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SCALAR MESON WIDTHS

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1. Introduction

The scalar mesons were introduced in theory some years ago, although their exictence is still doubtful from the experimental point of view. The best established scalar mesons seem to be η_v (1050) and π_v (1008). To form an octet Watanabe[1] added the K_v(1080) meson, but its evidence and quantum numbers are not yet well established[2].

The purpose of this paper is to calculate the widths of the mesons η_v π_v and K_v in the widely used approximation of soft-meson emission.

It is assumed that the field S which represents the scalar meson is given by the following relation suggested in σ model[3] and chiral SU(3) × SU(3) theory[4]

$$\delta(x^0 - y^0)[A_i^0(x), \partial_\mu A_j^\mu(y)] = \sum_{k=1}^{9} 2i\, d_{ijk}\, g_{jk}\, S_k(x)\, \delta^4(x-y) + S.T., \qquad (1)$$

where $i\,j\,k$ are unitary indices running over 1,2...9, d_{ijk} are Gell-Mann's completely symmetrical coefficients, $A_i^\mu(x)$ are the densities of axial currents, g_{jk} is an unknown constant and S.T. means possible Schwinger terms. It is easy to understand the dependence on j of the constant g_{jk} if the Gell-Mann's commutation relations[4] and partial conservation of axial current (PCAC)[3] are simultaneously taken into account.

2. Equations for the Decay Amplitudes

In order to calculate the decay amplitudes of scalar mesons we start with the following identity

$$\int d^4x\, d^4y\, e^{-iqx} e^{iky} \langle f | T(\partial_\mu A^\mu_a(x)\, \partial_\lambda A^\lambda_b(y)) | i \rangle \equiv$$

$$\int d^4x\, d^4y\, e^{-iqx} e^{iky} \langle f | \Big[\delta(x^0 - y^0) \{ -\tfrac{1}{2} [A^0_a(x), \partial_\lambda A^\lambda_b(y)] -$$

$$- \frac{1}{2} [A^0_b(y), \partial_\mu A^\mu_a(x)] + \frac{i}{2}(q_\mu + k_\mu)[A^0_a(x), A^\mu_b(y)] \} + \tag{2}$$

$$+ q_\mu k_\lambda T(A^\mu_a(x) A^\lambda_b(y)) \Big] | i \rangle$$

We take at first $\langle f | = \langle c(p) |$ and $| i \rangle = | d(p') \rangle$, where p and p' are the momenta of some pseudoscalar mesons with unitary indices c and respectively d running over 1,2...8. The reduction formula/ 5/ together with PCAC enable us to express the left hand side of identity (2) in terms of the meson scattering amplitude $\langle a(q)\, c(p) | M | b(k)\, d(p') \rangle$.

$$\int d^4x\, d^4y\, e^{-iqx} e^{iky} \langle c(p) | T(\partial_\mu A^\mu_a(x)\, \partial_\lambda A^\lambda_b(y)) | d(p') \rangle =$$

$$= \frac{i(2\pi)^4 \delta^4(p+q-p'-k) F_a F_b m_a^2 m_b^2}{(q^2+m_a^2)(k^2+m_b^2)(2\pi)^8 (4p_o p_o')^{1/2}} \langle a(q)c(p) | M | b(k) d(p') \rangle, \tag{3}$$

where F_i is defined by

$$(2\pi)^{3/2}(2q_0)^{1/2} \langle 0 | \partial_\mu A^\mu_i(0) | j(q) \rangle = F_i\, m_i^2\, \delta_{ij}\, .$$

Using the definition (1) the first terms on the right hand side of identity (2) may be related to the decay amplitude $T(i \to cd)$ of a scalar meson.

$$\frac{1}{2} \int d^4x\, d^4y\, e^{-iqx} e^{iky} \delta(x^0 - y^0) \langle c(p) | [A^0_a(x), \partial_\lambda A^\lambda_b(y)] | d(p') \rangle = \tag{4}$$

$$= \sum_{i=1}^{9} \frac{(2\pi)^4 \delta^4(p + q - p' - k)}{\left((k-q)^2 - M_i^2\right)(2\pi)^3 (4 p_o p_o')^{1/2}} d_{abi} \, g_{bi} \, T(i \to cd) , \quad (4)$$

where M_i is the mass of the scalar meson. The dependence of $T(i \to cd)$ on the meson momenta is always neglected.

Let now q_μ and k_μ go to zero together. From the relations (2) and (3) it follows the expression for the meson scattering amplitude in first order in q and k /6/

$$\langle a(q) c(p) \,|\, M \,|\, b(k) d(p') \rangle \to$$

$$M^{(0)}_{ac,bd} - \frac{2}{F_a F_b} (q+k)(p+p') \sum_{i=1}^{8} f_{abi} \, f_{cdi} \,, \quad (5)$$

where f_{ijk} is the Gell-Mann's completely antisymmetrical coefficient and $M^{(0)}_{ac,bd}$ is a zero order term due to the contribution of scalar mesons. The first three terms on the right hand side of identity (2) yield the right hand side of eq. (5). The fourth term in eq. (2) is of second order in the vanishing momenta because of the absence of poles near $q = k = 0$ and hence it is omitted from eq. (5). All the considerations are made for S-wave meson-scattering. It was also assumed that the Schwinger terms do not contribute in first order in q and k /6/.

Using the relation (4) $M^{(0)}_{ac,bd}$ may be expressed as

$$M^{(0)}_{ac,bd} = - \sum_{i=1}^{9} \frac{T(i \to cd)}{M_i^2 F_a F_b} (g_{ai} + g_{bi}) d_{abi} \,. \quad (6)$$

In the identity (2) we take now $\langle f | = \langle 0 |$ and $| i \rangle = | S_j(k-q) \rangle$, where S_j is a given scalar meson. After similar considerations it follows from eq. (2)

$$F_a F_b T(j \to ab) = - d_{abj} (g_{aj} + g_{bj}) \quad (7)$$

in zero order of the vanishing momenta q and k.

Now $M^{(0)}_{ac,bd}$ takes the form

$$M^{(0)}_{ac,bd} = \sum_{i=1}^{9} M_i^{-2}\, T(i \to ab)\, T(i \to cd). \qquad (8)$$

In order to obtain the equations for the decay amplitudes of scalar mesons, some considerations should be made about $M^{(0)}_{ac,bd}$.

Following Weinberg, we propose an expansion up to second order in momenta of the off-mass shell scattering amplitude $\langle a(q)c(p)|M|b(k)d(p')\rangle$ in accordance with the general requirements of the crossing symmetry, Bose statistics and conservation of unitary spin

$$\begin{aligned}\langle a(q)c(p)|M|b(k)d(p')\rangle &= d_{ab9}\, d_{cd9}(A + B(s+u) + Ct + \dots) + \\ &+ d_{ad9}\, d_{bc9}(A + B(s+t) + Cu + \dots) + d_{ac9}\, d_{bd9}(A + B(u+t) + Cs + \dots) + \\ &+ \Delta_{ab}\Delta_{cd}(A' + B'(s+u) + C't + \dots) + \Delta_{ad}\Delta_{bc}(A' + B'(s+t) + C'u + \dots) + \qquad (9) \\ &+ \Delta_{ac}\Delta_{bd}(A'' + B''(u+t) + C''s + \dots) + \Delta'_{ab}\Delta'_{cd}(A'' + B''(s+u) + C''t + \dots) + \\ &+ \Delta'_{ad}\Delta_{bc}(A'' + B''(s+t) + C''u + \dots) + \Delta'_{ac}\Delta'_{bd}(A'' + B''(u+t) + C''s + \dots),\end{aligned}$$

where $s = -(p+q)^2 = -(p'+k)^2 \quad t = -(q-k)^2 = -(p'-p)^2 \quad u = -(p-k)^2 = -(p'-q)^2$

The coefficients $A, B, \dots$ are unknown constants,

$$d_{ab9} = \left(\frac{2}{3}\right)^{1/2} \delta_{ab}, \quad \Delta_{ab}\Delta_{cd} = \sum_{i=1}^{8} d_{abi}\, d_{cdi} \text{ and } \Delta'_{ab}\Delta'_{cd} = \sum_{i=1}^{27} D_{abi}\, D_{cdi}$$

where D_{abi} is the Clebsch-Gordan coefficient of the coupling $[8 \otimes 8]$ 27.

In the limit $k = q \to 0$ the relations $s = m^2 - 2pq \quad t = 0, \quad u = m^2 + 2pq$ and the comparison between eqs. (5) and (9) give

$$(B-C)(d_{db9}\, d_{ac9} - d_{bc9}\, d_{ad9}) + (B'-C')(\Delta_{bd}\Delta_{ac} - \Delta_{bc}\Delta_{ad}) + (B''-C'')(\Delta'_{bd}\Delta'_{ac} - \Delta'_{bc}\Delta'_{ad}) =$$

$$= \frac{4}{F_a F_b} \sum_{i=1}^{8} f_{abi}\, f_{cdi} \qquad (10)$$

$$\begin{aligned}M^{(0)}_{ac,bd} &= d_{ab9}\, d_{cd9}(A + 2m^2 B) + (d_{bc9}\, d_{ad9} + d_{ac9}\, d_{bd9})(A + m^2 B + m^2 C) + \Delta_{ab}\Delta_{cd}(A' + 2m^2 B') + \qquad (11) \\ &+ (\Delta_{bc}\Delta_{ad} + \Delta_{ac}\Delta_{bd})(A' + m^2 B' + m^2 C') + \Delta'_{ab}\Delta'_{cd}(A'' + 2m^2 B'') + (\Delta'_{bc}\Delta'_{ad} + \Delta'_{ac}\Delta'_{bd})(A'' + m^2 B'' + \\ &+ m^2 C'').\end{aligned}$$

In order to avoid some ambiguities related to the vanishing momenta, the mesons a,b,c,d must have equal masses.

If we assume the existence of only nine scalar mesons and compare the expressions (6) and (11) we find that $M^{(0)}_{ac,bd}$ must be proportional only to $\Delta_{ab}\Delta_{cd}$ and $d_{ab9}\, d_{cd9}$ so eq. (11) gives

$$\begin{aligned} A + m^2 B + m^2 C &= 0 \\ A' + m^2 B' + m^2 C' &= 0 \\ A'' + m^2 B'' + m^2 C'' &= 0 \qquad (12) \\ A'' + 2m^2 B'' &= 0 \end{aligned}$$

If now we consider that the scalar mesons form an unitary nonet it is appropriate to require that their contribution to $M^{(0)}_{ac,bd}$ must be proportional to $d_{abi}\, d_{cdi}$, where $i = 1, \dots 9$ and thus to put $A = A'$, $B = B'$, $C = C'$. Then from the eqs.(10) and (12) it follows

$$M^{(0)}_{ac,bd} = \frac{4}{F_a F_b}\, m_a^2 \sum_{i=1}^{9} d_{abi}\, d_{cdi} \qquad (13)$$

3. *Results*

In order to evaluate some of the decay amplitudes of scalar mesons we consider eqs. (8) and (13) for three sets of unitary indices: $a = b = c = d = 1$, $a = b = c = d = 4$ and $a = d = 4$, $b = c = 6$.

It follows then

$$\frac{T^2(8\to11)}{M_8^2}+\frac{T^2(9\to11)}{M_9^2}=\frac{4m_\pi^2}{F_\pi^2}(d_{118}^2+d_{119}^2) \qquad (14)$$

$$\frac{T^2(3\to44)}{M_8^2}+\frac{T^2(8\to44)}{M_8^2}+\frac{T^2(9\to44)}{M_9^2}=\frac{4m_k^2}{F_k^2}(d_{448}^2+d_{448}^2+d_{449}^2). \qquad (15)$$

$$\frac{T^2(2\to46)}{M_2^2}=\frac{4m_k^2}{F_k^2}d_{462}^2 . \qquad (16)$$

It is in the spirit of our considerations about the scalar nonet to propose the following solution for the eqs. (14), (15), (16)

$$\frac{T^2(i\to aa)}{M_i^2}=\frac{4m_a^2}{F_a^2}d_{aai}^2 . \qquad (17)$$

On the other hand, the comparison of relation (17) with eq. (7) suggests for the constant g^{ai} the factorization

$$|g^{ai}|=F_a m_a M_i . \qquad (18)$$

From eq. (7) it follows then the general expression of the decay amplitude of a scalar meson

$$M_i^{-2}T^2(i\to ab)=d_{abi}^2(m_aF_a+m_bF_b)^2(F_aF_b)^{-2} . \qquad (19)$$

It is very easy now to obtain from eq. (19) the widths of scalar mesons. We take/1,7/ $F_\pi=\sqrt{2}\,m_\pi$, $F_K=1{,}16\,F_\pi$, $F_\eta=1{,}18\,F_\pi$ and for a better agreement with the present experimental data concerning scalarmesons we neglect the mixing between the eighth and ninth scalar mesons. The ninth scalar meson is supposed to be ϵ (700).

With these assignments from eq. (19) it follows

$$\begin{aligned}&\Gamma(\eta_v\to\pi\pi)=20\text{ MeV}\\&\Gamma(\eta_v\to K\bar{K})=22{,}5\text{ MeV}\\&\Gamma(\epsilon\to\pi\pi)=26\text{ MeV}\\&\Gamma(\pi_v\to K\bar{K})=30{,}6\text{ MeV}\\&\Gamma(\pi_v\to\eta\pi)=54\text{ MeV}\\&\Gamma(K_v\to K\pi)=136\text{ MeV}\end{aligned} \qquad (20)$$

4. Discussions

The experimental data/2/ give about 50 MeV for the width of η_v meson and a branching ratio $\Gamma(\eta_v\to\pi\pi)/\Gamma(\eta_v\to K\bar{K})\leq 2{,}3$. The experimental Γ_{π_v} is 70 ± 15 MeV/2/ and the branching ratio $\Gamma(\pi_v\to\eta\pi)/\Gamma(\pi_v\to K\bar{K})<5$. For ϵ(700) a width of about 50 MeV is indicated.

Our results (20) are in satisfactory agreement with them.

The assignment of the ninth scalar meson is in agreement with the Gell-Mann's considerations/4/ about the mass-splitting between scalar octet and singlet.

Although the scalar meson nonet cannot be uniquely determined because of the unconclusive data the presented results seem to justify the considered assignment.

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