ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 3793

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K - K AND π - K SCATTERING LENGTHS

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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Introduction

The commutation relations of the current algebra /1/ together with the assumption of partial conservation of the axial current (PCAC) /2/ give one of the most powerful methods of calculation for processes involving pseudoscalar mesons. Most of the results are obtained in the approximation of vanishing meson momenta, but a careful application of this method allows one to estimate the on-mass-shell matrix elements from the off-mass-shell ones. For instance, as it was emphasized by Weinberg /3/, the on-mass-shell amplitude of the pion scattering on a much heavier target may be obtained in a good approximation from the off-mass-shell scattering amplitude by retaining only first order terms in the vanishing momenta. The extension of this method to the more difficult case of pion-pion scattering is made by assuming a general expansion for the low-energy scattering amplitude in terms of some unknown parameters and the kinematical variables. This form of the scattering amplitude taken in the limit of two vanishing pion momenta is compared with that obtained by current algebra in the approximation of soft-pions. Adding the Adler self-consistency argument /4/ and a physical assumption about the scalar meson contribution, an expression for the low-energy pion-pion scattering amplitude is obtained.

The purpose of this paper is to give a generalization of the method of Weinberg /3/ to the case of $K-K$ low-energy scattering and to add the scalar meson contribution to the $\pi - K$ low-energy scattering.

K – K Low-Energy Scattering

The element of S matrix is related to the meson scattering amplitude $\langle a(q)\,c(p)\,|M|\,b(k)\,d(p')\rangle$ by

$$\langle a(q)\,c(p)\,|\,S\,|\,b(k)\,d(p')\rangle =$$

$$= \frac{-i(2\pi)^4\,\delta^4(p+q-k-p')}{(2\pi)^6\,(16\,p_0\,p_0'\,q_0 k_0)^{1/2}}\,\langle a(q)\,c(p)\,|M|\,b(k)\,d(p')\rangle, \tag{1}$$

where $a(q)$ denotes a pseudoscalar meson of unitary index a and momentum q.

Following Weinberg /3/ we propose an expansion up to second order in momenta for the $K-K$ low-energy S-wave scattering amplitude in accordance with the general requirements of the crossing symmetry, Bose statistics and conservation of unitary spin

$$\begin{aligned}
\langle a(q)\,c(p)\,|M|\,b(k)\,d(p')\rangle &= d_{ab9}\,d_{cd9}\,(A+B(s+u)+Ct+\ldots)+\\
&+ d_{ad9}\,d_{bc9}\,(A+B(s+t)+Cu+\ldots)+d_{ac9}\,d_{bd9}\,(A+B(u+t)+Cs+\ldots)+\\
&+\sum_{i=1}^{8} d_{abi}\,d_{cdi}\,(A'+B'(s+u)+C't+\ldots)+\sum_{i=1}^{8} d_{adi}\,d_{bci}\,(A'+B'(s+t)+C'u+\ldots)+\\
&+\sum_{i=1}^{8} d_{aci}\,d_{bdi}\,(A'+B'(u+t)+C's+\ldots)+\sum_{i=1}^{27} D_{abi}\,D_{cdi}\,(A''+B''(s+u)+C''t+\ldots)+\\
&+\sum_{i=1}^{27} D_{adi}\,D_{bci}\,(A''+B''(s+t)+C''u+\ldots)+\sum_{i=1}^{27} D_{aci}\,D_{bdi}(A''+B''(u+t)+C''s+\ldots),
\end{aligned} \tag{2}$$

where $s=-(p+q)^2=-(k+p')^2 \quad t=-(k-q)^2=-(p-p')^2 \quad u=-(p'-q)^2=-(p-k)^2$, unitary indices a,b,c,d, run over 4,5,6,7, d_{abl} are the Gell-Mann's completely symmetrical coefficients with $d_{ab9}=\sqrt{\frac{2}{3}}\,\delta_{ab}$, D_{abl} is the Clebsch-Gordan coefficient of the coupling $[8\times 8]\ 27$ and A,B... are unknown constant coefficients.

As Weinberg emphasized such an expansion holds when the momenta k, q, p, p' are of order of the meson mass or less. For S-wave $K-K$ scattering, the Bose statistics requires that the scattering amplitude preserves in the physical region only terms symmetrical in a and c. This is the reason for which only SU(3) symmetrical coupling are considered in eq. (2). On the other hand, considering the identity

$$\int d^4x\, d^4y\, e^{-iqx}\, e^{iky} < f \,|\, T(\partial_\mu A^\mu_a(x)\,\partial_\lambda A^\lambda_b(y))\,|\, i > =$$

$$= \int d^4x\, d^4y\, e^{-iqx}\, e^{iky} \{ -\frac{1}{2}\,\delta(x^0-y^0)\,(< f\,|\,[A^0_a(x),\, \partial_\lambda A^\lambda_b(y)]\,|\,i> +$$

$$+ < f\,|\,[A^0_b(y),\, \partial_\mu A^\mu_a(x)]\,|\,i> - i(q_\mu+k_\mu) < f\,|\,[A^0_a(x),\, A^\mu_b(y)]\,|\,i>) \tag{3}$$

$$+ q_\mu k_\lambda < f\,|\, T(A^\mu_a(x)\, A^\lambda_b(y))\,|\, i> \},$$

where $A^\mu_a(x)$ are the densities of axial currents, and i, f are one-mass states, using the PCAC assumption and letting the momenta k and q tend to zero, we obtain an expression for the scattering amplitude in the first order of the vanishing momenta /3/

$$< a(q)\,c(p)\,|\, M \,|\, b(k)\,d(p') > = M^{(0)}_{ac,\,bd} + \frac{8}{F_K^2}\,(qp)\sum_{l=1}^{8} f_{abl}\, f_{cdl}\,, \tag{4}$$

where f_{abi} are the completely antisymmetrical coefficients of Gell-Mann. The first term on the right hand side of eq. (4) is of zero order in the vanishing momenta and is symmetrical in a and b. It comes from the equal time commutation between an axial charge and an axial divergence and may be related to the contribution of scalar mesons /5/.

From the comparison of eq. (4) with the expression (2) taken in the limit $k = q \to 0$ it follows /3,5/

$$(B-C)(d_{bd9}d_{ac9}-d_{bc9}d_{ad9})+(B'-C')\sum_{i=1}^{8}(d_{bdi}d_{aci}-d_{bci}d_{adi})+$$
$$+(B''-C'')\sum_{i=1}^{27}(D_{bdi}D_{aci}-D_{bci}D_{adi}) = \frac{4}{F_K^2}\sum_{i=1}^{8}f_{abi}f_{cdi} \tag{5}$$

$$M^{(0)}_{ac,bd} = d_{ab9}\,d_{cd9}\,(A+2m^2B)+\sum_{i=1}^{8}d_{abi}d_{cdi}(A'+2m^2B')+$$
$$+\sum_{i=1}^{27}D_{abi}D_{cdi}(A''+2m^2B'')+(d_{bc9}d_{ad9}+d_{ac9}d_{bd9})(A+m^2B+m^2C)+ \tag{6}$$
$$+\sum_{i=1}^{8}(d_{bci}d_{adi}+d_{aci}d_{bdi})(A'+m^2B'+m^2C')+\sum_{i=1}^{27}(D_{bci}D_{adi}+D_{aci}D_{bdi})(A''+m^2B''+m^2C'').$$

Assuming the existence of an unitary scalar nonet it follows /5/ that $M^{(0)}_{ac,bd}$ must be proportional only to d_{abi}, where $i=1,2,..9$ and hence it is appropriate to require

$$\begin{aligned} A &= A' \\ B &= B' \\ C &= C' \\ A''+2m^2B'' &= 0 \\ A+m^2B+m^2C &= 0 \\ A''+m^2B+m^2C'' &= 0. \end{aligned} \tag{7}$$

The Adler self-consistency argument /4,6/ generalized to the case of $K - K$ scattering requires the vanishing of the scattering amplitude at $s = t = u = m^2$, and hence it follows

$$
\begin{aligned}
& A + 2m^2 B + m^2 C = 0 \\
& A'' + 2m^2 B'' + m^2 C'' = 0 .
\end{aligned}
\tag{8}
$$

Using the equations (5), (6), (7) and (8) the expression of the low-energy $K - K$ scattering amplitude is

$$
\begin{aligned}
\langle a(q)\, c(p) \,|\, M \,|\, b(k)\, d(p') \rangle = & \frac{4}{F_K^2} \{ (m^2 - t) \sum_{i=1}^{9} d_{abi}\, d_{cdi} + \\
& + (m^2 - u) \sum_{i=1}^{9} d_{adi}\, d_{bci} + (m^2 - s) \sum_{i=1}^{9} d_{aci}\, d_{bdi} \} .
\end{aligned}
\tag{9}
$$

It is now very easy to evaluate the four distinct $K - K$ and $K - K$ S-wave scattering lengths

$$
\begin{aligned}
& a^{Y=0}_{I=0} = 0 && a^{Y=2}_{I=0} = 0 \\
& a^{Y=0}_{I=1} = 1{,}14\, m_\pi^{-1} && a^{Y=2}_{I=1} = -0{,}28\, m_\pi^{-1} .
\end{aligned}
\tag{10}
$$

The actual experimental data /7/ give for the $a^{Y=0}_{I=1}$ scattering length a positive real value between 2 and 6 fermi, which is of the same order of magnitude with our result.

$\pi - K$ Scattering Lengths

Taking the identity (3), where $a(q)\, b(k)$ are pions and i, f are K mesons in the limit $q = k \to 0$, Weinberg /3/ obtained an expression similar to eq. (4) for the $\pi - K$ scattering amplitude.

Expressing $M^{(0)}_{ac,bd}$ in terms of the scalar meson decay amplitude /5/ the scattering amplitude $\langle a(q) f(p) \,|\, M \,|\, b(k) i(p') \rangle$ takes the form

$$< a(q)\, f(p) \,|\, M \,|\, b(k)\, d(p') > = \sum_j \frac{T(j \to ab)\, T(j \to cd)}{M_j^2} + \frac{8}{F_\pi^2}(pq) \sum_{j=8}^{8} f_{abj}\, f_{cdj} \quad , \tag{11}$$

where $T(j \to ab)$ is the decay amplitude of the scalar meson j and M_j is its mass.

It is easy now to give the values of the $\pi - K$ scattering lengths $a_{1/2}$ and $a_{3/2}$ using eq. (11) and the expressions of the scalar meson decay amplitudes /5/

$$a_{1/2} = 0{,}14\; m_\pi^{-1}$$

$$a_{3/2} = -0{,}12\; m_\pi^{-1} \quad .$$

Conclusions

Comparing the experimental data /7/ and the theoretical predictions /3,6 / with our results, it is easy to see that the signs of the scattering lengths are always the same, but the values differ considerably. For instance, our scattering length $a^{Y=2}_{I=1}$ is about two times larger than that predicted by Cronin /6/ and the scalar meson contribution in $\pi - K$ scattering modifies sensibly the results previously obtained by Weinberg, without taking it into account. On the other hand, the results of Weinberg agree better than ours with those of Cronin /3,6/.

Of course, it is impossible to discuss the various theoretical predictions in the absence of more complete experimental data.

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Received by Publishing Department
on April 8, 1968.