel is symmetric under $s \to u$ then $V_i^- = 0$. For any plannar $\pi\pi$ -kernel $2b_i = a_i$ but for any plannar πn- or nn -kernel the sum of any two graphs obtained from each other by the $s \to u$ exchange has a definite signature. In table I we give also the types of potentials which can contribute to a given amplitude, but none of the kernels may be odd.

Consider now the divergences. All divergent subgraphs are viewed as regularized. However, the $\pi\pi$ -kernels (which are themselves divergent) are automatically regularized by the Mellin transformation (3.1). Unlike the other divergent subgraphs, their contribution to the asymptotic is calculated in just the same way as the contribution of other kernels [1,2]. So, we may forget the divergency of such subgraphs.

The remaining divergences are taken into account according to the following scheme. First of all we look for the asymptotics of kernels when $s \to \infty$ (and "interkernel" lines with respect to their momenta p^2). In Appendix we show that the contribution of the internal divergent parts of any kernel are simply factorized. Summing over the different kernels we obtain in this case the potential as a series in powers of invariant charges [3], rather than in powers of g and h. These invariant charges absorb into themselves all the divergences (except the divergences of internal lines). The most simple and especially interesting is the case when the asymptotical limits of the invariant charges, which correspond to bare charges, are some finite numbers. Simultaneously, the asymptotics of the propagators (up to renormalization) are naturally supposed to be the same as free propagators. Some other possibilities of the propagator and invariant charges asymptotics will be considered elsewhere. The asymptotics of the sum of the kernels which differ by the divergences alone in this case have the form (2.2) and the asymptotics of the potentials (the sum of all the kernels, if any) can be written (up to 1/s terms) as follows

$$V^{+}_{\pi\pi} = -v_{\pi\pi}\,\frac{\log s + \log(-s)}{2}, \qquad V^{-}_{\pi\pi} = i\pi\, u_{\pi\pi}$$

$$V^{+}_{\pi n} = \frac{v_{\pi n}}{\sqrt{2}}; \qquad V^{-}_{\pi n} = -i\pi\,\frac{u_{\pi n}}{\sqrt{2}} \tag{2.3}$$

$$V^{+}_{nn} = \frac{v_{nn}}{2}; \qquad V^{-}_{nn} = -i\pi\, u_{nn}/2 .$$

We always disregarded here, terms 1/s . It seems natural that their contribution to the amplitude does not exceed that of the senior terms taken into account (for more details see Discussion). In addition , we disregarded the junior logarithm. We hope that this approximation corresponds to the summation up to $(\log s)^{-1}$ but this hypothesis seems to be less realistic than the first one concerning $1/s$). However, if the potentials v and u in (2.3) are small enough then in the energy region $1 \ll \log s \lesssim 1/v$ our summation is equally well grounded as the "doublelogarithmic" one in electrodynamics (see[6] and bibliography there).

3. *Asymptotic of Amplitudes*

It is more easy to investigate the asymptotical behaviour of the amplitudes using the Mellin transformation. The leading singularities in the complex μ plane, coincide with that of the usual j-plane (see[1])

$$T^{\pm} = -\frac{1}{2i} \int_{\delta - i\infty}^{\delta + i\infty} d\mu \, (-s)^{\mu} \, A(\mu) \, \frac{f^{\pm}(\mu, t)}{\Gamma(1+\mu)} \, \frac{1 \pm e^{-i\pi\mu}}{\sin \pi\mu} , \tag{3.1}$$

where for convenience the factor A depending on the definite process is introduced:

$$A_{\pi\pi} = 1 , \quad A_{\pi n} = -\mu/\sqrt{2} , \quad A_{nn} = \mu/2 . \tag{3.2}$$

According to /1/ we know that the leading singularity of $f_i(\mu, t)$ for any graph T_i is the pole of some order at the point $\mu = 0$. We know also from Appendix 2 ref. 1 that the residue of this pole does not depend on t and for the finite bare charge $f^{\pm}$ is the product of the kernel contributions and a certain factor depending only on the number of kernels n .

This allows us to sum up the leading singularities of all graphs with a given number of kernels n and obtain the iterated solution of the B-S equation with n potentials V. The contribution of the potentials to the amplitudes $f^{\pm}(\mu, t)$ according to (2.3), have asymptotically the form

$$v_{\pi\pi}/\mu; -v_{\pi n}/\mu; \; v_{nn}/\mu \quad \text{into} \quad f^{+}(\mu)$$

$$u_{\pi\pi}/\mu; \; -u_{\pi n}/\mu; u_{nn}/\mu \quad \text{into} \quad f^{-}(\mu)$$

Besides, the factor $\sqrt{\mu}$ has to be added to each potential $V_{\pi n}$ and each pair of external spinor lines. Two of these factors for πn - and nn -amplitudes are taken into account by $A(\mu)$. Notice, the number of such factors is to be always even so there is no branching point at $\mu = 0$.

The factorization of the kernels reduces the whole problem of summation to the problem of the type of "ladder" graphs. The equation for $f^{\pm}$ is similar to the equation for the ladder graphs [7,8]. The asymptotic of any graph is determined by the number of the kernels and the number of unions of kernels (including the graph itself). It can be deduced as follows. Any union of kernels is again a scattering graph, therefore denoting by f_m the amplitude with m kernels we have[x)] asymptotically for $m \geq 2$

$$f_m = \frac{1}{\mu} \sum_{k=1}^{m-1} f_k \, f_{m-k} \; .$$

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x) *The asymptotic of a graph in the α -representation is a result of integration over the maximal number of parameters λ which arises from the barycentrical transformation of α's belonging to the kernels and the unions of kernels in the region $\lambda \approx 0$. One of these 2λ's corresponds to the graph as a whole. After integration over it, at least one pair of α's corresponding to the interkernel lines is to be finite. This just corresponds to the division of the graph into two parts. The exp. (1.4) is the sum over all possible division of the kind.*

The factor μ^{-1} on the right-hand side corresponds to the unions f_k and f_{m-k}. Notice now, that the sum over f_m from 1 to ∞ is simply the amplitude f and f_1 is the potential v or u of (3.3). So the addition of f_1 to the left- and right-hand sides and the summation over m gives a nonlinear equation for the amplitude.

a. "Vacuum Group"

For the amplitudes of the so-called "vacuum group" (see table I) the division of the graph can proceed either by two pion or two nucleon lines. For the latter case the factor $\sqrt{\mu}$ (and $A(\mu)$), which we mentioned above, is to be taken into account. So, the summation over f^+ results in the system for

$$\mu(f^+_{\pi\pi} - v_{\pi\pi}/\mu) = f^{+2}_{\pi\pi} + \mu f^{+2}_{\pi n}$$

$$\mu(f^+_{\pi n} + v_{\pi n}/\mu) = f^+_{\pi n}(f^+_{\pi\pi} + f^+_{nn}) \tag{3.5}$$

$$\mu(f_{nn} - v_{nn}/\mu) = f^{+2}_{nn} + \mu f^{+2}_{\pi n}$$

(In analogous equations of the papar[8] the factor μ on the right-hand side has been forgotten). Solving the system we easily obtain

$$f^+_{\pi\pi} = \frac{\mu}{2} - \frac{\mu^2/4 - v_{nn} + \sqrt{\rho^+}}{R^+}; \quad f^+_{\pi n} = \frac{v_{n\pi}}{R^+}; \quad f^+_{nn} = \frac{\mu}{2} - \frac{\mu^2/4 - v_{\pi\pi} + \sqrt{\rho^+}}{R^+} \tag{3.6}$$

where

$$\rho^+ = \left(\frac{\mu^2}{4} - v_{nn}\right)\left(\frac{\mu^2}{4} - v_{\pi\pi}\right) - \mu v^2_{\pi n}\ ; \quad R^+ = \sqrt{\frac{\mu^2}{2} - v_{\pi\pi} - v_{nn} + 2\sqrt{\rho^+}}\ .$$

All the amplitudes have a square-root branchpoints on μ - (or j -) plane when

$$\rho^{+} = \left(\frac{\mu^{2}}{4} - v_{nn}\right)\left(\frac{\mu^{2}}{4} - v_{\pi\pi}\right) - \mu\, v_{\pi n}^{2} = 0\,. \tag{3.7a}$$

In the vicinity of these points, up to a nonessential additive constants

$$\begin{aligned} f_{\pi\pi}^{+} &= \tilde{\tilde{c}}^{+}\left(\frac{\mu^{2}}{4} - v_{\pi\pi}\right)\sqrt{\rho^{+}} \\ f_{\pi n}^{+} &= \tilde{\tilde{c}}^{+}\, v_{\pi n}\sqrt{\rho^{+}} \\ f_{nn} &= \tilde{\tilde{c}}^{+}\left(\frac{\mu^{2}}{4} - v_{nn}\right)\sqrt{\rho^{+}}\,. \end{aligned} \tag{3.8}$$

The equations for f^{-} and their solutions can be obtained from (3.5) (3.8) by replacement $v_{\pi\pi} \to u_{\pi\pi}$, $v_{\pi n} \to u_{\pi n}/\mu$, $v_{nn} \to u_{nn}/\mu$. So, the branchpoints of f^{-} are determined by

$$\left(\frac{\mu^{3}}{4} + u_{nn}\right)\left(\frac{\mu^{2}}{4} - u_{\pi\pi}\right) - u_{\pi n}^{2} = 0\,. \tag{3.7}$$

Let us denote by $\mu_{\pm}$ the root the most on the right of (3.7), and suppose it to be positive. This can happen when $v_{\pi\pi}$, $v_{\pi n}$ and $v_{nn} > 0$, which agrees with the first order of perturbation theory. With the help of (3.1) (3.2) we can obtain

$$\left.\begin{matrix} T_{\pi\pi}^{+} \\ \\ T_{\pi n}^{+} \\ \\ T_{nn}^{+} \end{matrix}\right\} = c^{+} R^{+}(s) \times \left\{\begin{matrix} \mu_{+}^{2}/4 - v_{\pi\pi} \\ \\ \mu_{+} v_{\pi n}/\sqrt{2} \\ \\ \mu_{+}(\mu_{+}^{2}/4 - v_{nn})/2\,, \end{matrix}\right. \tag{3.9a}$$

$$\left.\begin{array}{l} T^-_{\pi\pi} \\ \\ T^-_{\pi n} \\ \\ T^-_{nn} \end{array}\right\} = c^- R^-(s) \times \begin{cases} \mu_-^2/4 - u_{\pi\pi} \\ \\ u_{\pi n}/\sqrt{2} \\ \\ (\mu^3/4 - u_{nn})/2\,, \end{cases} \tag{3.9b}$$

$$R^{\pm}(s) = \left(\frac{s}{d_0}\right)^{\mu\pm} \left(\log\frac{s}{s_0}\right)^{-3/2} \frac{1 + e^{-i\pi\mu\pm}}{\sin\pi\mu_{\pm}}\,. \tag{3.10}$$

Notice, that the exact value of the constant $c^{\pm}$ and $\tilde{c}^{\pm}$ is of no importance for us because they are connected with a normalization energy s_0 .

In addition to the leading singularity μ_+, which corresponds naturally to the vacuum trajectory P, eq.(3.7a) gives three branch-points one of which being positive when $v_{\pi\pi}$ and $v_{\pi n}$ are positive. This positive branchpoint can correspond to the second vacuum trajectory P′ . It is natural to compare the singularity μ_- with the ρ -meson trajectory.

More detailed analysis of this solution and comparison with experiment will be given below. Here we do stress the following interesting properties. The leading singularity is the same for all the processes with the same quantum numbers in the crossing channel. The asymptotical behaviour, which this singularity respon-ses for, is such that with the account of the Froissart theorem ($\sigma < \log^2 s$) the total cross sections decrease not slower than

$(\log s)^{-3/2}$ and the ratio $\frac{\mathrm{Re}\,T}{\mathrm{Jm}\,T} \approx (\ln s)^{-1}$. Another interesting property is the factorization of the amplitudes

$$T_{\pi\pi} T_{nn} = T^2_{\pi n}\,; \tag{3.11}$$

which is valid in the neighbourhood of any of the mentioned singu-larities. To check this it is enough to notice that (3.11) coincides with the equation for branching point (3.7).

b) Other Amplitudes

For each of the remaining amplitudes in table I the system (3.5) degenerates to only one equation because of the sole type of kernel responses for the asymptotic of this amplitudes. So, instead of (3.5) we have

$$\mu f^+ - v = f^{+2}\;;\quad \mu f^- - u/\mu = f^{-2} \tag{3.12}$$

the solution of which has the form

$$f^+ = \frac{\mu}{2} - \sqrt{\frac{\mu^2}{4} - v}\;;\quad f^- = \frac{\mu}{2} - \sqrt{\frac{\mu^2}{4} - \frac{u}{\mu}}\,. \tag{3.13}$$

This gives the asymptotical behaviour (for $v > 0$)

$$T^{\pm} = C^{\pm} \frac{s^{\mu\pm}}{(\log s)^{3/2}} \frac{1 + e^{i\pi\mu\pm}}{\sin\pi\mu_{\pm}}\,, \tag{3.14}$$

where

$$\mu^+ = 2\sqrt{v}\;;\quad \mu^- = \sqrt[3]{4u}$$

we believe the conclusion about the asymptotical behaviour, when v or u are negative, to be meaningless if the junior logarithms are not taken into account. The singularity of $F_{\mathbf{v}}$ with $I=0$ among the others seems to be the most interesting because it corresponds to the ω-meson trajectory which plays an important role in nucleon-nucleon scattering [9].

c) Deuteron Amplitudes

The nucleon-antinucleon backward scattering shown in fig.2 is especially interesting. The quantum numbers of the u-channel here correspond to the ground or excited states of a deuteron or to a resonance in the system of two nucleons. (That is why

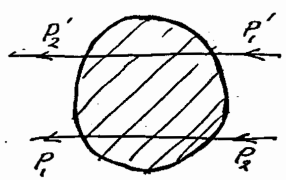

Fig.2

we call it deuteron singularity). All the calculations for this process differ only by the exchange $t \to u$ and the kinematical analysis [4,5] is similar to the nucleon-nucleon one in Sec.2

The equation for the deuteron amplitudes with different quantum numbers is similar to (3.12) which results in the asymptotics (3.14) (3.15) with the corresponding replacement of potentials.

Because of the only type of kernel to be allowed for this process (fig.2), the connection between kernels with different quantum numbers must emerge. In addition the potentials of this process must be connected with the potentials of processes with negative G-parity in t-channel (see table 1) which ruled by similar kernels. That is why the backward nucleon-antinucleon differential cross section must decrease not faster than the combination of nn- and $n\bar{n}$-forward cross sections which is conditioned by the ρ-meson trajectory that is, roughly speaking, as $(\frac{s}{s_0})^{-0,6}$ (see, for instance [9]). The absence of experimental data for backward nucleon-antinucleon scattering makes more detailed calculations difficult. We hope to return to this process when we learn to take the junior logarithms into account.

4. Discussion

So, having learned to extract the asymptotic of any Feynman graph we risk to sum this asymptotics up bearing in mind, of course, that this procedure may seem to us doubtful. We cannot dispel completely these doubts without summing all the perturbation theory. But everybody knows what this problem is! Maximum, what we can hope to do (and what we planned to do in future) is to sum up the junior logarithms. As to the next terms (for instance, $s^{-1} \log^N s$ in $\pi\pi$-scattering) the example of summing of the graphs of fig.3, perhaps the most dangerous,

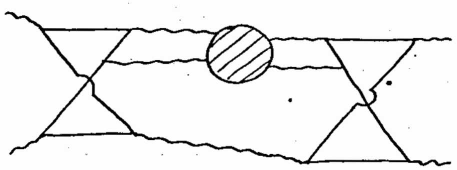

Fig.3

shows that for $\mu_+ = 1$ and $t \approx 0$ their contribution does not exceed the senior logarithm. This happens because we are dealing with the square-root branch point at $\mu = 1$ instead of the pole. But this sum can give a contribution to the junior logarithms.

As an excuse, we can only say now that the potentials $v_{\pi\pi}$, $v_{\pi n}$, v_{nn} which play the role of "interaction constants" in our consideration, appear to be small from the experiment. So in the energy region where $\log s/m^2 \gg 1$ but $v \log(\frac{s}{m^2}) < 1$ our summation is as legal as "doublelogarithmic" summation in quantum electrodynamics. Really, we know from the experiment the total cross sections approximately to go to the constants (so we fix $\mu_+ = 1$ in expression (3.9a)) and the ratio $\frac{\sigma_{\pi n}}{\sigma_{nn}} = 0{,}6$. This means (see (3.9a) again) that $v_{\pi\pi} = 0{,}25 - 0{,}5\ v_{\pi n}$ and $v_{nn} = 0{,}25 - 2\ v_{\pi n}$. The positiveness of $v_{\pi n}$ follows simply from its proportionality to the total cross section with a positive coefficient (as not difficult to check). Let us suppose now that $v_{\pi\pi} > 0$ and $v_{nn} > 0$, as is indicated by the first order of perturbation calculations. Then we obtain immediately that $v_{\pi\pi}$ and $v_{nn} < 0{,}25$ and $v_{\pi n} < 0{,}12$. This gives

the following region of validity of our considerations

$$1\ \text{Gev} \ll E_{Lab} \lesssim 200\ \text{Gev} \ .$$

By the way, this estimate is in good agreement with the values needed for the second positive root of $\rho^+ = 0$ to be the second vacuum trajectory P' when $t = 0$.

The small values of potentials allow us not only to trust the summation of senior terms but to indicate also small values of bare charges. This gives in turn the possibility of calculating the potentials from the lowest graphs of perturbation theory.

Now, believing at least a part of the doubts to be dispelled let us turn to the results. The most interesting is the square-root branchpoints appearance instead of the Regge poles. The leading branchpoint ("vacuum") gives with assumption $\mu^+ = 1$ the slow decrease of the total cross sections as

$$\sigma_{tot} \approx (\log s)^{-3/2} \quad \text{and} \quad \frac{\mathrm{Re}\,T}{\mathrm{Im}\,T} \approx (\log s)^{-1} \ .$$

(Is this not the true reason for effects which one try to explain now by a small deflection of the vacuum trajectory from unity !?).

It is not difficult to understand the origin of these branchpoints. In the "quasipotential" Schrödinger equation (10) the asymptotical behaviour for large s corresponds to the $1/r^2$ behaviour of the potentials in the r-space. This leads, as is well known, to the square-root branchpoints. Generally speaking, there seems to exist some intimate relationship between summation of the asymptotics and the quasipotential which we can understand and evaluate only in the future.

These branchpoints are universal for all the processes just as the Regge poles and for "residues" at each of these points the factorization theorem is automatically valid. This fact is closely connected with the "kernel" structure, determining the asymptotic of any graphs which means in fact, two-particle unitarity.

Our approximation gives us no possibility to notice the movement of singularities. Only junior logarithms can indicate something about this. But we do not know definitely whether this is a moving branchpoint or the Regge pole appears from under the standin branchpoint or something else. The second possibility seems to us more probable because the junior logarithm, by all means, will give the Regge poles, and the values T, calculated here, are playing the role of interaction constants (depending on j!) of the "reggions" with the usual particles. It is interesting to notice in this connection that for the tensor amplitude F_T each kernel gives, probably, only one logarithm (similar to the usual "ladder" graphs in the ϕ^3 -theory). This leads to a pole singularity of j -plane even for the sum of senior terms.

Another problem arises when one tries to take into account other members of the meson and baryon octets for the scheme of dynamically broken SU(3). The system (3.5) will greatly increase but the square-root character of singularities appears to be kept.

In conclusion we would like to say that beginning the summation of the senior terms, we hoped to answer some questions high energy physicists are interested in. In reality, our work seems to be "arrowed to future" that is it generates many more questions than it gives answers. We would like to believe that after summation of the junior logarithmic terms " a pan of questions" will not be as overloaded as now and equilibrium will be more stable.

We are very grateful to everybody who has discussed this work and especially to D.I.Blokhintsev, A.T.Filippov, P.T.Matthews, V.V.Serebryakov, D.V.Shirkov and K.A.Ter-Martirosian. One of us (I.G.) is constantly thankful to D.Stelmach.

Appendix

Notice first of all that the factorization of the divergences on the external lines of the kernel (of the self-energy type) which gives a factor dependent on p^2 is selfevident because such subgraphs are weakly connected with the rest of the kernel.

Turn now to the internal divergences. We show in paper [1] that the presence of r internal divergences increases the power of logarithm (or the order of the leading pole in μ -plane) by r in the asymptotic of the graph. This fact is due to the additional parameters ρ which appear in the expression for regularized contribution of the graph. (Remember, that R-operation[3] is equivalent to the following procedure in the α-representations. We replace each α_σ of nonregularized contribution by $\rho_{i_1}\cdots\rho_{i_q}\alpha_\sigma$ in the line σ enters divergent subgroups $\Gamma_{i_1}\ldots\Gamma_{i_q}$ simultaneously and next we take the residual term of the Maklourin series in ρ of the power of divergency of the subgraph and put $\rho=1$). These give the possibility for introducing r additional sets of parameters which let vanish the coefficient for s in the exponent and determine the leading singularity. To form these sets it is enough to take instead of α's of a divergent subgraph the corresponding ρ .

Let, for instance, the kernel V contain divergent subgraphs $\Gamma_1 \ldots \Gamma_n$. Then together with the usual barycentrical transformation for $\{\alpha_6 \in V\}$ we have the possibility of introducing the sequence of the transformations

$$\{\alpha \in V-\Gamma_1, \rho_1\}, \{\alpha \in V-\Gamma_1 \beta-\Gamma_2, \rho_1, \rho_2\} \ldots \{\alpha \in V-\sum^n \Gamma_i, \rho_1 \ldots \rho_n\}.$$

But this possibility is not unique. For example, if Γ_1 and Γ_2 do not intersect with each other we can introduce a sequence, where $\rho_1 \leftrightarrow \rho_2$. (This does not change the asymptotic itself but doubles the coefficient of it).

How many possibilities of the kind exist in the general case? Let us denote each of them by a set of nembers 1,2...n. with a definite order (for example { 1,2,3...n } or { 2,1,3... n }). First of all, it is clear that if $\Gamma_i \subset \Gamma_j$ then the set $\{\ldots j \ldots i \ldots\}$ is forbidden. Really, let $\Gamma_1 \subset \Gamma_2$, for instance, then after the transformation $\Sigma\ \alpha \in V-\Gamma_1 + \rho_1 = \lambda$ it is useless to introduce the transformation $\Sigma\, \alpha \in V-\Gamma_1-\Gamma_2+\rho_1+\rho_2=\lambda'$ because λ and λ' cannot vanish simultaneously. According to this we must keep the definite subordination for the intersecting subgraphs.

When there are no interesting subgraphs all $n!$ sets are possible. When we have i nonintersecting groups each of them containing mutually intersecting Γ's . In this case the number of possible sets with definite order inside each of the groups is equal, as is well known, to $\dfrac{n!}{n_1! \ldots n_i!}$. Denote by m_i the number of possible sets inside to i-th groups. Then the total number of all sets appears to be $m = \dfrac{n!}{n_1! \ldots n_i!} m_1 \ldots m_i$. Going from the variable μ to s , according to (3.1), we find the additional factor due to the divergent parts to have the form

$$\left(\frac{m_1}{n_1!} \log n_1 s\right) \ldots \left(\frac{m_i}{n_i!} \log n_i s\right) .$$

that is it consists of i factors each of them being exactly the asymptotic of the divergent subgraph.

References

1. I.F.Ginzburg, A.V.Efremov, V.G.Serbo. Preprint IM SO AN SSSR, TΦ-44 (1968);
   I.F.Ginzburg, V.G.Serbo. Preprint TΦ-45 (1968).
2. I.F.Ginzburg. A.V.Efremov, V.G.Serbo. JINR Preprint P2-3833, Dubna (1968), (submitted to Phys. Lett).
3. N.N.Bogolubov, D.V.Shirkov. Introduction to the Theory of Quantized Fields, Interscience Publishers, New York. 1969 .
4. D.V.Volkov, V.N.Gribov. JETP 44, 1068 (1963).
5. D.H.Sharp, W.G.Wagner. Phys. Rev., 131, 222 (1963).
6. V.G.Gorshkov et al. Jorn. of Nucl. Phys., 6, 129 (1967).
7. I.F.Ginzburg, V.V.Serebryakov. Jorn. of Nucl. Phys., 3, 164 (1966).
8. V.G.Vasiov, I.F.Ginzburg. Jorn. of Nucl. Phys., 5, 669 (1967).
9. W.Rarita, R.I.Riddel Jr., C.B.Chin. R.I.N.Phillips. Preprint UCRL -17523, 1967.
10. A.A.Logunov, A.N.Tavkhelidze. Nuovo Cim., 29, 380 (1963).

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